Production and Operation Management

Professor Rajat Agarwal
Department of Management Studies
Indian Institute of Technology, Roorkee

Lecture 41

Statistical Concepts in Quality Control-3 (c-charts and Examples)

Welcome friends, in our last session, we were discussing about statistical quality control. We discussed that different types of quality control charts can be plotted for determining the upper and lower control limits. And we discussed with the help of one example, that, how can we plot a very important quality control charts for the variable. And the name of the chart was X bar R-charts.

So we have to plot these X bar R-charts mostly as a set, one chart gives the variation amount the mean values and the other chart gives you the variation about the range of values. So, both these charts are actually used to determine whether your products whether your processes are in control or not in control.

The other type of charts, which we use are the charts for controlling the attributes. Now for controlling the attributes, we use normally the p-charts and we have already discussed the theoretical part of p-charts in our previous class, where we discussed that in a particular sample, how many defective products are there and based on the percentage or ratio of those defective pieces, you determine a p value and that p value is going to help you in determining upper and lower control limits for the p–chart.

So in this particular session, we are going to discuss about the development of p-chart that how are we going to plot the p-chart and how do we interpret the results of p-chart.

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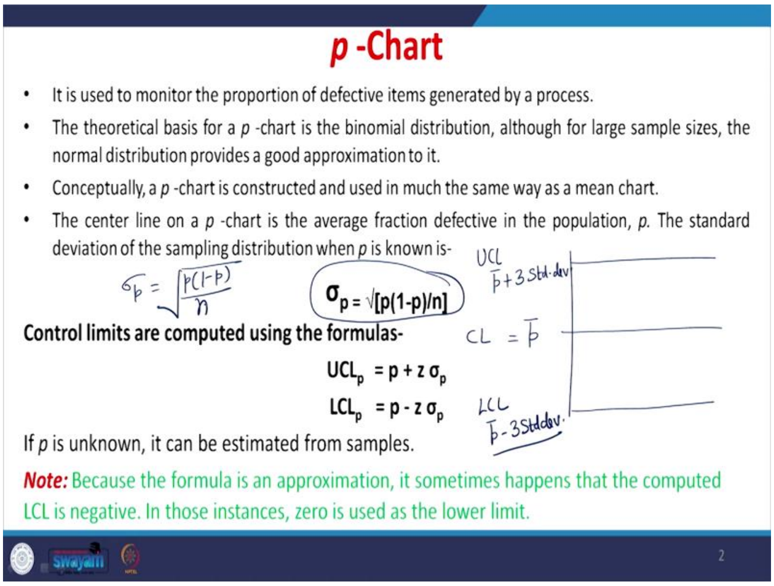

So in our previous session, we have already discussed this particular concept that in case of a p chart, the central line represents the average p value or p bar value, then upper control limit and lower control limit this p bar is the central line value or average value. The upper control limit is generally p bar plus 3 standard deviation and lower control limit is p bar minus 3 standard deviation.

So, you have to determine the values of a standard deviation and the values of a standard deviation can be calculated from this formula that in a particular sample, if I have application of this formula, so this gives you the value of a standard deviation of the p value and the plus minus 3 of standard deviation will give me the upper and lower control limits. Now, how are we going to determine these values? Let us understand this with the help of one example.

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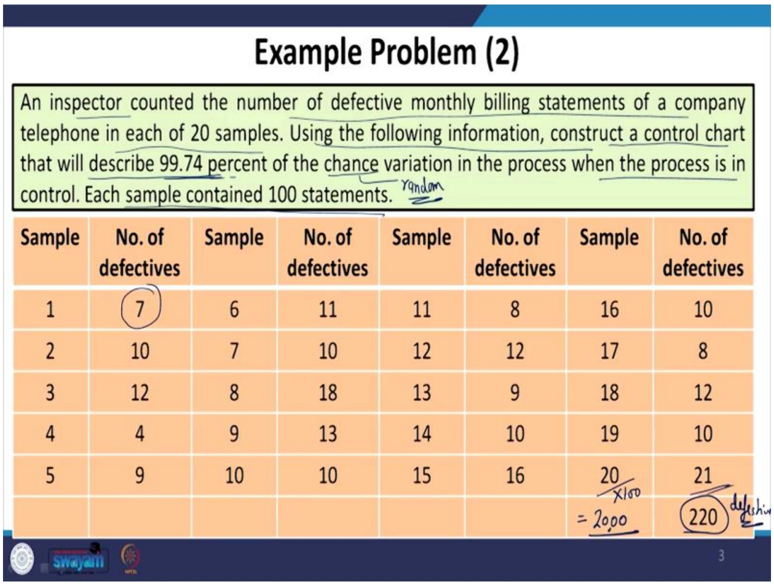

