Production and Operation Management
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Lecture 40
Statistical Concepts in Quality Control-2 (p-Chart and Examples)

Welcome friends. In last few sessions, we were discussing about quality management. And in last two sessions you recall, we focused on total quality management and as a very important part of total quality management to make TQM implementable in your organization the concepts of statistical quality management are very important.

The tools and techniques which probability and statistics offer us; these tools will help you in measuring the performance of your processes, these will help you in measuring the performance of output of your processes, and these measurements will help you to keep your processes within the limits. We discussed that variations are natural; you cannot eliminate variation from your processes. But variations can be classified in two categories; one variations which are random, which can happen because of some chance.

So those things which are happening because of some chance you do not have any specific reason of their happening, these are random variations. But there are non-random variations also, which are happening because of some particular issue. Maybe the coolant is not properly working in your machine, and therefore overheating is taking place, and because of overheating some defects are generated.

So, now you know that because of improper functioning of coolant pump coolant is not properly coming and it is resulting into overeating, and that overeating is giving you the defects. So, this is a assignable causes and you can eliminate these assignable causes and then, the variations related to all these assignable causes can also be eliminated.

So, two types of variations are there; assignable variations and random variations. Random variations are acceptable, we should know the limits of random variation for our processes and we try to keep our processes within the limits of natural variation, the random variations. And whenever our processes cross these limits of natural variation, whenever processes enter into those reasons of assignable variations, we need to stop the process, we need to debug, and then after fixing the problem we can again start our process.

So, this is the advantage of statistical quality control that how you can continuously improve. If you are able to minimize the variations, if you are able to minimize the stoppage of your processes because of assignable reasons, that is a very, visible indicator of improvement of your process. So, we have discussed that this type of quality control require making of some important type of QC charts. And we discussed in our previous class, that these QC charts have a very typical kind of arrangement where you have a central line, the upper control line, and the lower control line.

And in this class, we will see with the help of some data, that how we are going to plot these different types of quality control charts. So now let us see, some data and see, how we are going to use this data to plot the quality control charts.

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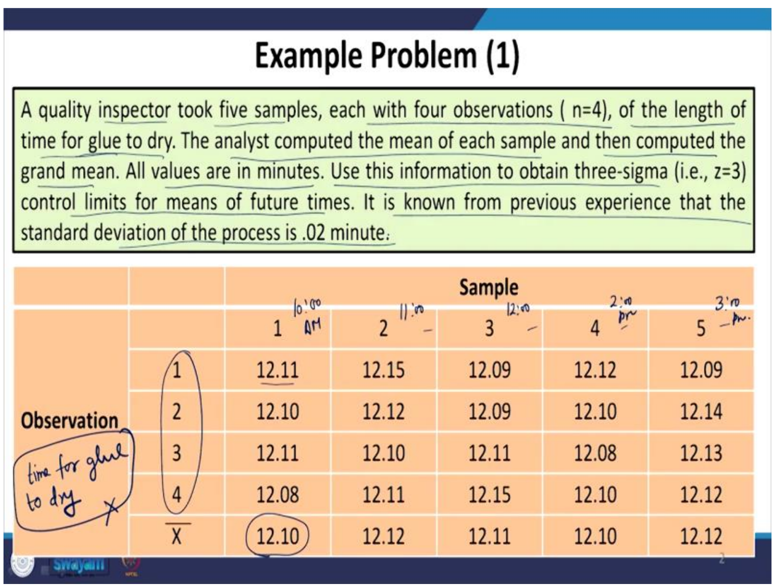

Now here, we have a situation that a quality inspector is taking 5 samples and out of these 5 samples, in each sample 4 observations are made. So, whenever you are taking a sample you are taking 4 units, there may be a lot of let us say, 50 units. Out of 50 units you are randomly picking 4 units in a particular sample. And what you are measuring the length of time for the glue to dry, how much time a glue is taking to dry.

