Production and Operation Management
Professor Rajat Agrawal
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 34
Production Planning Problems using LP

Welcome friends. In last few sessions, we are discussing about production planning, we discussed that how we need to merge the forecast for individual items into aggregate forecast. And then based on the aggregate forecast, we take lot of intermediate decisions and these decisions may be for 4 months period, 6 months periods, 8 months periods and so on.

And most of these decisions are with respect to how to manage your inventory, how to minimize the inventory of your production and how to manage the time availability of various resources. And time availability particularly with respect to machines and with respect to manpower. So these are the 3 important components of our production planning that how to ensure the availability of the machines, how to ensure the availability of manpower and then to take decisions about the inventory labels.

And all these decisions are based on their relative cost parameters. And for that purpose we discussed that there are certain strategies and these strategies are chase strategy and level strategy. And we finally discussed that in practice, we use a combination of chase and level and for that purpose we find some kind of alternatives and we compare those alternatives and this is a trial and error method that you can some alternatives and then you calculate the total cost of those alternative and whichever alternative gives you the minimum cost that becomes your strategy.

Then we have more accurate methods, which are based on the mathematical analysis. In our last class we introduced the concept of linear programming for the production planning problems. And in this particular task, we will do some examples with the help of those examples, we see that how LP that is linear programming that is one of the very powerful tool of operations research that how this particular tool can help us in better production planning or to take more accurate decisions with respect to various decision variables in our production planning problem.

So for that purpose we start directly with one example. And this is a very simple example, but it will give us fairly good idea that how to formulate the problem and then how to solve this problem using some of the latest software like Excel Solver.

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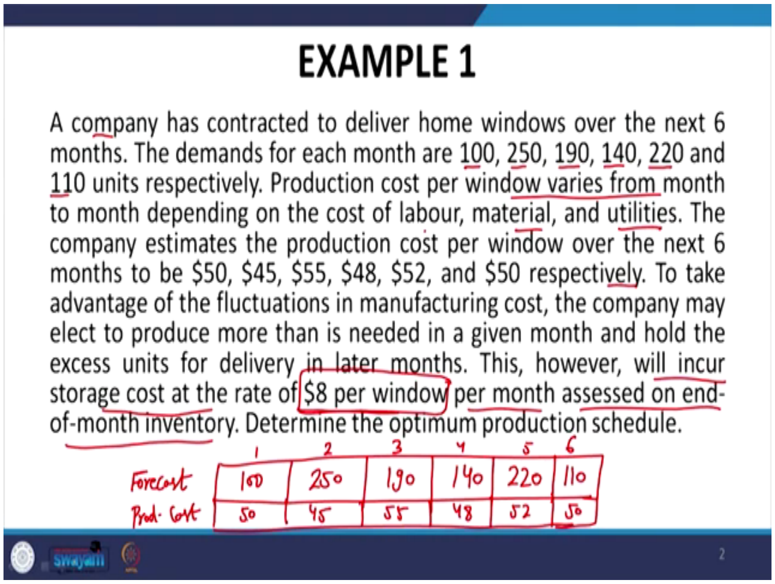

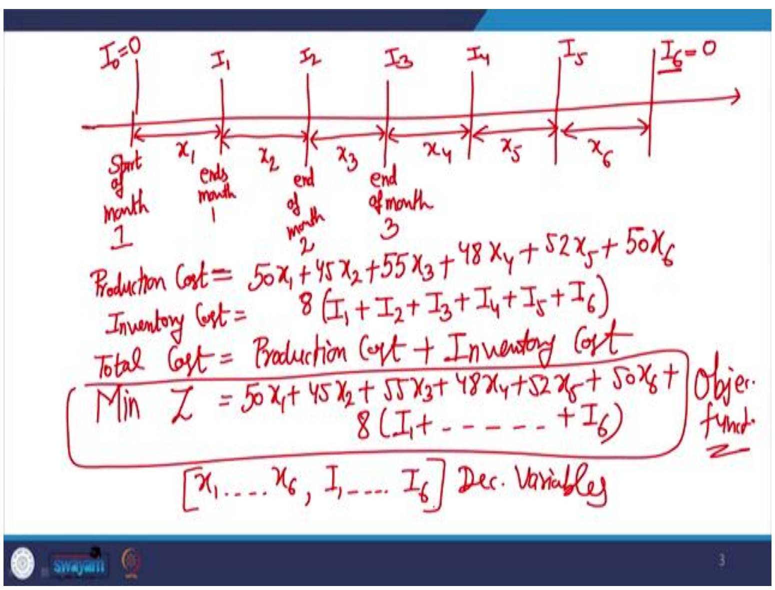

Now let us first see the problem, the problem is a company has contracted to deliver home windows over the next 6 months. The demand for each months are 100 units, 250, 190, 140 and 110 units. So you have the demand level that is over a 6 month period you can write that way, 1st, 2, 3, 4th, 5th and 6th. 100 units, 250 units, 190 units, 140 units, 220 units and 110 units. Then production cost per window varies from month to month depending on the cost of labour, material and utilities.