Example Problem (2)

An inspector counted the number of defective monthly billing statements of a company telephone in each of 20 samples. Using the following information, construct a control chart that will describe 99.74 percent of the chance variation in the process when the process is in control. Each sample contained 100 statements. random

Sample	No. of defectives	Sample	No. of defectives	Sample	No. of defectives	Sample	No. of defectives
1	7	6	11	11	8	16	10
2	10	7	10	12	12	17	8
3	12	8	18	13	9	18	12
4	4	9	13	14	10	19	10
5	9	10	10	15	16	20	21
						×100 = 2000	220

Now in this example, the situation is like this, there is an inspector who is counting the number of defective monthly billing statements of a company telephone in each of 20 samples. So that means, every time the sample which this inspector is taking in that 20 items are randomly selected or since it is a question of monthly billing.

So, you can say that this inspector selects 20 bills randomly every time, out of that in the first sample 7 defective pieces are there 7 defective bills are there in the second 10 then 12 then 4 then 9 then 11, 10, 18, 13, 10 and so on up to 20th sample we have total 220 numbers of defective bills. And how many total number of samples you have taken?

Because in each sample you are taking 20 statements and using the following information, we have to construct a control chart that will describe 99.74 percent of chance variation. Chance variation are the random variation, chance variations are the random variations, which are you have no reason for them, we have discussed them and when the process is in control, now each sample contained 100 statements.

So in these 20, you have taken every time 100 statements, so the total number of statements which you have studied are 2000 and out of 2000 statements to 220 are defective, so here we are going to see how we are going to calculate.

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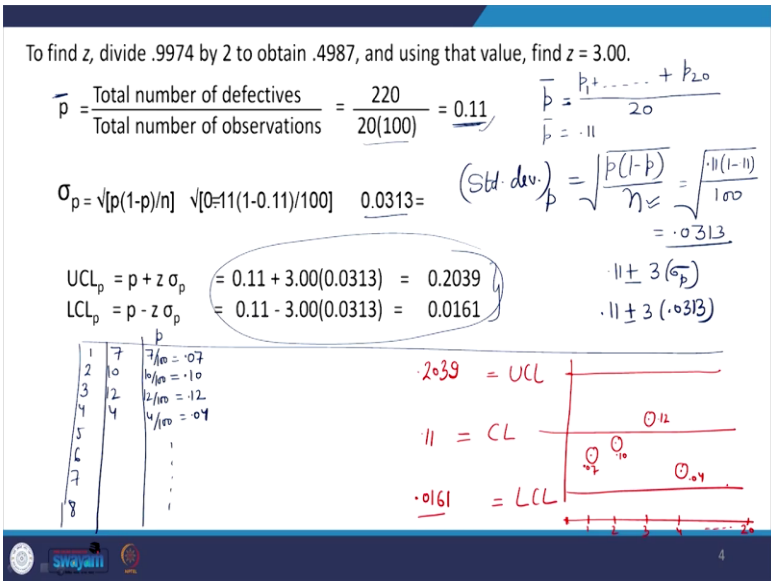

