Production and Operation Management
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Lecture 39
Statistical Concepts in Quality Control-1 (Overview of Controls Charts)

Welcome friends. In our last session, we discussed in detail about the concepts of total quality management. The philosophy of total quality management is based on 3, 4 important pillars, that it involves all the functional area and it involves people at all the levels, from top level to bottom level.

Then it is a continuous process, it is a never-ending process. And you are continuously looking to exceed the customer expectations and also, you try to empower your employees to achieve all these 3 things. Now, we also discussed that, for implementation of total quality management we need to have good knowledge of tools and techniques. Now what are these tools and techniques, which are required for implementing the total quality management.

It is very important to understand that there are certain natural phenomenon, which are applicable everywhere. And one such natural phenomena is that under all circumstances where everything is identical, your all conditions are identical, but it will be very difficult, it is impossible to produce 2 products, which are completely alike. So, you have all similar kind of input, similar person is working, same machine, same raw material, everything is same, all ambient conditions are also same but 2 products, 2 continuous products will not be completely alike.

In this world also, we see we have a population of around 6 billion, but it is a rarest of rare incidence that 2 persons are of similar type. There are always variations between 1 to 2, 2 to 3, 3 to 4, and so on. So, variations are natural phenomena. So now, to summarize my these statements, let us understand that variations are natural phenomena. You cannot eliminate variations from products, from processes. And therefore, this becomes one of the important criteria for using some tools in implementing the total quality management.

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And this is known as the concepts which are based on this issues related to variation, known as Statistical Concepts in Quality Control. How we have implemented the laws of probability in managing the quality, that is statistical concepts in quality control. So, in this particular session, we are going to discuss, what statistical concepts are used in quality control. So, let us start our discussion.

Now, the discussion of quality control or to use statistical concepts in our quality management, this is very seriously advocated by Deming. Deming was the person, in our one of the session we discussed some modern quality gurus and one very important name was Deming in that list. And Deming developed various concepts related to statistical quality control. And because of his tremendous contribution in the field of statistical quality control, and these principles were applied in the Japanese industries first.

So, the development of Japanese automobile industry, the Japanese electronic industry is because of their ability to implement SQC and to honour the or to recognize the contribution of Deming in the Japan, the Japan government started Deming prize, which is given annually for those organizations who have successfully demonstrated the application of total quality management. But it is basically, SQC which is driving TQM in the application form of Deming prize.

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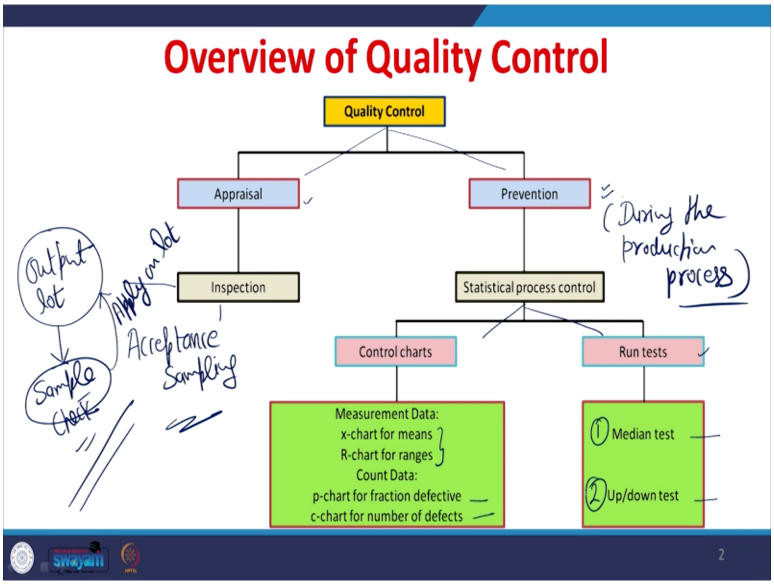

Now, if I see the diagram of quality control, now there are 2 broad categories of quality control; one is the appraisal, appraisal means the inspection of the final product. So, you are going to check the quality of your finished product and in that particular case, your concepts of acceptance sampling because it is not possible nowadays, that you are going to inspect your 100 percent output, because number of outputs are increasing, because of increased productivity, so it is almost impossible that you go for 100 percent inspection.