So the production cost, this is the forecast the production cost is per window that is per window that is given to us 50, 45, 55, 48, 52 and 50 Dollar per window that is the production cost. So this is the second data. Now, to take advantage of the fluctuations in manufacturing cost the company may elect to produce more than is needed in a given particular period and hold excess units for delivery in a later period.

This, because when you are holding the excess produced in a previous month and delivering it in a later period, so you will incur the inventory holding cost, the storage cost at the rate of 8 dollar per window, per month assessed on end of the month inventory. So depending upon how many units are left at the end of a particular period you have to pay the carrying cost, inventory holding cost or storage cost all these are the different terminologies you may find, but the meaning is same that you will incur 8 dollar per window kind of holding cost in the coming period. So you have to determine the optimum production schedule for this particular situation.

So like in this case the situation is like a flow diagram. This is the start of month 1 and during this month you are producing some number of units x1, month ends, ends month 1. And at the end of month 1, you have some inventory available that is i1, or you can say this is the beginning inventory for month 2 then month 2 comes.

During month 2, you produce x2, then month 2 ends, end of month 2 and at the end of month 2 i2 inventory is available with you. The 3rd month is going on and in the 3rd month you produce x3 units, end of month 3 and i3 inventory is available with you and so on, 4th month you are producing x4 and i4 unit is available, x5 in the 5th i5 unit is available then x6 and then i6 inventory is available at the end of 6 month.

Now one thing is very obvious, which is very simple to understand that we will plan in such a manner, so that at the end of 6 period this i6 should be equal to 0, why should you like to keep any inventory at the end of your planning period. So obviously, we will plan in such a manner that at the end of the 6th period there should not be any inventory left with us. So i6 should be 0.

And in the similar way it is not mentioned in the data. So therefore, we also assume that at the start of month 1, we have I knot, initial inventory that what is available to you at the start of month 1, and at that time also since there is no mention of this particular term, so initial inventory is also considered to be 0. So at the beginning of the plan period and at the end of the plan period inventory levels are 0, so I knot and i6 are 0. And during the intermediate periods at the end of 1, at the end of 2, at the end of 3, end of 4, end 5 and end of 6 we have i1 to i6 level of inventory though i6 is 0.

Now based on this our cost issues are like that in the first period our production cost is 50, second is 45, so you are producing x1 unit in the first period. So cost of producing x1 unit is so one expression I will write the production cost. So production cost will be because I am producing x1 unit in the first period, so 50 x1 is the production cost for the first month. Then 45 x2, then 55 x3, plus 48 x4, plus 52 x5, plus 50 x6.

So for 6 periods this is going to be my production cost because the production cost is varying from period to period so I am producing x1, x2, x4, x5, x6 different number of units in different period, so this becomes by total production cost. The second cost which I am going to incur that is the inventory cost. Now inventory cost I am going to pay on the end of the season inventory, end of the period inventory. So at the end of the first period I am having i1, then at the end of second I have i2, end of third I have i3, end of fourth I have i4, end of five I have i5, end of six I have i6, though i6 is 0.

And for that the rate at which I am paying the inventory holding cost is 8 dollar per unit per month, so this is multiplied by 8. So this is my inventory cost, so my total cost of plan is production cost plus inventory cost.

And that is what I want to minimize, so your expression becomes 50 x1, plus 45 x2, plus 55 x3, 48 x4, plus 52 x5, plus 50 x6, plus 8 into i1 to i6. And I want to this is equal to let us say z, this is equal to z and my objective is to minimize the z, so this becomes my objective function. And now, I think you all will be able to understand the decision variables also because we have already formed the objective function. So the decision variables are x1 to x6 and i1 to i6, these are the decision variables. And the number of this decision variable you can count these are 12 in numbers.

