Production and Operation Management
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Lecture 33
Aggregate Planning Techniques -3

We were discussing in our last session about numerical problems to handle the aggregate the planning. And in that we started with trial and error method where we discussed that how we need to compute the cost of a selected plan, a selected strategy, we discuss that two types of strategies are possible, either you can have a level strategy or you can have a chase strategy.

We solved a numerical example with the level strategy and in that also we had a steady rate of output during regular time. We can also introduce the concept of over time production, we can also introduce the concept of some contracting and if we go for a chase strategy in that particular case then we can hire some employees in a particular period, we can remove some employees in the later period.

So all these variations are possible but because of human limitations you cannot make infinite number of plans and then you can compare the cost of all those plans. You can only have some limited number of plans that you can compare. So it is always possible that while you are comparing the cost of different plans in trial and error method you may not be able to get the best solution. So, therefore mathematical techniques are used.

And in the mathematical techniques the most common technique which we use is linear programming. We go to operations research and in operation research we use linear programming because the assumptions if you recall in our last 2 sessions we discuss that what are the assumptions to develop a particular aggregate plant? In that assumptions we discussed the second assumption where the all cost parameters and the number of units which we are producing they are having a linear relationship.

Because the relations between the number of units which we are producing and the cost parameters are linear we can very well apply linear programing to find the solution of our aggregate plan problems.

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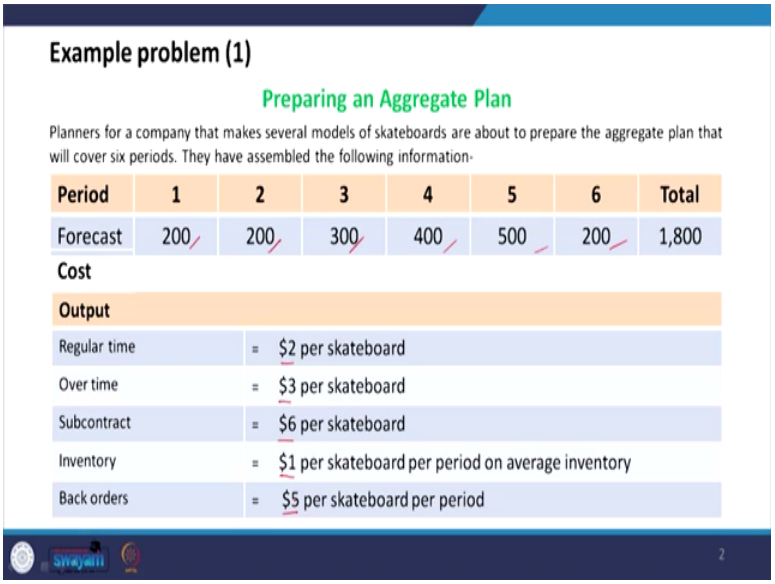

Now we will use the same data which we had in our previous case, previous class and here you have the different type of forecast for different periods for 6 periods and this is a case of skateboards. And some cost are given to us that the cost of regular time production, cost of over time production, cost of sub-contracting, cost of inventory holding and cost of back order. Now we will use this data again for developing a plan which is not based on trial and error.

We use this data for developing a plan where we had some fix number of employees and that were 15 if you remember and each employee were able to produce 20 number of units in a particular period. And we developed the solution that in each period they will produce 300 units and that is how we will fulfill the total demand. In some periods we developed some inventories but in one period if you remember in 5th period we had a backlog of 100 units also.

So sometime some backlog was there, sometime some inventories were there but finally the ending inventory at the end of 6th period was 0. And that particular system had a total cost of 4700 US dollars. Now in this particular situation I do not know how many employees I am going to have in a particular period, how many employees are going to work for overtime, how many units I am going to produce through sub-contracting.

How many number of employees I am going to hire in a particular period, how many employees I am going to layoff in a particular period, and all these how manys are my decision variables when I am going to have a linear programing problem. So now let us see that how to formulate a linear programing problem out of this.