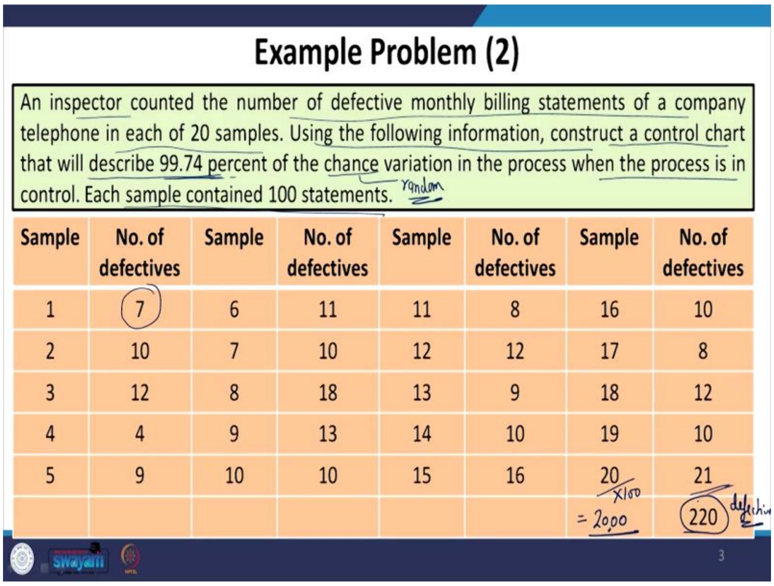

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						= 2000	220

So first is, we need to calculate the p value. Now, it is always better that you make this table and in this table, these are sample number 1, 2, 3, 4, 5, 6, 7, 8 like that. And let us see how many defective pieces are there 7, 10, 12, 4, 7, 10, 12, 4 like that. So, you can calculate p value for each sample and p value for each sample is number of defective pieces divided by number of items taken in that sample.

So you have taken 100 items in a particular sample, so 7 divided by 100 that becomes 0.07 10 divided by 100 that becomes 0.10 12 divided by 100 that becomes 0.12 4 divided by 100, 0.04 and so on. So for each sample, you can calculate the p value, then you can calculate p bar value by taking the average of these p 1, 2, since we have 20 samples total. So from p1 to p20 you can take the average of these 20 samples or you can do it directly also, because the total number of defective pieces are 220 the total number of samples which you have taken in these 20 samples are 2000, so 220 divided by 2000, so 0.11 is the p value. So, that p bar either you calculate from this method or by this method, this is p bar, it is going to be 0.11. I am recommending that we should take the p value for individual samples, because when we are going to plot it on the chart, we will be plotting the P values.

So, in any case, we require the values of p for individual sample then only you can plot these values on the p chart. So, just for initial development of my control limits this calculation is okay. But these individual p values p1 to p20 are also required, also necessary, otherwise it will not be possible to make a p chart. Now, once you have developed the formula for p bar and you have calculated the p bar value. The next is to determine the standard deviation in p, this is a standard deviation of P value and that is sigma p and that is under root of p into 1 minus p divided by n.

Now, it is very very important to remember that many a times we get confused with this n value. Now, you have taken 20 samples, you have taken 20 samples and in each sample 100 units were studied. So, here the n value is 100 that what is the sample size what are the number of units in a particular sample, not the total number of samples. So total number of samples normally are written with capital N, that how many number of samples you have taken, so that we write normally with capital N, it is a small n here so that means, how many number of units you have taken in a particular sample.

So now, this calculation gives me that p is 0.11 and into you can do it this way, 0.11 into 1 minus 0.11 divided by 100. So this calculation is coming 0.0313 this calculation is coming 0.0313 and therefore, you will calculate the values of upper control limit and lower control limit and that upper control limit and lower control limits are calculated that 0.11 plus minus 3 into sigma p and 0.11 into plus minus 3 into 0.0313.

So we have calculated the standard deviation of p values, and this is coming to be 0.0313. And when we are doing this calculation of adding and subtracting of this a standard deviation from the mean value, we get 0.30 0.2039 as the upper control value, and 0.0169 as the lower control value, and then you can plot these things on a p chart. The central line, the upper control line, the lower control line, the center line value is 0.11, the upper control line value is 0.2039 the lower control line value is 0.0161.

