

Production and Operation Management
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Lecture 32
Aggregate Planning Techniques-II (Examples)

Welcome friends, in our last session we introduced that how we can apply different kind of computation techniques for knowing the aggregate plan. We discuss about 2 important techniques one is based on trial and error and another is more analytical in nature that is mathematical techniques.

In trial and error method we use different type of combinations of our decision variables and based on which combination gives us minimum cost of our plant that becomes the suitable plan for our case. The only drawback of this particular system is that, because large number of combinations are possible.

So, we may not be able to get the best combination we can compare the results of some limited number of combinations and out of those possible combinations whichever is giving us minimum cost we will select that, but it is not guaranteed that the selected one is the optimal one also.

So, therefore we go for more advanced techniques where we may use linear programming or simulation for getting a better solution. But we will first discuss trial and error method because in simple problems many a times solutions are possible simply by trial and error method also. And sometime you can by intuition can understand that whether you are going into the right direction or not in the right direction once you have some good exposure of planning practices.

Now, when we are going for trial and error method, we need to compute some important things if you recall we discussed a spreadsheet in our previous session, and in that spreadsheet we had some important output and depending upon those output we calculate the cost and one important output is how much you are producing in the regular time and how much you are producing in the overtime.

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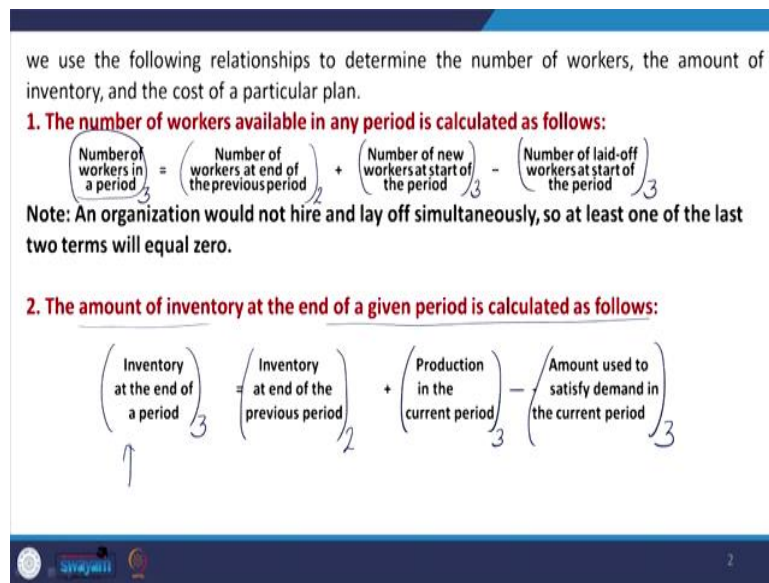
we use the following relationships to determine the number of workers, the amount of inventory, and the cost of a particular plan.

1. The number of workers available in any period is calculated as follows:

$$\left(\begin{array}{c} \text{Number of} \\ \text{workers in} \\ \text{a period} \end{array} \right)_3 = \left(\begin{array}{c} \text{Number of} \\ \text{workers at end of} \\ \text{the previous period} \end{array} \right)_2 + \left(\begin{array}{c} \text{Number of new} \\ \text{workers at start of} \\ \text{the period} \end{array} \right)_3 - \left(\begin{array}{c} \text{Number of laid-off} \\ \text{workers at start of} \\ \text{the period} \end{array} \right)_3$$

Note: An organization would not hire and lay off simultaneously, so at least one of the last two terms will equal zero.

2. The amount of inventory at the end of a given period is calculated as follows:

$$\left(\begin{array}{c} \text{Inventory} \\ \text{at the end of} \\ \text{a period} \end{array} \right)_3 = \left(\begin{array}{c} \text{Inventory} \\ \text{at end of the} \\ \text{previous period} \end{array} \right)_2 + \left(\begin{array}{c} \text{Production} \\ \text{in the} \\ \text{current period} \end{array} \right)_3 - \left(\begin{array}{c} \text{Amount used to} \\ \text{satisfy demand in} \\ \text{the current period} \end{array} \right)_3$$


And for that purpose the number of workers in a particular period is important calculation because depending upon the number of workers in a particular period, you know that how much total time is available. And in that time how any units you will produce and based on your cost of regular time based on your cost of your overtime you will calculate the cost of your production in regular time and in overtime.

The calculation of this number of workers in a particular period we explained in our previous session, so just now to have a quick glance of that understanding the number of workers at the end of the previous period that means, how if I am calculating the number of workers in a period that is 3. So, in third period how many workers are available with me?

So, for that purpose number of workers at the end of previous period that is period 2, and number of new workers which you are hiring at the start of this period 3. And number of workers you are laying off at the start of this period 3. We have already discussed that it is not possible practically that in a particular period you are hiring and laying off simultaneously. Either you will hire or you will lay off.

So, out of this second and third term one term will always be 0, either you will be hiring net or you will be laying off, but to have a general understanding we have used both these terms so that you can understand that hired workers are added and laid off worker are to be subtracted. So, it gives you the number of workers in a particular period.