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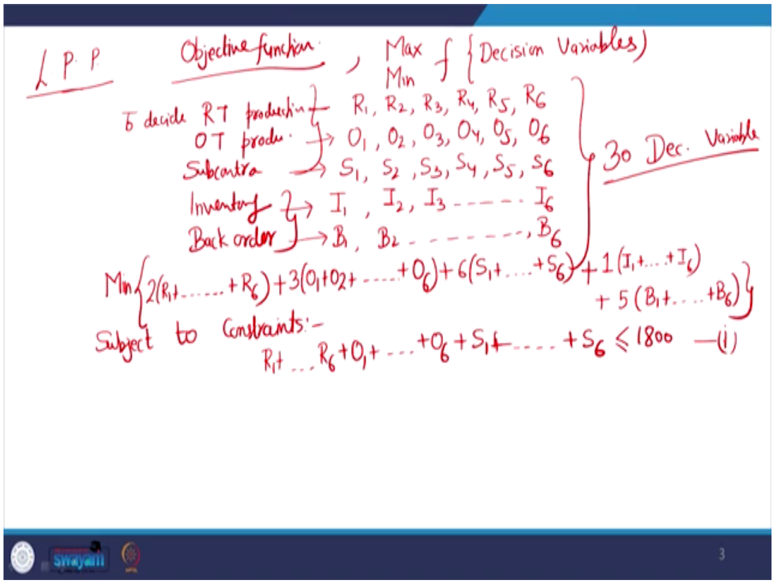

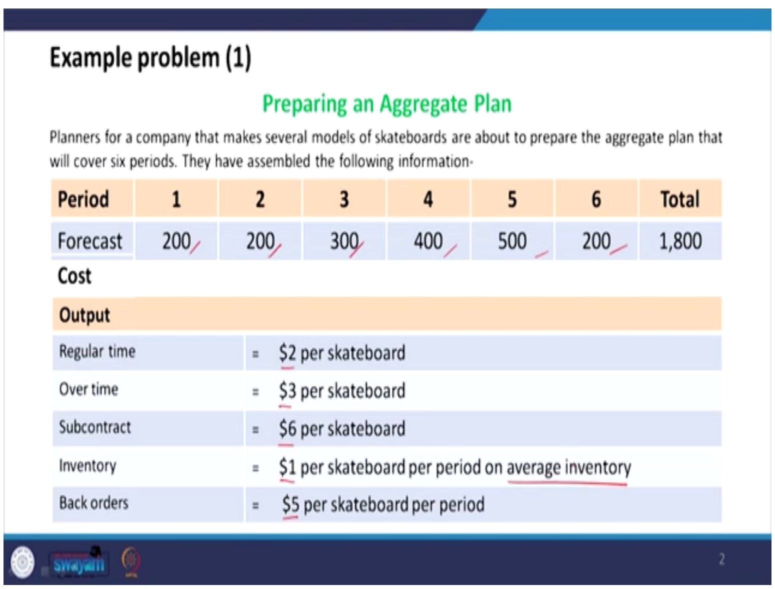

So I am going to formulate the LP, using this data. Now when we are going to formulate linear programing problem LPP, there are certain requirements. The certain requirements are we should have the objective function, the first is the objective function we need to have. Now for developing the objective function what we want to do? Either you want to maximize your profit, you want to maximize your time or you want to minimize the cost.

So objective function is either of maximization you want to increase the profit, you want to increase the market share, you want to increase the sales, et cetera et cetera, or you want to minimize the cost, you want to minimize the distance travelled, so some time it is a maximization, sometime it is a minimization kind of problem. Then the second important thing so first you have to decide whether your objective function is a minimization function or a minimization function.

Then the second thing is objective function will be composed of some variables and these variables which objective functions use these are known as decision variables, so objective function is a function of these decision variables which we are going to use. So like if it is a problem of product mix, if a company is making 3 types of products A, B, C so how many of A should a company make? How many of B should a company make? How many of C a company should make?

So the number of units to be produced for A, B and C are the decision variables, so it will have 3 decision variable that how many units of A, how many units of B, how many units of C a company should make. So many a times we do lot of mistake in identification of right decision variables. In this particular case for us to identify the decision variables will be that who decide regular time production.

To decide overtime production, to decide sub-contracting production these are the 3 important things we want to have, then okay, two more things were given to us inventory and back order. Minimize the cost of inventory and minimize the cost of back order, so these are the 5 important things about which we need to develop a comprehensive expression and the cost of that comprehensive expression should be minimum.