Now that time is measured in minutes, now the analyst computed the mean of each sample, and then computed the grand mean. Now for this simple one; let us say these samples are taken at different intervals. The first sample is taken let us say, at 10 a.m., the second sample is at 11, third at 12, fourth at 2 p.m., and then it is at 3 p.m. So, that is how these 5 samples are taken and each sample we have 4 observations. Now, in the first sample these values are indicating that one particular product is taking 12.11 minutes for drying the glue. The second is taking 12.10, third is taking 12.11, and fourth is taking 12.08.

So, this is the mean of this sample, 12.10 is the mean of first sample. Similarly, 12.12 is the mean of the second sample, 12.11, and 12.10, 12.12 to these are the means of different samples. Now, because we are interested in the time to, time for glue to dry. So, this is my X, this is the variable in which I am interested now. Some of you may be interested that how much glue is being applied, how much glue is being applied, so that may become a second X, and that units may be in grams, or milligrams, etc.

So, you can make as many X charts as possible, that depends that how many variables you want to actually control. So, depending upon the important parameters, important variables, which you want to control, that many number of X bar R-charts are required. For the purpose of understanding how to draw X bar R-chart, we are considering only one particular variable, and that particular variable here is time for glue to dry. And these are the values in minutes and we have taken the average of each of these samples.

Now, further it says, use this information to obtain 3 sigma control limits for means of future times. It is known from previous experience that the standard deviation of the process is 0.02 minutes. So now, based on this information we have to make a X bar R-chart. Now, we have already seen the calculations of, so you can say that this is X1 bar, this is X2 bar, this is X3 bar, this is X4 bar, this is X5 bar. So, the average of X1 bar to X5 bar is X double bar.

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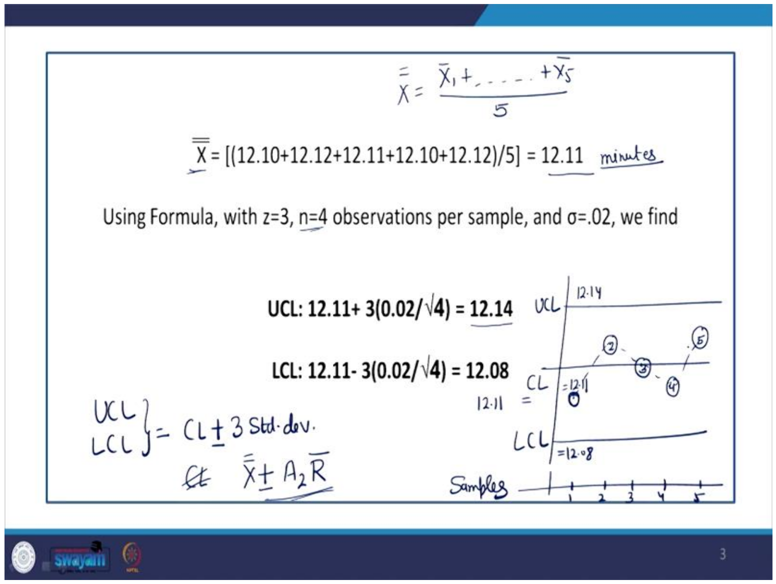