So, you take a random sample, you take a random sample of your production lot and you can decide the frequency of taking the sample, and based on results which you have obtained for the sample you will generalize those results for the entire lot. And that is one type of quality control which is appraisal and inspection. But because we are following the laws of probability, we will not do 100 percent inspection, we will do inspection of a smaller sample, a very small sample as compared to population.

So, the population means, the 100 percent output which you have produced in a particular lot but we take a smaller number of units which is known as sample. So, in that case you have the entire output and from this output, which is the lot, you take a sample, and then you are checking the quality, you are doing the inspection of this sample. And based on principles of probability, you apply your decision on this, you check it and apply on lot. So, this is the appraisal part of quality control.

But this appraisal can happen only when you have finished products with you, this happens at the end of the production process. But you can also do quality control during the production process, and this is known as prevention approach. This is appraisal, this is prevention. Now in the prevention, you are doing quality control during the production process, or you can say during the production process. And here, statistical process control is going to help us in prevention approach of quality control.

Now in this statistical process control, we have 2 very important methods. One is use of control charts or someone ones can say, quality control charts, some people can say that statistical quality control charts, so there are different names but control chart is a smaller words. And there are different types of control charts like, x-bar, R- charts, these are one type of quality control chart which are used for various type of variables, which you want to control. Then for attributes, we have a different type of control charts, which are either p-charts or c-charts.

So, in our discussions, we will discuss in detail about p, c, x bar, R-charts. But the concept of these charts is that, you want to keep the production process within some limits, so these charts help you in determining whether the production process is within those limits or it is crossing the limits. So, if it is crossing the limit, then you have to be careful, you have to stop the process look for the reason, and after correcting those problems you again start your production process. So, that is the role of control charts.

Similar to control charts, we have another type of statistical process control mechanism, these are known as run test. There are 2 very popular type of run test; one is median test and the second is up and down test. In median test, we have some median value of the variable which is under observation. And in your observation, you see that how many times that particular observation is above to median value, and how many times it is below to median values, and based on that you make the different types of run.

And then you see, that those runs should be in random order, if those runs are not in random order, if these runs are showing some kind of biasness it means, your process can go out of control. Same thing is up and down test. Here, either values continuously increasing or values continuously decreasing. So, one continuous increase of values is known as one run of upside, the continuous decrease of values is known as one run of downsize.

So, we should have the random runs. So, if random runs are there, it means sometime values are increasing, some time values are decreasing. So, that will help us in saying that the process is under control. But if values are either continuously increasing or continuously decreasing that again we will have a system that your process is going out of control, your process is going to produce products which may be out of the specification limits. So, either run tests you use or control charts you use.

Many a times we use all these things simultaneously, because they have some inherent advantages and some inherent limitations. So, to take a better decision it is always useful that you use all these tools available to you simultaneously. Earlier it was slightly difficult to use multiple tools simultaneously, because we used to do these things with the help of graph papers, paper, and pencil, scale etc.

But nowadays, computer screens are available you can have a monitor installed at your shop floor and on that monitor, you can divide screen for different types of control charts, run test results and you can simultaneously see, that all parameters are in right direction or not. Like, for monitoring the health of a person, you measure the temperature, you measure the oxygen level, you measure the blood pressure, you measure the sugar level, so all these vitals should be within the control limits, then you say that person is healthy.

Same thing is in this case of process control, that we measure all vitals of the process and when all vitals of the process are in control, we say that process is under statistical limits. Otherwise, we say that there are some problems or there may happen some problems and we need to be proactive in resolving those problems with the team efforts. So, that is about the overview of our quality control, particularly statistical quality control.