Now the total units which we are going to produce in regular time, so in period 1 it is R1, period 2 it is R2, period 3 it is R3, period 4 R4, period 5 R5, period 6 R6. So these are the number of units I am going to make in regular time in different periods. Number of units which we are going to produce in over time are these, number of units in different periods are these then the inventory available, average inventory because we are incurring the cost as per the data says on average inventory.

So average inventory in each period are i1, i2, i3 and up to i6 and back orders in each period are b1, b2 up to b6 so you see these 30 are decision variable and I have to minimize the total cost coming because of these decision variable, so now what is the expression? I have to minimize, what I have to minimize let us see that.

First is the cost of regular production, now the cost of regular production is 2 dollar per board and I am producing R1 plus R2 plus R3 plus R4 plus R5 plus R6 units in the regular production, so 2 into R1 to R6 that is the total cost of production of regular units, the second is overtime production.

The overtime cost of production is 3 dollar per board, so number of units which I am going to produce in overtime O1 to O6, so 3 into O1 plus O2 and up to O6 then the sub-contracting, in each period I am giving sub-contracting of s1, s2, s3 to s6 and that is at the rate of 6 dollar per unit, 6 into s1 to s6 then the inventory cost, this is average inventory I am having in different period, so the cost of average inventory which we have incurred at the rate of one per period.

So i1 to i6 and then the cost of back order is incurred at the rate of 5 dollar per period per unit, so 5 into b1 to b6, so this is the overall my objective expression that 2 into units produce in the regular time 3 into units produced in the overtime 6 into units produced through sub-contracting 1 into units in your average inventory, 5 into units in back order. Now this is the development of objective function.

But this objective function is subject to constraints, it has some limitations and without limitation it is not going to full fill the purpose. Now one important constraint is that our total production should be less than 1800 because the total demand of 6 period is 1800, so this is the total value, so one constraint which you will see that you are producing R1 to R6, O1 to O6, S1 to S6 so R1 to R6 plus O1 to O6 plus S1 to S6 should be less than equal to 1800. That is one very important constraint, now for developing more constraints we need to understand situations more clearly.

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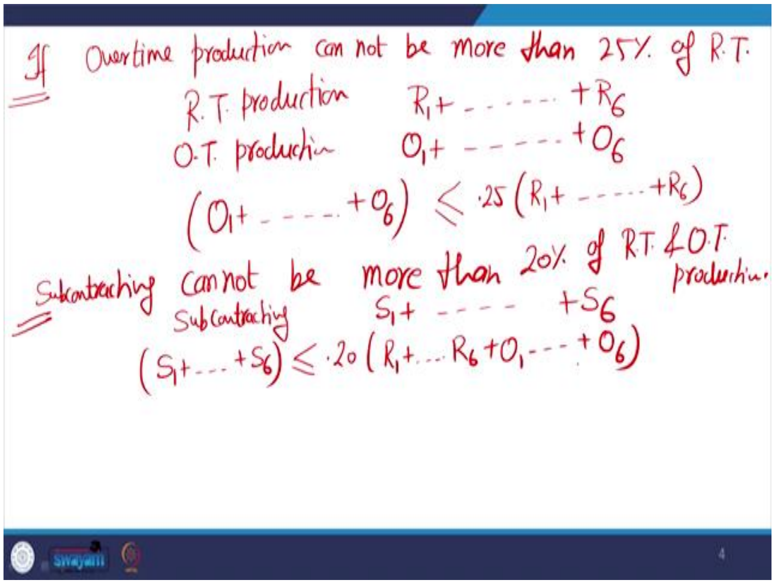

For example, now if the condition is that the over time production, if over time production cannot be more than 25 percent of regular time, there may be a condition that over time production cannot be more than 25 percent of regular time, it is quite possible that employees may behave in a manner that they do not work in regular time but they work in more in the over time.

Because over time production are higher than the regular time, so you may have such kind of policy that over time production cannot be more than 25 percent of the regular time. Now regular time production is regular time production for 6 period is R1 to R6, over time production is O1 to O6, now with this condition, with this constraint your O1 to O6 should be less than equal to 25 percent that 0.25 of R1 to R6.