Now on this, you will take the number of samples, so first, second, third, fourth and up to 20th sample you will go. The p value since it is a p chart, so you will plot p values on this chart. Therefore, the individual calculation of p values are required. The first p value is 0.07 that will come somewhere here, this is 0.07, the second p value is 0.10, third is 0.12, fourth is 0.04.

So in this way, different p values we will plot same rule as we did in the case of X bar R-chart, same rule will apply here also that if anytime, if you get any p value at the upper control limit beyond the upper control limit or any p value below the lower control limit, then you need to check your process it is not desirable to get p values above or lower than UCL and LCL.

Because if p values are more than UCL, it means your process is producing more number of defective pieces which is undesirable because if more number of defective pieces are there, it means you have a very loose control, your system is generating a lot of scrap, you have to incur a lot of rework and maybe if those defective pieces go into the hands of the customer, this will incur the cost of a guarantee warranty repair or goodwill will also be affected.

So, we have already discussed different types of cost of quality, so that will majorly be affected if you have more number of defective pieces and that has to be checked. If you have defective pieces, which are less than NCL. So, that is a matter of happiness, that is a matter of satisfaction. That now my process has improved. And it is producing actually, if you understand the physical meaning of points which you are getting above or lower, if you are getting points above to UCL, it means that you are producing more defects, if you are getting points below LCL it means you are producing very minimal defects, that is also not desirable.

So you must wonder that why points below LCL are not desirable, it is rather a happy sign, it is a matter of satisfaction that now my system is improving. So the answer is that, if you are consistently getting points below LCL that is a meaning of improvement in your process that now, your process is producing less number of defects, but 1 or 2 random points below LCL are not good, that means you have some casual inspection, it means something is not properly done and therefore, you are not able to check the defective pieces and therefore, most of the pieces are going okay.

So, the random occurrence of points below LCL is not desirable, but yes, if continuously you are getting points close to LCL or below LCL that means, your process have improved and when your processes have improved, now you are producing less number of defective pieces. And in that case, you need to revise your p bar value, the average value and your UCL value and LCL value, you need to have a new set of average UCL and LCL value. Because, if you remember in our TQM class, we discussed that we want to have continuous improvement system.

Now, how that continuous improvement system will be exhibited? That continuous improvement will be exhibited, when my p bar value will continuously be decreased when my LCL will touch to 0 levels that products are no product is defective, no product is defective that is the best thing I am able to achieve. So when my values are continuously showing the sign of improvement, then only I will say that we have in implemented to clear in letter in spirit.

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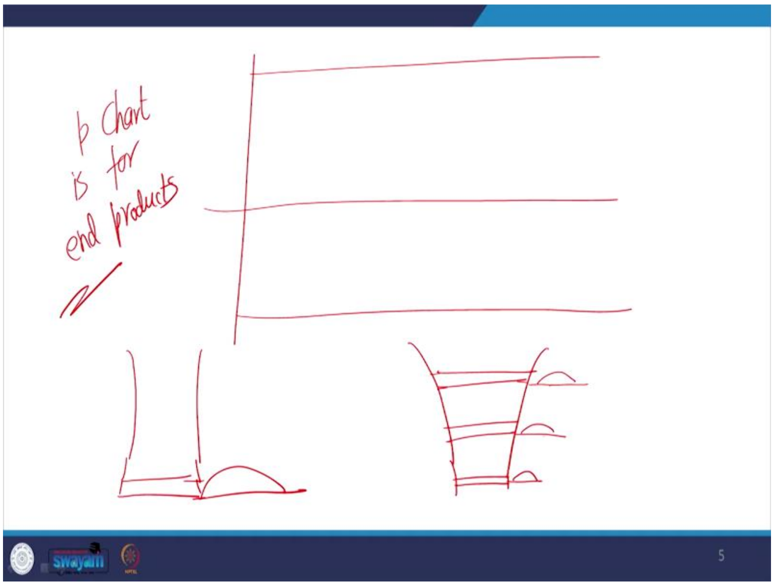

So this type of calculation is done for the p chart and you have in detail this availability of entire p data and since it is for attribute, so you will not have p charts, it is for the attributes, so you will not have multiple features, it is not normally whether it is defective or it is not defective, whether it is accepted whether it is failed. So, these are the type of two characteristics on the basis of which you will differentiate your products and based on that, you will prepare the p chart, like in the case of variable charts, we have many charts, because we may be interested in controlling the process parameters for different types of variables.