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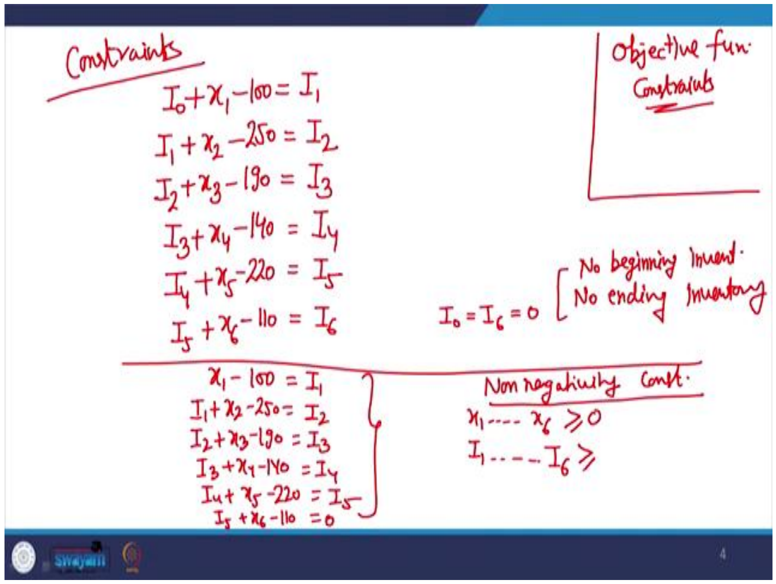

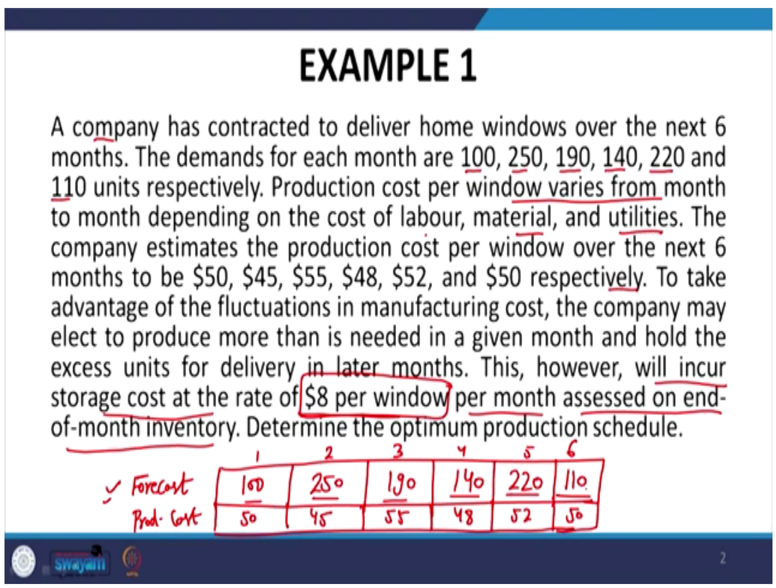

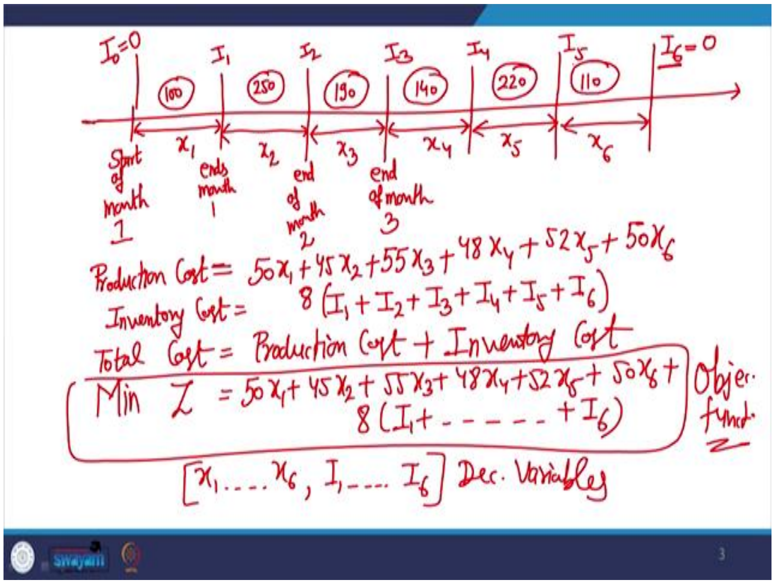

Now, the next issue is to formulate the constraints. Because in any LP problem you remember yesterday in our previous session, we discussed that you have the objective function, which will be either maximization function or minimization function. So in this case it is a minimization function and this objective function is subject to some constraints. The boundary lines on the basis of which you will have this development of the model.