So this is a constraint which system may impose, similarly another constraint can be sub-contracting cannot be more than 20 percent of regular and overtime production. Sub-contracting cannot be more than 20 percent of regular time and overtime production. Now regular time production we have just seen R1 to R6, overtime production is O1 to O6 and sub-contracting production is S1 to S6.

And with this condition this is another condition, so with this condition our equation will be that S1 to S6 should be less than equal to 0.20 of R1 to R6 plus O1 to O6, so this becomes another condition, another constraint for this situation.

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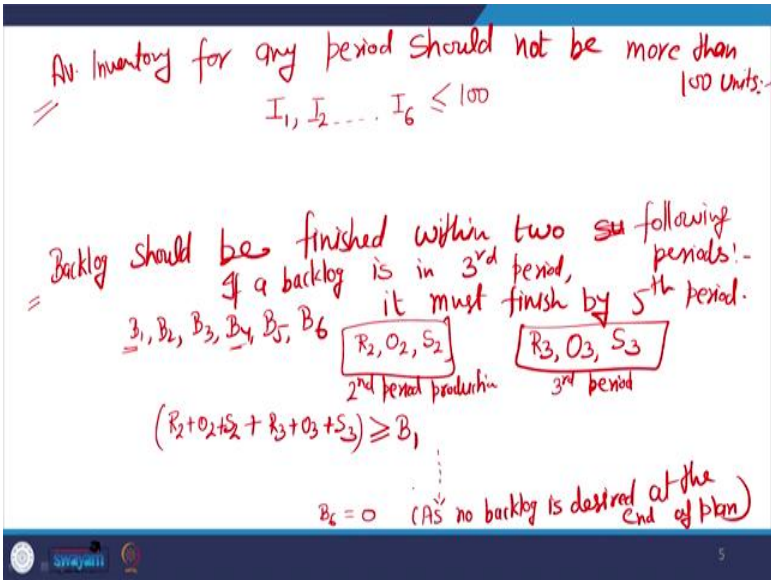

Then another constraint can be imposed that average inventory for any period should not be more than 100 units, so in that case i1, i2 and up to i6 should be less than equal to 100, that is another constraint we will impose.

Similarly another constraint can be backlog should be finished within two successive periods or within two following periods, so if there is a back log if a backlog is in third period it must finish by 5th period, it cannot go beyond 5th period. A new backlog may develop in 5th period but you have to finish the previous backlog by next two period. Though that is again a condition, so if backlog we represent it by b1, b2, b3 so b1, b2, b3, b4, b5, b6 etc.

So now your b1 whatever is your backlog in this particular period your production of second and third period should take care that this b1 is able to finish in that particular period, so if I am talking of b1 and the production in the second period is R2 then overtime is O2, sub-contracting is S2 and in the third period this is the second period production, 3rd period production is R3 O3 S3.

And during this second and third period production you also need to fulfill the demand of their periods an there may be a new backlog which may come from these periods also, but you have to fulfill the backlog of previous period in these two periods, so the equation will become in that way that b1 that R2 plus O2 plus S2 and the R3 plus O3 plus S3, so these two productions must sufficiently fulfill the requirement of b1.

Though the point which I am trying to say that because of fulfilling the requirement of b1 they may not be able to fulfill the requirement of their current periods, so that will develop b2, that will develop b3 and we will take care of b2 and b3 in the next periods but at least these two periods production should be able to fulfill the backlog of my previous period.

So should be more than equal to b1, because the two successive, two following periods production should be fulfilling the backlog of my first period and in that way we will have more such equation up to last period that b4 backlog should be fulfilled by b6 and b5 backlog has to be fulfilled by 6th period demand. And there should not be any backlog at the end of the 6th period, so your b6 should be equal to 0.

Because we do not want as no backlog is desired at the end of plan, so when no backlog is desired at the end of the plan we have b6 equals to 0 and before that we will have multiple such equations where you have different equations for different levels of backlog, so now I have developed some equation.