So p charts are much simpler as compared to our variable charts, which are large in numbers and, but the benefit of this chart is that when you are giving, finally the products to the customer, or to your next person in the supply chain, you may use these attribute charts, show to show that how many of your products are in acceptable limits.

So that p chart is normally for end products because you are taking samples at the end stage. So this is from one point of view, it is a advantage that it is a very simplified process, but from the other point of view, it is a limitation also that you have already done a lot of value addition.

Yesterday in one of our class, we discussed that type of diagram that now, in our modern quality system, we are having a lot of intermediate checks in the processes and at each stage you take some defects out in the conventional system, we used to have inspection only at the last and you have already added a lot of value in already defective pieces, the defect was generated at stage 1, but you are carrying operations of stage 2, 3, 4, 5, 6, 7 all these stages were in already a defective pieces.

So, that all value addition which you are doing in a defective pieces is going to create a lot of waste for you. So, this is one very important measure of Japanese system of quality management that we continuously need to have superior process capabilities. And we need to have more systems that we do not add value in already a defective pieces. So at intermediate we did stages, if you can filter out your defective pieces, so you will have only those remaining pieces, those remaining units which are okay from that point and only in the okay items, you are doing some next stage value addition.

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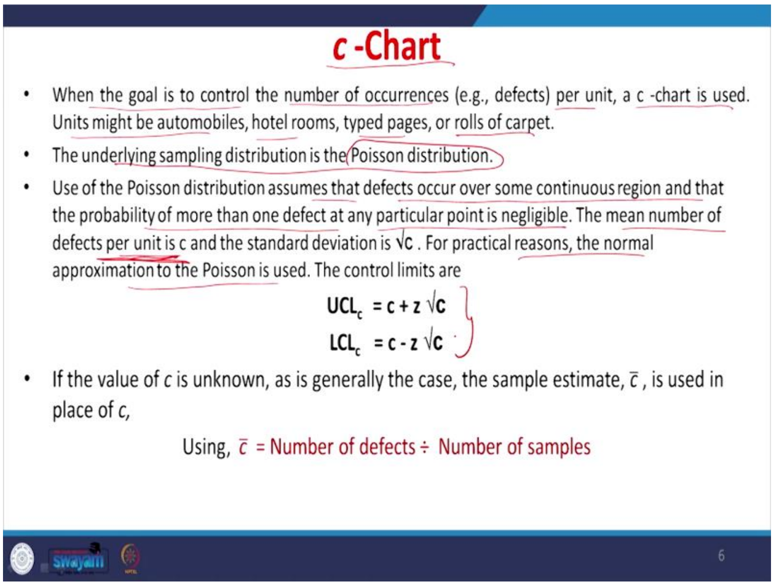