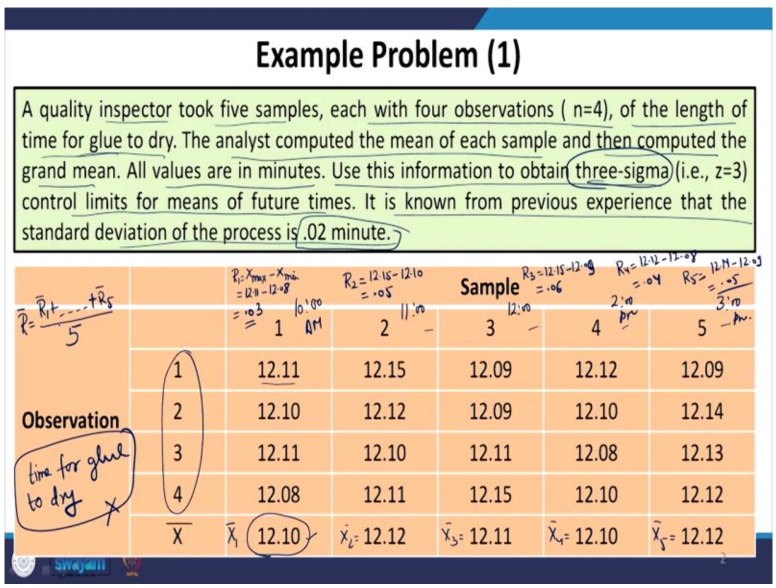

So now we are doing the calculation of X double bar, so X double bar is actually the X1 bar to X5 bar divided by 5, and that calculation is done here, and that is coming 12.11. And all these values, you please remember, are coming in minutes. So, these are the calculations for X bar, X double bar for plotting the central value or central limit, for the purpose of this chart. Now, the important thing is, you have to find 3 important values in the chart.

And in this case, we have CL, UCL, and LCL. CL, we have found that is 12.11, CL is 12.11. UCL and LCL, UCL and LCL these are equal to CL plus minus 3 standard deviation. That is given in this problem that we are going to follow 3 sigma control limits. So, as we have discussed in our previous sessions also, that all the natural variations all random variations or all variations because of chance are considered within the 3 sigma natural limits. So, if though in this problem, it is mentioned, but even if it is not mentioned you will take 3 sigma.

Normally, if you want to have some other limits, if you are going to have more stricter norms, then you can have a plus minus 2 sigma limits or plus minus 2.5 sigma limits. So, that may be a very specific case, depending upon some very crucial items but in general, we take plus minus 3 sigma limits though, it is mentioned but it may not be mentioned also. So, you are going to calculate plus minus 3 standard deviation. Now, in this particular case, the standard deviation of process is given as 0.02 minutes, standard deviation is given as 0.02 minutes.

But otherwise, if you remember we can also get the control limit central values, that is the X bar, here I can write X bar plus minus A2R bar that is also a way to determine 3 standard deviation in case of X bar R charts. Now here, for that purpose I have to calculate the value of R-bar for each of these samples. Now, how do I calculate the R first, like for the first sample, when values are 12.11, 10, 11, 0.08. So, the maximum value R1, let me write R1 here, that will be X max minus X min, X max is 12.11 and X min is 12.08. So, this becomes 0.03 that is the value of R1.

Similarly, R2 will be 12.15 that is X max for this sample, and 12.10 will be X min for this sample. So, R2 will be 0.05, R3 will be again, X max minus X min, X max will be 12.15 minus 12.09 that is 0.06. Then R4 will be, 12.12 minus 12.08 that is 0.04, and this is R5 that is a 12.14 minus 12.09 that is 0.05. And you will take average of all these R values, average of R1 to R5 will give you R bar. So, if I calculate the R bar and the value of A2 I can take, if you remember we discussed a quality handbook data and in that based on the sample size different parameters are given A2, D3, D4, etc.

So, if I take that value of A2 from that table and use this R bar value, I can also get the value of upper control limit and lower control limit, using this formula. But since, directly in this particular problem, the standard deviation is given as point 0.02 minutes. So, I need not to go for this lengthy calculation of determining the upper control limit and lower control limit directly using this formula and the number of observations are given to me as 4 in this.

So, 3 into 0.02 that is the standard deviation for each sample divided by number of observations under root you will have, and this is plus, this is minus, so this becomes 12.14 that is the value here, at this LCL this is the value of CL 12.11 and this is 12.08. So, this becomes the valuation of upper control central limit and lower control. So now, you will plot different values of X bar on this chart. Now the first value of X bar is 12.10. So, let us say this is samples, so first sample, second sample, third sample, fourth sample, and fifth simple.