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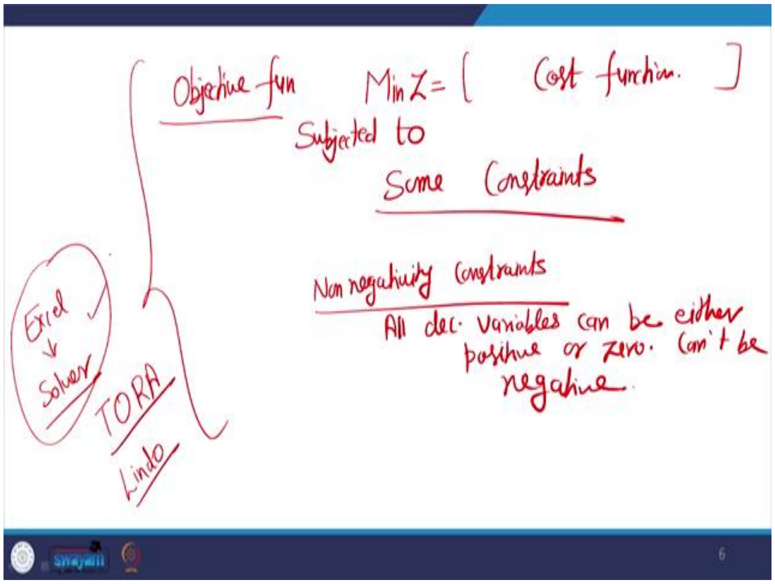

So you can understand that now in LP you have the objective function which is minimization of z and that is in the form of a cost function which is subjected to some constraints and these constraints and objective function and then you have some non-negativity constraint also that means all these decision variables can be either positive or 0, cannot be negative.

You cannot produce negative quantity in a particular period, you cannot have negative cost of inventory, you cannot have negative backlog, so to ensure that model does not produce any negative values we put those things in the form of non-negative constraints. So here our model is developed with very large number of decision variables and depending upon how many constraints we want to build into the problem we will have those many equations in the constraint list.

Now it will not be possible humanly to solve such a complicated model, we have the system of simplex method which help us in solving LP problems of 3, 4, 5 decision variables but here we have seen that we have 30 decision variables, so such type of problems cannot be solved with the paper and pencil using our normal simplex method, so here we take the help of various computer algorithms and in that we can take the help of some software which are very easily available to all of us that is excel.

So you can go to solver of excel and excel solver can help you in solving this problem that you can model this entire problem under the excel solver and using that solver tool of excel you can solve it then another prop is TORA, that is also available and TORA is a very simple software which is easily usable and that is quite customized to handle such kind of LP problems.

So you can feed your problem and we have not discussed the algorithmic part of solution of this particular case but you can and again when you are using TORA et cetera there is no need to have the algorithmic understanding you just know how to formulate the problem that is more important in our whole purpose and once you formulate the problem correctly then different types of tools available.

Then TORA is there, then you can also use Lindo, that is also a very popular operation research software and using the Lindo also you can solve your problem, so multiple solutions software are available depending upon the availability with you, you can take I think excel solver all of us can use easily and we can have a next class where we can show you that how to use excel solver for solving such type of LP problems so we can feed the data here in this class.

And you can copy it step by step at your places so that you can understand that in which particular cell what data to be fed and how that will be useful in solving our such type of large LP problems. And the different type of constraints which you can we have taken only some 1, 2, 3, 4 constraints in this problem. But many more constraints you can think of we did not take any constraint with respect to hiring and laying off of the employees.

So you can take constraints with respect to hiring and laying off also and for that purpose you should know what is the cost of hiring and what is the cost off laying off. Because that also need to be built into the objective function, so you can make this problem as complex as possible. And you will have finally a very fine tuned solution of the problem which will help you to get the optimal solution which we are again and again saying since last two classes that our trail and error method may not be able to give us the accurate solution.

But this particular method which we have just done and the solution part of this we have so far formulated the problem, this was problem formulation in our LP method. Now in our next class we will do the solution of this using excel solver and that will help us that how we have solve this, how we have fed the data and once you have that kind of template available in the excel solver we can play with the solution that we can change the parameters and we can see that how solution is being effected by those change in parameters. So we stop here in this class and in our next class we will see the solution by excel solver. Thank you very much.