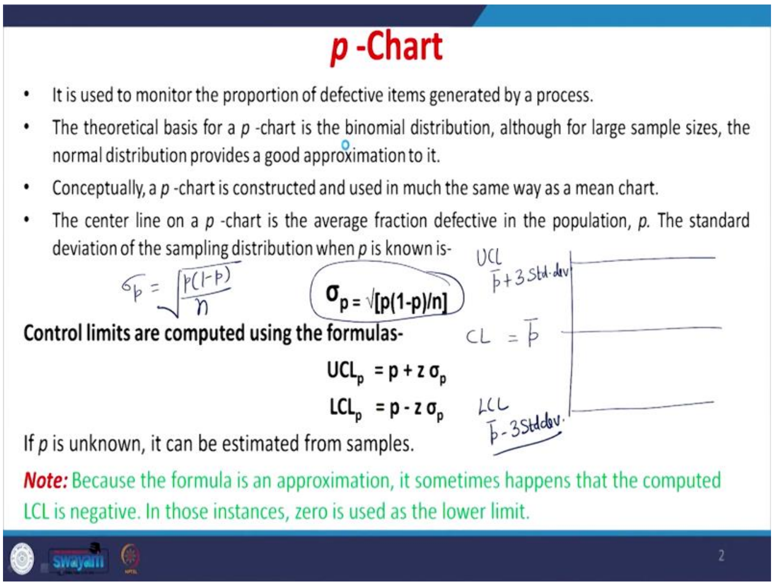

Now, another chart for doing this kind of charting for attributes is c-chart. So p-chart is one type of chart, another very popular chart is c-chart. Now, let us understand about c-chart and what is the difference between p and c-charts. Now, when the goal is to control the number of occurrences per unit c-chart is used and these units may be number of automobiles, hotel rooms are type two pages, rolls of carpet, so, any kind of unit depending upon your requirements, your organization's requirement, you can decide.

Now, the sampling distribution here is the Poisson distribution. Did you remember what type of distribution we followed for a p-chart. Now, just to because I did not discuss at that time, so that we can have a better comparison in case of a p-chart, the distribution is binomial, but for the larger samples like in this example, which is having a sample size of 100 each a normal distribution can also provide a good approximation of this binomial distribution, in c-chart, it is the Poisson distribution, which is the underlying distribution of so, that also you will understand that, how it is going to differentiate between binomial and Poisson distributions.

Now, using the Poisson distribution assumes that defects occurs over some continuous regions, and that the probability of more than one defect at any particular point is negligible the mean number of defects per unit is C. So, defects per mean number of defects per unit is C and the standard deviation is under root c. So for practical reasons, the normal approximation of poison is used and accordingly the control limits are c and plus minus 3 standard deviation of c.

So if the value of c is unknown, normally which is not given, so we will determine these c from the given sample data and number of defects divided by number of samples, that is the formula for determining the c value, now we do a numerical problem to understand the calculation of c-charts.

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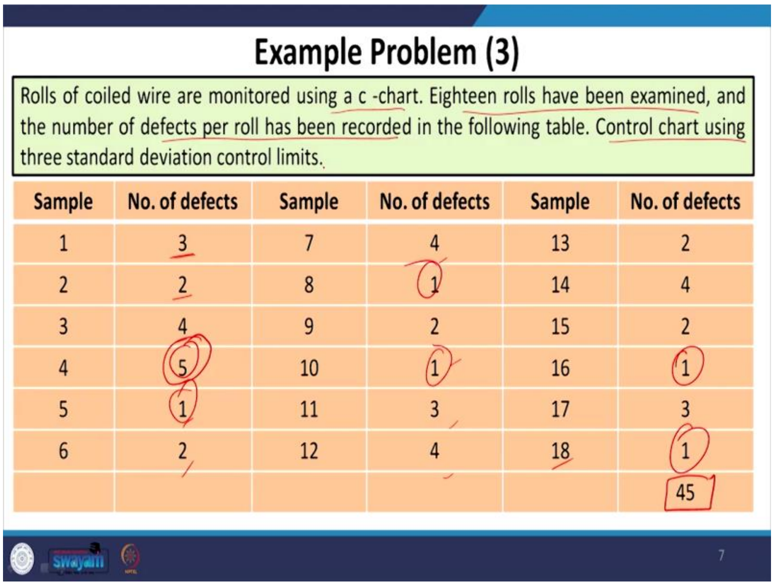