So, first sample is 12.10, it will come somewhere here, because our central line is at 12.11 so it will be slightly below the central line. Second sample is 12.12 it will be slightly above, this is 2. The third is 12.11 which is going to be at central line, third. Fourth is 12.10, it is here. Fifth is 12.12, it is here. So you see, if I join these 1, 2, 3, 4, 5, all these 5 points are within UCL and LCL.

So, you can conclude that the process is in statistically controlled limits, no point is above to UCL and no point is below LCL. If some point is above UCL, it means more variations are happening, if some point is below LCL that also means more variations are happening. So, both these are undesirable that the production process is not in the control, if it is beyond UCL and LCL, we immediately stop and see why it is happening. So, this is how my X bar R-chart is being plotted.

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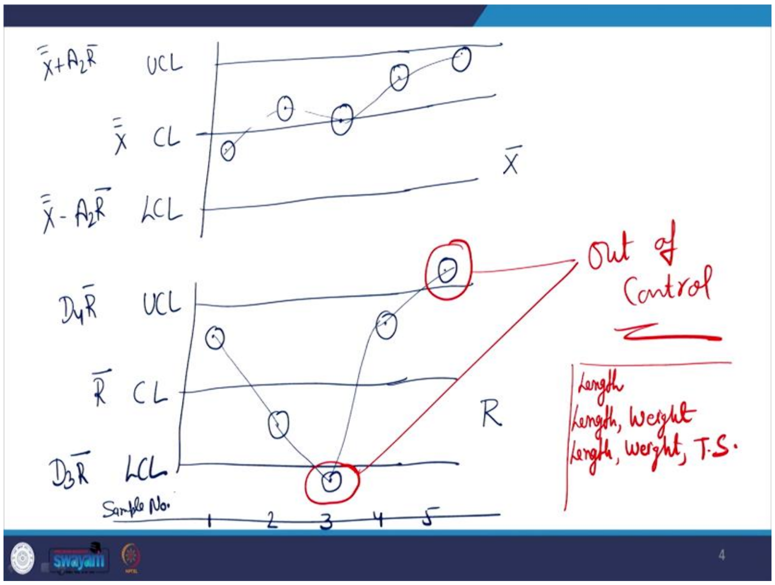

Sometime you need to have two charts together. Like in this case, we had X bar chart, and then you can have R-charts also. So here, X bar chart and R-chart, you calculated central line, upper control line, lower control. Similarly, for R-chart central, upper, and lower. It is X double bar, X double bar plus A2R bar, X double bar minus A2R bar. This is R bar, this is D4R bar, this is D3R bar.

So, you have to plot. In our this example, if I say that, and on the bottom of this you can write the sample numbers, 1, 2, 3, 4, 5 let me slightly extend them, this way. You will have the value of 1 coming here, and value of 1 may come here, value of 2 is coming here, value of 2 may come here. Value of 3 in my X bar chart is on the central line but in this case, it may be here, maybe I am just imaging, I have not done the calculation. Fourth it is here, it is here. Fifth in this case it is here, and fifth in this case it is here.

Now, I need to check both these diagrams simultaneously. And here I see, that these two points are out of control. You have some expected range of deviation, that deviations will have within upper control and lower control limits. But you found that at third sample and at fifth sample, at third sample the range is less than the desirable range. You expect that there can be a deviation of some 0.02 minutes, but actually the deviation came of 0.01 minute, which is looking much better.

But you know that, my process is not capable of producing products of that lesser deviation. So, either it is a poor measurement, or some faulty entry, or some overlooking the inspection process and as a result, this low deviation is there. So, it is mostly because of some human issues because of which you are getting the value below LCL. Sometime you get a point at number 5, which is above to UCL, and this is above to UCL maybe because of some problems in your process that the quality of glue which you are getting it has more water content, and because of more water content it is taking more time to dry.

Or maybe, if you are drying that glue in front of a fan, so fan is speed not appropriate, now fan is not operating up to the optimum speed and because of that reason it is taking more time to dry. And these type of issues, you can resolve by identifying the reasons that why these problems are happening, and therefore you can resolve these issues. But both these points are alarming points, that your overall value of X bar is within the limits, but there are some issues with respect to variability in the process.