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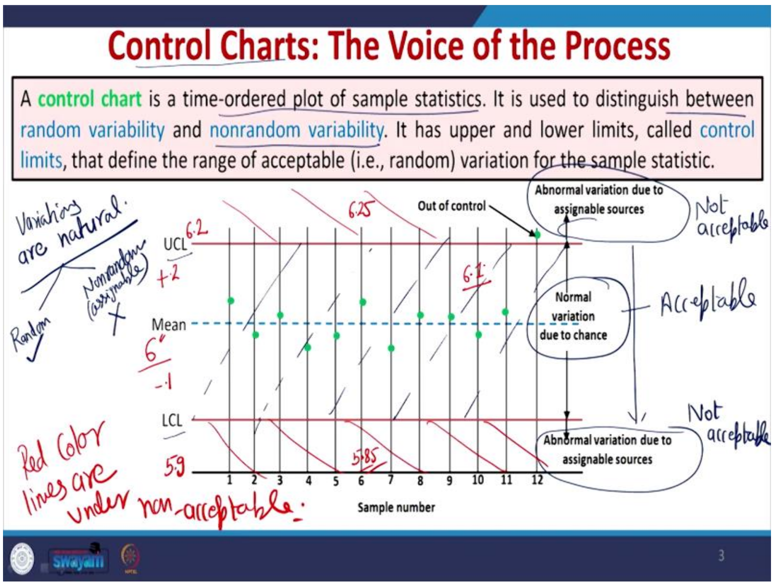

Now about control charts, which we just discussed, that is going to be the essence of our statistical quality control in this particular class. Now, the general shape of a control chart is of this type, where you will prepare a control chart for some sample statistics. Now, there are 2 types of variations, as I said variations are natural, you cannot eliminate variations. But variations can be of 2 types; these variations can be random variations or non-random variations.

Now random variations are acceptable, random variations are acceptable. Variations, which are happening because of some chance and you cannot assign any reason for those variations, so these variations are okay, you are going to accept these variations. But non-random variations are those variations which are because of some particular reason, for which you can assign some cause. So, these are assignable variations, and assignable variations are non-acceptable. We are not going to accept those variations which are non-random, which are assignable there is a cause behind these variations.

So, those type of variations are not acceptable. So, the non-random variability is not acceptable. And with the help of these quality control charts, we expect that if a product is under this limit of UCL and LCL; UCL stands for Upper Control Limit, LCL stands for Lower Control Limit. So, if a particular variable, which is under our observation, if the values of that particular variable are within the limits of UCL and LCL, so that particular variation is attributed that it is because of random variation.

And when it is crossing UCL or LCL, then we say that this additional variation is because of non-random variation, this is because of assignable reasons that is to be avoided. You cannot avoid random variation, but you can certainly avoid non random variability. And that non-random variability is in this particular reason, this is a non-random variability, that is non-random variability.

And whatever variability is there between UCL and LCL, you see this is the reason, which is between UCL and LCL. So, this is normal variation due to chance, this is the variation because of randomness. So, you are okay, acceptable, this is not acceptable, this is again not acceptable. So, the area between UCL and LCL, this area is our acceptable area. And the area which is a like this, this is, which is red colour area.

So, these red colour lines are representing that area which is non acceptable. So, we try with all our efforts to keep our process within the UCL and LCL, as long as process is producing products, which are within UCL and LCL, we say that our process is under statistical control. When process is producing like, for an example just, just to understand the meaning of this, I am having this pen, now the height of this pen should be 6 inch, the height of this pen should be 6 inch.

Now the variations are possible of 0.2 inch, and minus 1 inch. So, you can produce up to 6.2 inch and 5.9 inch. Now if anytime the height of a particular pen, which is randomly selected from a particular output, comes to be 6.25 it means this is the additional variation of 0.05 is because of some kind of non-random variation. If the some other sample it comes to be 5.85, now it is also because of randomness, non-randomness.

But if, in some sample the height of the pen is coming 6.1 it is, this is acceptable because this due to randomness. So, this is how, how many parameters you are going to control depending upon that you need to make that many number of control charts. So, this is in generic terms, we understood that, what is a control chart, how it look likes, and how to interpret the results of control charts.