Otherwise, if it is a minimization problem, so minimum cost will be 0, so if I do not produce anything the cost will be 0 and that is the best solution, but that cannot be there because I have to fulfill certain conditions also. One condition we already know that we have to meet these forecast 100, 250, 190, 140, 220, 110 so we have to meet these forecast and we need to meet these forecast in such a manner that we should incur the minimum cost.

Otherwise if there is no condition, if there is no constraint, no boundary lines, then the obvious choice is do not produce anything and that will give you the zero cost. But that is not possible it is impractical thing which I am saying, so now we need to see.

And this particular diagram will help us in understanding the development of these constraints. Now in the first period, you see the demand was 100, in the second period demand is 250 then 190, 140 then 220 and 110. This diagram will give us fairly good idea that how to develop the constraint equations.

You have initial inventory and that initial inventory is i0, then you are producing something in this first period that you are producing x1 number of units and then you have used units of 100 for consuming or for fulfilling the demand of that particular period and what is left with you is the inventory at the end of the period that is i1.

Now, when we are starting the second period i1 is the beginning inventory, you are producing x2 quantities in this period and the demand during this is 250. 250 you are consuming and then you are left with i2 units. In the third period the beginning inventory is i2, you are producing x3 units then you have some demand that is 190 units and then something is left over that is i3, i3 will become the beginning inventory for the next period, i4 is the, x4 you are going to produce during this period the demand is 140 and i4 is the left over.

Similarly, i4 is the beginning inventory for the next period, x5 you are going to produce and then 220 is the consumption and i5 you are going to left, i5 then x6 minus 110 equals to i6. And you know that i knot equals to i6 equals to 0. No beginning inventory, no ending inventory, so therefore i knot and i6 is 0.

So you can simplify these constraints in such a manner that your final model will help us that x1 minus 100 equals to i1, i1 plus x2 minus 250 equals to i2, i2 plus x3 minus 190 equals to i3, i3 plus x4 minus 140 equals to (i5) i4, i4 plus x5 minus 220 equals to i5 and then i5 plus x6 minus 110 equals to 0. So this is the constraints. Objective function, we have already developed these are the constraint. And then the last thing, the non-negativity constraints.

Our decision variables are from x1 to x6 and x1 to x6 are the quantities you are going to produce in a particular period. It is quite possible that when we are solving this model for some value may be for x3 it may come 0, but you just think over that the value of x for any period cannot be negative, you cannot produce negative quantities in any period. Either you will produce or you will not produce. So if you produce that is a positive value , if you are not producing that is a zero value, but you cannot have a negative value of production.

So all these x1 to x6 are non-negative. They are either 0, they can be 0, if you are not deciding to produce in a particular period or they will have some positive value. Similarly, other decision variables are from i1 to i6 these are the inventories, which are available to you at the end of the period. So inventories are available at the end of the period so that means either there will be some inventory, if there is a inventory it has some positive value. May be i2 is 10 units so it is a positive value or there will not be any inventory. Let us say i4 is 0 and we have already discussed that i6 is 0, so in that case i1 to i6 will also have either 0 or positive values it cannot have a negative value.

Inventory cannot be in negative terms, so these are also non-negative. So these are also the restrictions that you cannot have the negative decision variables. There are some situations where negative decision variables are also possible, so those situations we call as unrestricted variables, where the directions can be either in the positive or in the negative. So we can those kind of situations also.

But in this particular case our all decision variables, all 12 decision variables are non-negative, they cannot take negative values, so this makes my model.