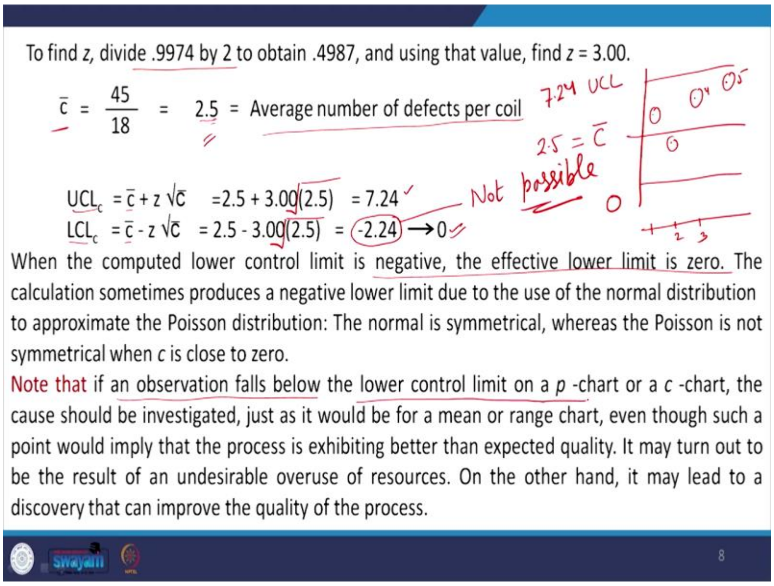

Now rolls of wire are monitored using a c-chart, 18 rolls have been examined, so now, the meaning of 18 rolls have been examined means, you have taken 18 samples and the number of defects per roll has been recorded. So, how many defects are there per roll that we have recorded per sample wise.

So, number of defects if you see are varying from 1 these are 1 and these may go as high as 5 in some cases. So like in sample 4, the number of defects are 5 that is the highest and the total number of defects are 45. Now, control chart using 3 standard deviation control limits need to be prepared, so here we have taken the values of number of defects per unit.

Now, the total number of c are 45 and total number of samples we have taken are 18. So, average value of c is 45 divided by 18. So 45 is the total number of defects, and the number of values are number of samples are 18. So 2.5 defects per sample are there, average number of defects per coin or per sample.

Now, upper control limit and lower control limit will be 3 standard deviation of this value of 2.5, so under root 2.5 will give us the standard deviation and plus minus 3 is considered this. Now, upper control limit is acceptable that is coming 7.24 the lower control limit is coming minus 2.24, now defects, occurrence of defect cannot be in minus.

So, the practical value which we will consider that is 0, because defects can be 1, it can be 2, it can be 3, but it cannot be minus 1 defect, it cannot be minus 2 defects. So this means, this minus 2.24 is not possible, it is impractical, so, you cannot have a negative value of defects, so therefore, this minus 2.24 is considered as 0.

So here, again you will have this type of chart, where c bar values are your average values. This is the value of your average line, this is the upper control limit. Here the average value is 2.5 as the data available to us, the upper control value we will consider as 7.24 and the lower control values, we will consider as 0, here I will like to mention that these are under root signs and then the calculation will come as 7.24 and 0.

Now, you have already the number of defects in each sample, so these are 3, 2, 4, 5, 1, 2 etc number of defects. So you will plot these values like here it is you can have samples 1, 2, 3, for first it is 3, for second it is 2 going below the average value, then for third it is 4, 5, it is 4 then it is 5, so like that.

So, you will plot your various values of c on the c-chart and this again have the same interpretation, if you are crossing the UCL or you are going below LCL that is not desirable, you need to have your points within upper control limit and lower control limit if it is between UCL and LCL that means, your process is under control, you are not producing too many defective pieces.

So, when the we have just given the explanation that why we are considering it as 0, note that now if an observation falls below the lower control limit on a p-chart or c–chart, the costs should be investigated, because as I am already made, mentioned that any point below LCL looks a very satisfying result, but the random occurrence is not good, the continuous availability of points below LCL may be a reason to satisfy, then you need to recalculate your average values, your LCL, UCL may change, it should change, but random occurrence should be avoided.

So in this way, we develop a p-chart or c–chart, both of these charts are for the purpose of attributes. And with this, we come to the end of this session. Thank you very much. In our next sessions, we will discuss more issues related to statistical quality control, which are related to acceptance sampling run chart, how to perform the run test, and the different kind of quality control tools. So with this, thank you very much.