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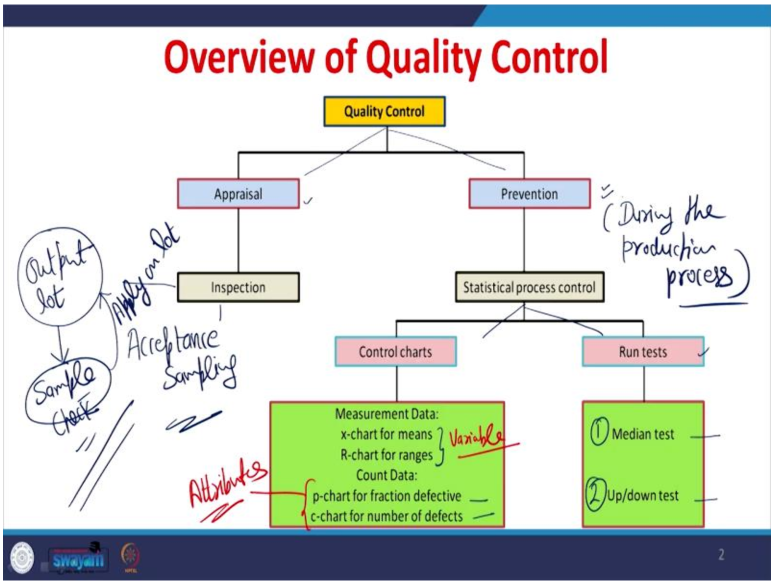

Now, control charts can be made for 2 types of things; control charts can be made for variables, and control charts can be made for attributes. As I told you, in this particular case that, x bar, R-charts are the control charts for variables, and p, and c, these are for attributes.

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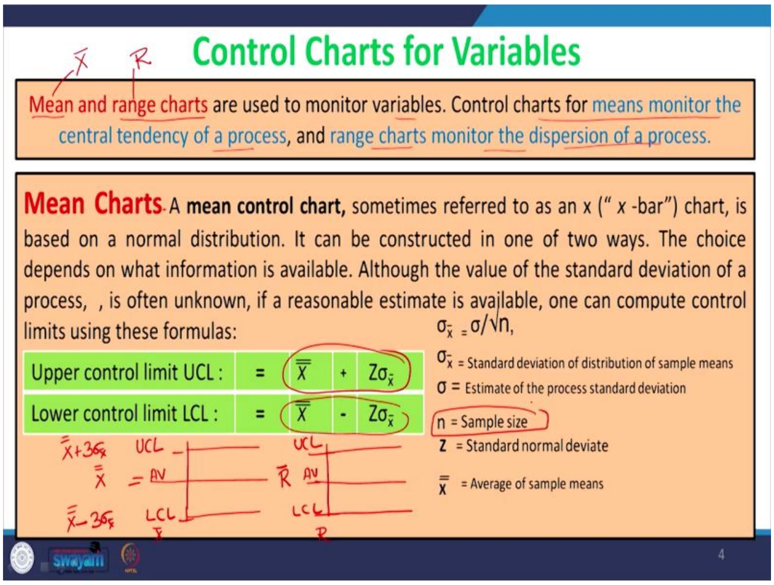

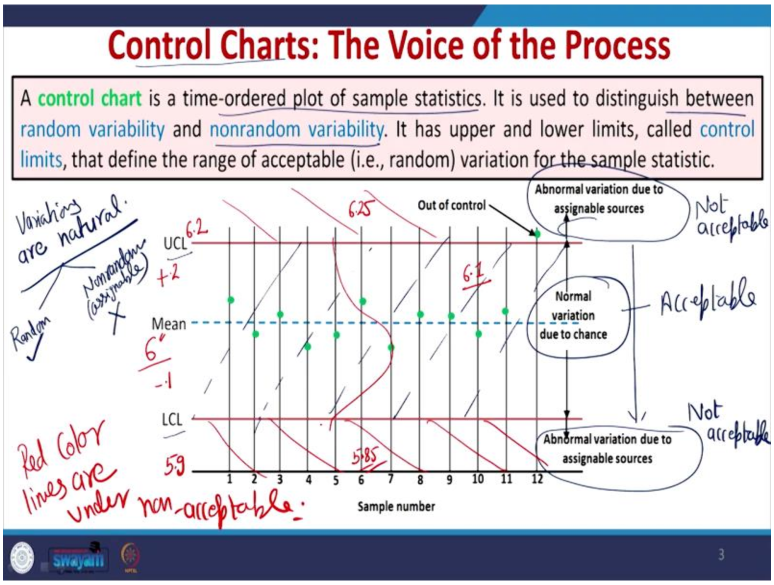

So here, let us see, what is a control chart for variable, and what is a control chart for attribute. So, we are starting first for a control chart with a variable. Now here, we need to make 2 charts together, for knowing whether the particular variable is in limit or not in limit. And these 2 charts are known as, 1 chart for the mean value, and another chart for the range value. The chart which is used for the mean value is known as, x-bar chart and chart which is used for the range purpose that is known as, R-chart.