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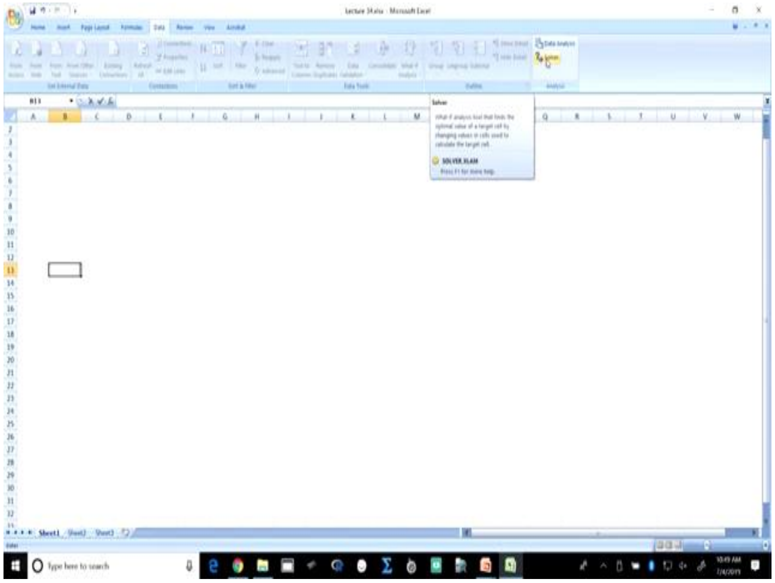
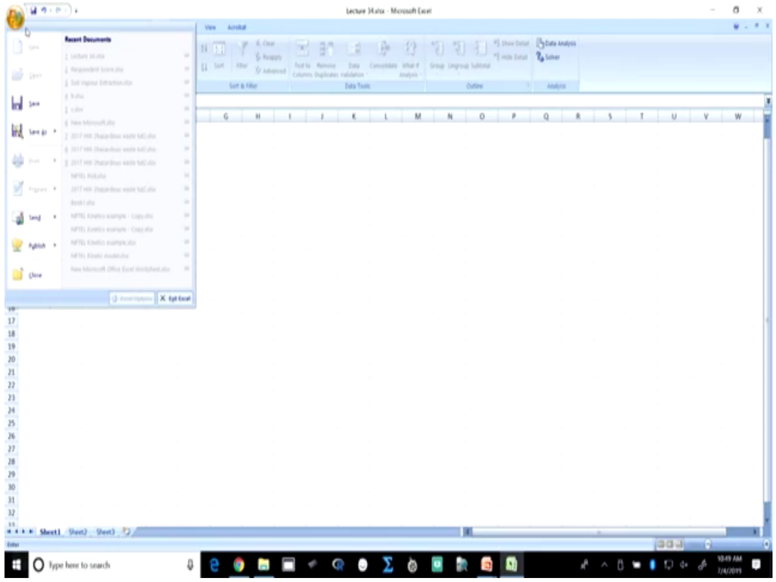
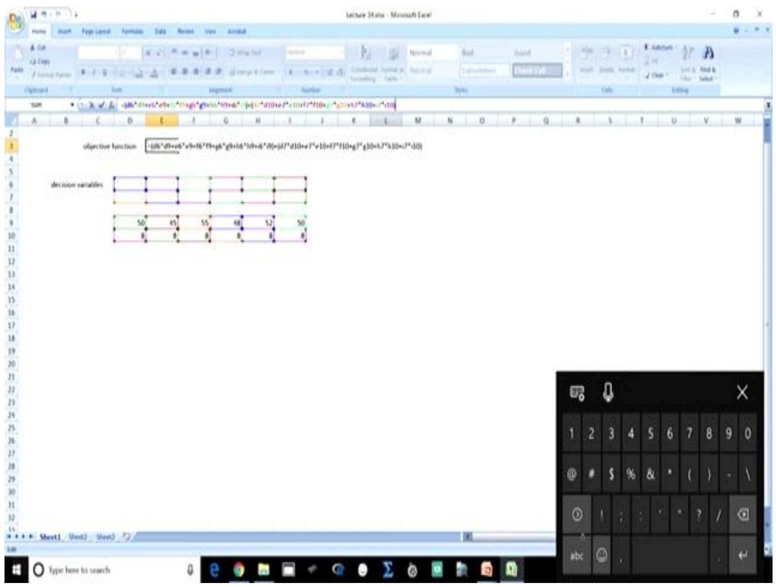
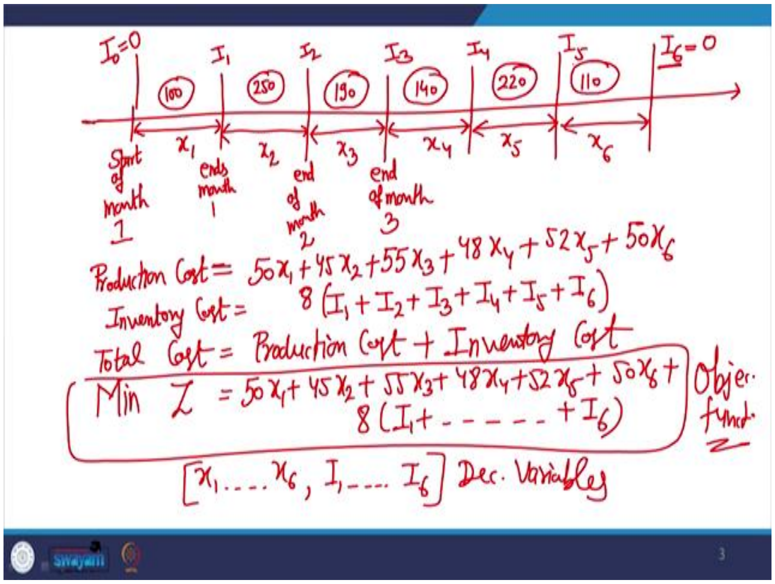