So, your process is either producing no variability, that is on theory looks very attractive. But practically, we know that variations are natural you cannot eliminate. So, if you are going below the natural variations it raises the doubt about your measurement process. And if, variations are more than the natural variations, that means some assignable reasons are present and you have to immediately solve those assignable reason. So, most of the time we use X bar R-chart together, so that we can make better decisions with respect to quality control in the processes.

Now, this is one point. The second point which is important to know, that we need to make as many X bar R-charts, as many variables we want to control. In a simple product, just to give you an example, if you are interested in controlling the length of your final product, you require only one set of X bar R-chart. Now with length, you want to also control weight of the product, you require two set of X bar R-chart, one set for length, another set for weight. If you want to control length, weight and tensile strength, then you require three sets of X bar R-charts.

So as many number of variables you want to control, that many number of X bar R-charts are required for controlling the process parameters. After understanding the X bar R-chart, and how do we plot this X bar R-charts, now we need, we move to another category of charts which are control charts for attributes. Control chart for attributes are very different than the control charts for variable.

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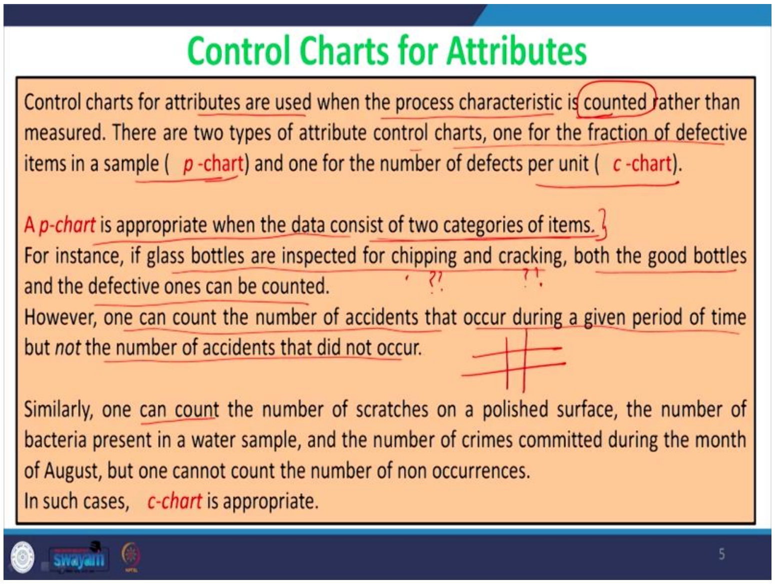

Now, first we need to understand that what is an attribute, what is an attribute, so attributes are used when these are the kind of you can say, process characteristics and the process characteristics which can be counted rather, measuring. For an example, in case of length of a pen. You are going to measure the length of the pen, whether it is 5 inch, 5 inch 5, 5.5 inch, or 5.9 inch, or 6 inch. So, you are measuring it and when you are measuring a particular characteristic it becomes variable.

But if I say, the length is acceptable or not acceptable same thing, but if I say that, the length is acceptable or not acceptable. So, I will give answer that out of these many pens, which I have produced the length of these pens are acceptable and these pens lengths are non-acceptable. So now it becomes an attribute. So, attributes are those things, those process characteristics which are counted, not measured. So, if you are measuring something then it is a variable, and if you are counting it, then it is a attribute.

So how many products in your finished product are acceptable. So, you are having a lot of 20 products and out of a lot of 20 products, you are counting that 4 products are defective, and 16 products are acceptable. So, this type of count is actually giving birth to the control chart for attributes. Now, in the control charts for the attributes the most common type of chart is the p-chart and the another chart is c-chart. So first we will see, what is a p-chart.

Now a p-chart is appropriate when the data consists of two categories of items; accepted, not accepted. Yes, no, these are the two categories. So, when you are data is divided in these two types of categories, a p-chart is more suitable. For instance, if glass bottles are inspected for chipping and cracking, both the good bottles and the defective ones can be counted. So, you have as an end product some glass bottles available with you, and you are seeing only these things in that glass bottle that whether it is chipped, some chip has come out of that glass bottle or is there any crack in the glass bottle.