So, x-bar and R-charts are always used in a pair. So, you make x-bar, R-charts together to understand the meaning of these 2 charts collectively. So, these are used to monitor variables. Control charts for means, monitor the central tendency of a process, and range chart monitors the dispersion of the process. So, whether you are able to produce as close to the mean value of specification as possible, this is checked by the mean chart. And how big is the dispersion, how big is the deviation in products specification, in a particular lot that deviation or that dispersion is measured or is controlled with the help of range chart.

So here, we see that how to prepare the x-bar and R-charts. So, upper control limit and lower control limit that is like this, you have 2 charts; this is your x-bar chart and this is your R-chart. Now, whenever we are making a chart, so first is the average value, the mean value is first decided. Then you have, for any kind of chart these 2 lines, one line above to mean value or the central value, and another is lower to central value.

The line which is above to central value or average value is known as upper control limit, a line which is below the average value is lower control limit. So, these are the upper control average value and lower control limits for 2 charts. Now, what are the values of these UCL, AV, LCL, for these 2 charts. Now for the x-bar chart, the value of average line is X double bar. Whatever is the mean value of a particular sample, that is X bar and the global mean of our samples that is X double bar, so that is the mean value.

Then it is assumed that these variations upper control limit and 6, and lower control limit are as per the normal distribution. So, in the normal distribution we take this aspect that plus minus 3 sigma, plus minus 3 sigma is the limit of natural variation. So here, X double bar plus 3 sigma, that is the upper control limit and X double bar minus 3 sigma, so you can say that X double bar plus 3 sigma of X bar and X double bar minus 3 sigma X, that is the LCL.

Similarly, in the range chart the average value is the R-bar, that is the average range of all the samples. And plus 3 sigma and minus 3 sigma of these average value will make your UCL and LCL. But for the sake of simplification, we have quality control handbooks, and in those quality control handbooks, we have some ready-made calculations available for plus 3 sigma and minus 3 sigma for these different samples and that calculation is based on the sample size. What is your sample size, using that you can directly determine the values of plus 3 sigma and minus 3 sigma in both these x-bar and R-charts.

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Table of Control Chart Constants

Sample Size = n	A_2	A_3	d_2	D_3	D_4	B_3	B_4
2	1.880	2.659	1.128	0	3.267	0	3.267
3	1.023	1.954	1.693	0	2.574	0	2.568
4	0.729	1.628	2.059	0	2.282	0	2.266
5	0.577	1.427	2.326	0	2.114	0	2.089
6	0.483	1.287	2.534	0	2.004	0.030	1.970
7	0.419	1.182	2.704	0.076	1.924	0.118	1.882
8	0.373	1.099	2.847	0.136	1.864	0.185	1.815
9	0.337	1.032	2.970	0.184	1.816	0.239	1.761
10	0.308	0.975	3.078	0.223	1.777	0.284	1.716
11	0.285	0.927	3.173	0.256	1.744	0.321	1.679
12	0.266	0.886	3.258	0.283	1.717	0.354	1.646
13	0.249	0.850	3.336	0.307	1.693	0.382	1.618
14	0.235	0.817	3.407	0.328	1.672	0.406	1.594
15	0.223	0.789	3.472	0.347	1.653	0.428	1.572
16	0.212	0.763	3.532	0.363	1.637	0.448	1.552
17	0.203	0.739	3.588	0.378	1.622	0.466	1.534
18	0.194	0.718	3.640	0.391	1.608	0.482	1.518
19	0.187	0.698	3.689	0.403	1.597	0.497	1.503
20	0.180	0.680	3.735	0.415	1.585	0.510	1.490
21	0.173	0.663	3.778	0.425	1.575	0.523	1.477
22	0.167	0.647	3.819	0.434	1.566	0.534	1.466
23	0.162	0.633	3.858	0.443	1.557	0.545	1.455
24	0.157	0.619	3.895	0.451	1.548	0.555	1.445
25	0.153	0.606	3.931	0.459	1.541	0.565	1.435

So, let us see that what are these plus 3 sigma and minus 3 sigma values. For that purpose based on these sample size, based on this sample size we have only a limited part of this entire table for a sample size of 2 to 25. But you can have a bigger table in any quality control handbook, where sample size may be up to 100 also, and you can find the values of various parameters.