objective function
decision variables



Now let us see that how to use solver for solving this particular case, so we go to excel solver and for that purpose you will open a excel spreadsheet like this. And in this excel spreadsheet you have different kind of things. In this if you see the initial tool bar, in that tool bar there are home, insert, page out, formulas, data, review, view, acrobat, etc are mentioned. Here you click on your data tool, here you click on your data and when you click on data you should be able to see under the analysis component one solver. If this solver is there then we can solve this problem using this solver because it helps us in optimization analysis.

If this solver is not there, if this solver is not installed then you have to go to this part and then you have to add this particular solver going through add-in. So since in our particular case the solver is already installed, so we do not require, but many times in a default setting solver may not be available, so you may go to initial this excel open sheet and through that you go to add-in. And in that add-in the solver add-in is available and you can run that solver add-in and that solver tool will appear under this data tab.

Now to start our discussion first, we need to feed that data of the problem which we have just formulated and then we will see how to develop the, this particular. And so now if you recall, we have the objective function. So here we are going to develop the objective function. So let me have, so what is this objective function? If you recall, so we have the objective function as 50 x1 plus 45 x2, so we will write all these terms in different cells because we have to devote some cells for different decision variables. So in our case these are the 12 decision variables, so we have to devote 12 cells for these different decision variables.

So what I am going to do, first, let me give the place for decision variables, so and the decision variables, are. So you just remember that this is x1, d1 is x1, e1 is x2, f is x3 this g6 is x4 so as on you go up to here for x1 to x6 and then in the next row you can go for i1 to i6. Now the value of our objective function, so the multiplying of these will be done by here, I will write 50, then in the second it is 45, then let me go back to the slide 55, 48. It is 55, then 48, then I need to see further, 52 and 50.

So these are the different cost of production for different periods. And then these are the cost of holding the inventory, so for all these periods it is 8. So now my objective function I am going to define in this particular cell, my objective function I am going to define in this particular cell e.

Now the objective function, if you remember will be the sum of production cost and inventory cost. The production cost will come by multiplying the cell, d6 with d9, cell d6 with d9. So now here the objective function will be equal to d6 plus d6 into d9. So this is d6 into d9, then I have to go further it is then e6 to e9, then further it will be f6 to f9, then it is g6, then it is h6 to h9, then it is finally i6 to i9.

I think we need to add one symbol here, so this is the total production cost we have. And before that also we need to add one bracket here and one bracket here. Now after that we will add the inventory cost and that is similarly the cost of inventory multiplied by number of inventories, number of units available is inventory, these are from d7 to i7. So you can have a very quickly the d7 multiplied with d10 plus again, it is better to put a bracket here.

Then we can go here, then it is e7 to e10, then it is f7 to f10, then it is g7 to g10, then it is h7 to h10 and finally it is i7 to i10 and you can control put the closing bracket. So this becomes your formula for calculation of the value of your objective function.

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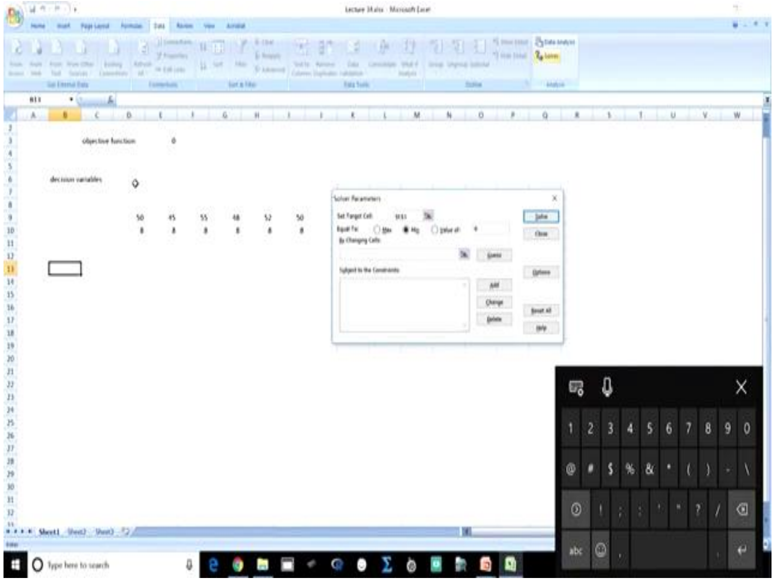