So, you are checking these bottles for chipping and cracking and you can determine easily by counting that in your sample how many bottles are okay, and how many bottles have either chipping or cracking. So, you can put both chipping and cracking in the same basket. So now, one can count the number of, similarly the accidents that occur during a given time period of time, but not the number of accidents that did not occur. So, there are situations that where you can count both the situations.

So, in case of this bottle example, you can determine that how many bottles are okay, and how many bottles are not okay. But on a intersection, where vehicles are continuously going in different direction, you can only count that how many number of accidents have happened here, but you do not know how many accidents have not taken place, you cannot count.

So, only record of, similarly in a city how many thefts have taken, so you have the record of how many thefts have taken place, but you cannot determine by any method that how many thefts did not occur, because that is never counted. No, no mechanism is available that these thefts have not occurred. So, in some cases you can know, in both the categories, yes, no, how many defective, how many non-defective, but in some cases, you cannot have data for both the category. So, we have some examples.

Similarly, so in these cases the, where we have data for both the categories, we can determine the defective number of pieces or defective number of occurrences, and based on that ratio of defective numbers we can plot the chart which is known as p-chart. This is about c-chart, so we are leaving it at the moment, we will discuss or we will commit, come to it later on. Now, when we are plotting a p-chart, the idea is very much similar what we discuss for X bar and R-chart.

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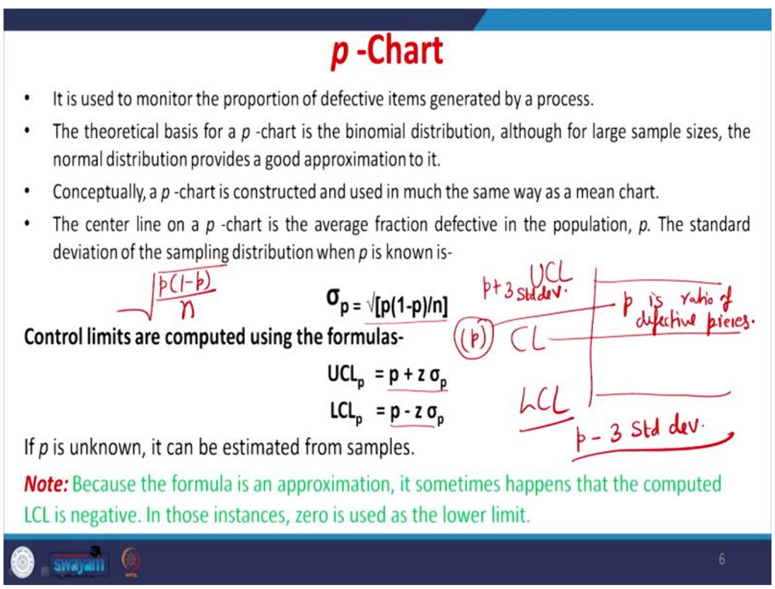

Again, we have a central line and we have upper control and lower control limit, UCL and LCL. Now, in a p-chart we will determine the values of p, that is the central value and p plus 3 standard deviation is the upper control, and p minus 3 standard deviation that is the lower control. And p is the value of, p is ratio of defective pieces. So here, the formula of upper control and lower control is p plus minus z into the standard deviation in p value. And this z is normally 3 sigma, because we want to have all these divisions within the natural range.

So, by default the value of this calculation of upper and lower control limits is plus minus 3 standard deviation. So, this is the formula for calculation of this standard deviation that, p into this is like this, if it is more clear if I write in this way divided by n, so this is the formula for calculation of the value of standard deviation in case of p-chart. And you will multiply this by 3, to get upper and lower control limit.

And we will do 1 or 2 numericals based on this formula, that how to plot a p-chart in our next class. With this, we come to end of this session. Thank you very much.