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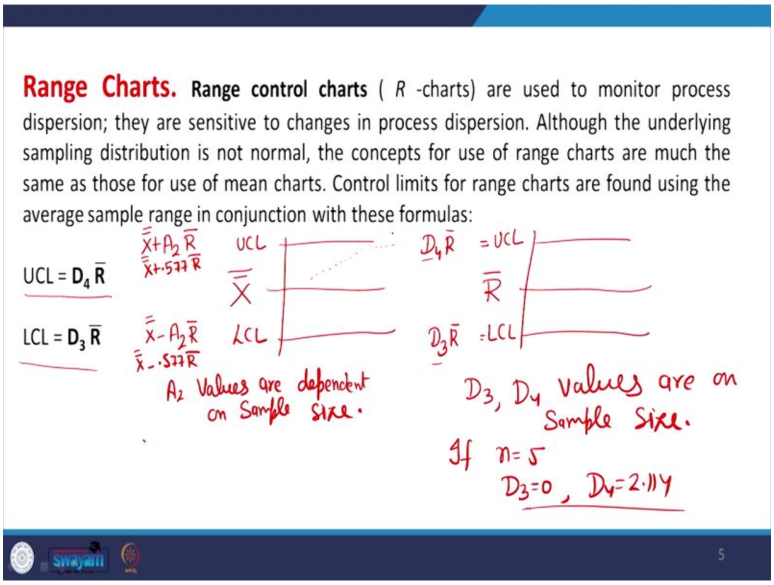

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So now, if I am talking of my range chart, so I talked of x-bar chart and now, I am talking of range chart, where this is the R-bar in the range chart, and the upper control limit UCL will be D4R bar, and the LCL, Lower Control Limit is D3R bar. Now these values of D3 and D4, values of D3 and D4 are dependent on sample size.

And this table gives you the idea that, if my sample size is 5, if sample size is 5 for an example, so you see the value of D3 is 0 and D4 is 2.114. So, in that case, if n is equal to 5 D3 is 0, D4 is 2.114. So accordingly, you will get the numerical values of upper control limit and lower control limit. Now coming back to our range chart, so in, the mean chart where the average value was X double bar.

So, the upper control limit and lower control limit in terms of, now with this table you can simplify, very simply calculate that upper control limit will be X double bar plus A2R bar. And the lower control limit will be X double bar minus A2R bar, and A2 values are dependent on sample size. So like, we have seen the value of D3, D4 for sample size 5, so similarly for A2 the value of, is 0.577 for the sample size of 5.

So, you can simply put that, UCL will be X double bar plus 0.577 R-bar and this is X double bar minus 577 R-bar. So, with this you automatically capture 3 sigma effect of deviation in the value of your, this particular variable and you have to plot the values of your variable on both these charts for all the samples.

Whenever any particular point is crossing UCL or LCL, it means some assignable reasons are there, some assignable variations are there, you have to stop your process and look for the reason. Otherwise, as long as you are within LCL and UCL, and another important thing you also need to apply run test on these values, which will show you that there is no trend available in the data. Sometime, it is quite possible that you will continuously get all these values within UCL and LCL.

But you may have a kind of trend in those values, so we do not want that trend also, because these variations are random in nature. So, some of the points should be above the mean line and some of the points should be below the mean line. So, there has to be a continuous random curve for the variables, if that curve is showing any kind of trend that is also not acceptable, which is more evident when we apply the run test.

So, in our next class, we will see the application of these principles of x-bar, R chart by actually plotting a x-bar, R-chart and doing the interpretation of that particular chart. Thank you very much.