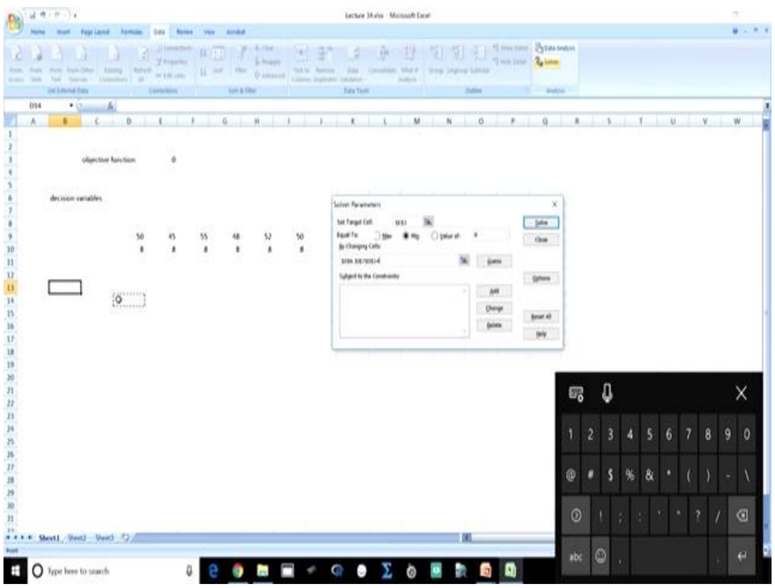

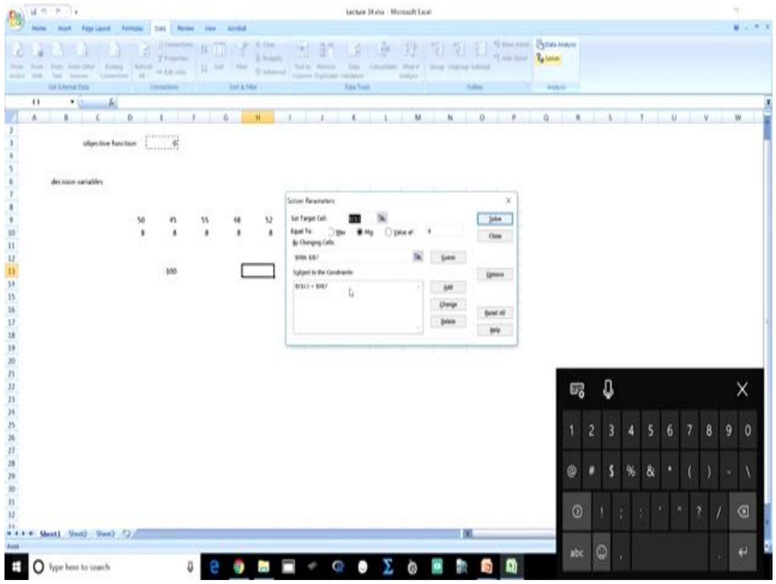
objective function
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Solver Parameters
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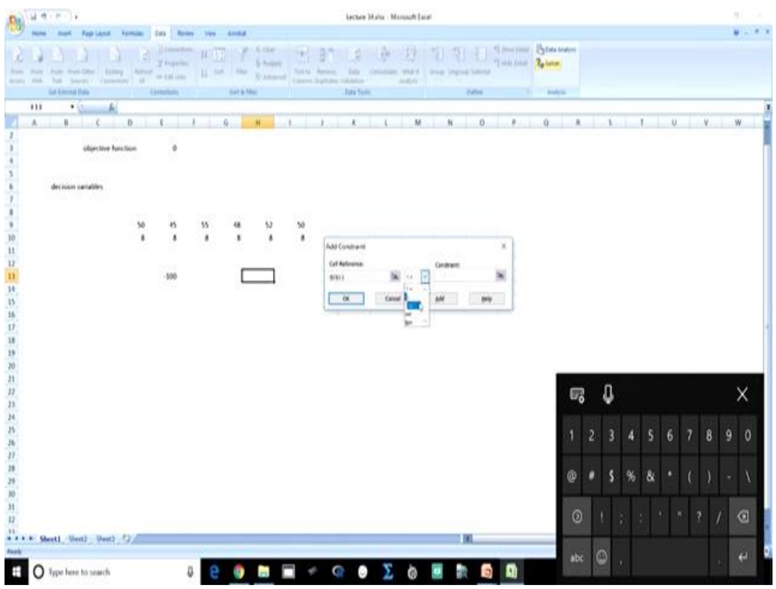
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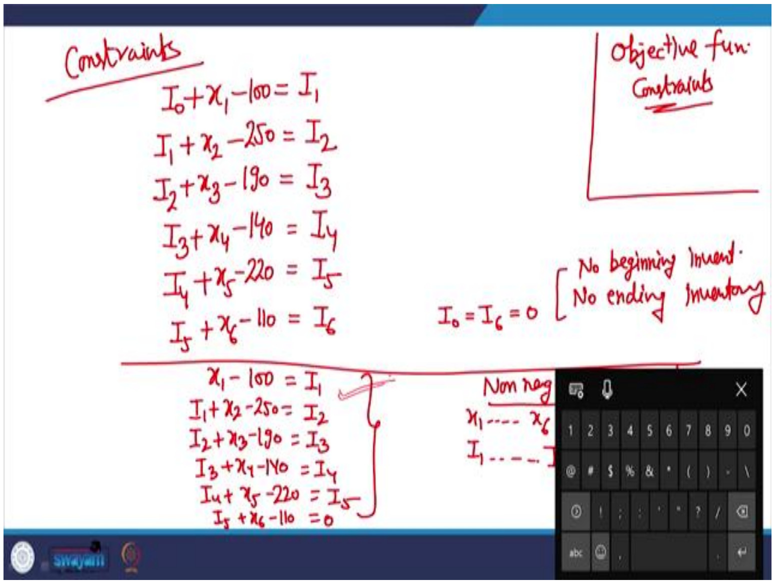

Now you see next is the development of the constraints. Since in our cells from d6 to i7, from d6 to i7, we have zero values, there are no values available, so therefore right now the value of our objective function is coming 0. All that products and the summation of those products are giving zero values. Now next part is we have to invoke the our solver calculation and then we go to this solver.

Now in this solver, our target cell is here, the target cell is this, e3 is the target cell. By changing the value of e3, we will get the, this solver parameter looks like that and we want to minimize it, so it is by default comes on the maximization, we have to minimize it. And then we need set the decision variables into it and decision variables in this particular case will be from d6 to i7, these are our decision variables. So we have set the decision variables.

By changing cells means that by changing the values of your decision value, parameters you will be able to get minimum value in your target cell. And then you have to subject to the constraints. So if you recall that we had 6 constraints based on the demand and inventory labels, so those constraints we will put here. And by substituting the values of changing cells the solver will automatically solve this problem.

So I request that I have told you that how to feed the problem. Now the only task, which I am asking you to complete because objective function we have developed, we have also seen that how to write the decision variables, now using that I am writing one constraint, I am writing one constraint and I expect that you will write remaining 5 constraints and put those constraints into this solver parameter.

Now one of the constraint, if you remember was x1 minus 100 equals to i1, this is one constraint, x1 minus 100 equals to i1. So let us see that how to put that x1 minus 100 equals to i1, x1 is the number of units you are going to produce in period 1 and that is being determined as d6, d6 actually is the number of units, which we are producing in period and that is to be kept in d6 value here.

Now when I am writing the constraints, so first constraint which I am going to write in this particular cell let us say so x1 minus 100 equals to i1. And that is this is going to come here and let us see how we write this constraint and for that purpose this is to be built in the form of a formula, so we start with the equal to sign, (this is oh no, this what happens, we have some, sorry, so this is our) now we have to come here.

So our formula we need to put, just I am closing it here. And the formula which we need to put here is going to come with this equal sign that is d6 minus 100 and this is going to equal to i1, the value of this particular column, this particular sign is d6 minus 100. So let us see this solver, so this is our target cell, this is the minimization and we have to set the values from here to here.

Now our (sorry), now for constraint purpose we are coming to this particular cell that is subject to the constraint and that is constraint that (d1) d6 that we have to add. So cell reference we have to give and that cell reference is of this cell, this is equal to and the right side value is the inventory available at the end of the period 1 that is d7, so this is d7, this, and we have added this constraint.

So this becomes the addition of constraint. Similarly, you will add remaining 5 constraints and then you put this solve button and automatically the result of this calculation will come. So with this you will be able to handle as many variables as possible in any production planning problem. So with this we come to end of this session. And I request all the participants to complete the solution process by doing it on their solvers and share the results may be in the form of a screenshot or may be in the form of the final values of cost and various decision variables, so that we can discuss the final correct solution of this problem in our discussion forums. Thank you very much.