The second is you have to calculate the amount of inventory available at the end of a given particular period, similarly let us calculate it for the third period. Inventory at the end of the third period. So, it is simple to understand that inventory at the end of the previous period. So, previous period in this case is second period.

How much you are producing in the current period that is the third period. And how much you have consumed in the current period that is to be subtracted. So, inventory the amount available at the end of the previous period. What you are producing in this current period that has to be added and then what have you consumed in this particular period that is to be subtracted.

So, that will give you the inventory at the end of period 3, many a times we have already discussed this issue that many a times the calculation of inventory cost is based on inventory available at the end of the period. So, in that case this value will be used if inventory cost, the holding cost is based on end inventory then this particular calculation will be useful.

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3. The average inventory for a period is equal to

$$= \frac{\text{Beginning inventory} + \text{Ending inventory}}{2}$$

is either on end inventory or on av. inventory

4. The cost of a particular plan for a given period can be determined by summing the appropriate costs:

Cost for a period	=	Output cost	+	Hire/lay-off cost	+	Inventory cost	+	Back-order Cost
		(Reg + OT + Subcontract)						
		<i>Regular Overtime</i>						

But sometime inventory cost is calculated on the bases of average inventory. So, average inventory is simply the average of inventory available at the start of period, inventory at the end of the period. And obviously plus divided by 2 that is the average inventory. So, inventory holding cost, the carrying cost is either on end inventory or on average inventory.

So, you have to see that which is more applicable, if in your system inventory accumulation is happening in steps and inventory accumulation is happening mostly all around the end of

the period. So, it is better that you calculate the inventory holding cost on the bases of end inventory. But if inventory is being developed over a period of time because of constant rate of accumulation, then it is better that you calculate the inventory holding cost on the basis of average inventory.

So, in particular situation where inventory is accumulated almost in the end of the period then it is better to have inventory calculation, otherwise the average inventory will unnecessarily be high in that case, though practically inventory is coming only in the last of the period but you will incur more inventory cost because of your average inventory formula.

So, you need to understand the physical practical implication also of these formula that how in real life because here we will develop... we will give you some readymade numerical data, but how you can develop the problems in real life on your own that is more required and that is the development of the subject.

Now, the fourth type of cost parameter is associated to a particular plan for given period can be determined by summing the appropriate cost. Finally the overall cost of a particular plan that we have use this much of overtime, we have use this much of sub-contracting, we hired these many total number of workers from period 1 to period 8, we laid off these many number of workers during this particular period.

So, that is the sum total of all types of cost for a particular plan, so you see that one cost is the output cost which is the regular this is overtime and sub-contracting. So, how much you have produced in regular period, how much you have produced in the overtime period and how much you have the sub-contracted. So, that is the output cost.

Then you have hired some employees, so hiring cost, you have laid off some employees, so the laying off cost so that is the second type of cost with respect to hiring and laying off. Then, either it is on the bases of average inventory or at the on the bases of end of the period inventory. So, you may have some inventory of level, particularly if you are using a level strategy certainly you will incur some kind of inventory, so the inventory cost.

Again because of your level strategy, backorders will also be there and the penalty associated with those backorder will be the backorder cost. So, you have to do sigma of all these different types of cost that is the cost of your plan for a particular period. So, that is the most compressive understanding of our plant.

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Example problem (1)							
Preparing an Aggregate Plan							
Planners for a company that makes several models of skateboards are about to prepare the aggregate plan that will cover six periods. They have assembled the following information-							
Period	1	2	3	4	5	6	Total
Forecast	200	200	300	400	500	200	1,800
Cost							
Output							
Regular time	= \$2 per skateboard						
Over time	= \$3 per skateboard						
Subcontract	= \$6 per skateboard						
Inventory	= \$1 per skateboard per period on 'average inventory'						
Back orders	= \$5 per skateboard per period						

Now, how to use it for that purpose let us have some numerical data and based on this data we will understand that how you can compare different types of plants. Now, let us try to understand the data first, planners for a company that makes several models of skateboards. Now, the first important thing if you recall in our previous session we discuss that we have to combine the demand of different types of products into single category.

So, models of skateboards are multiple, there are different types of skateboard models but we here for our purpose is considering them as single item. So, 200 there may be skateboards for the small children, for the (10:55), for the beginners, for the experienced people. So, different type of skateboards are possible.

But we are not considered the different varieties, we have combined all the varieties into the skateboard name only, or about to prepare the aggregate plan. That will cover 6 periods. So, from period 1 to 6 period the different values of forecast are available and the total demand, total expected forecast is 1800 units.

They have assembled the following information, now let us see the cost parameters. The regular time of production it cost you 2 dollar per skateboard, the overtime cost is 3 dollar per skateboard, please always remember the overtime cost will be higher then the regular time cost, because as per our loss personal law we have to pay higher rates if somebody is working at the overtime period.

Then the sub-contract, if you are giving it to some other vendor to prepare skateboards on your behalf, so it will cost you 6 dollar per skateboard. Then the inventory related cost is 1 dollar per skateboard, per period on average inventory. Now, it is clearly mentioned that this cost is on average inventory. Which we were discussing continuously that the inventory cost may be either on average inventory or end of the period inventory, so here it is average inventory.

And then the back order penalty is 5 dollar per skateboard per period, for example if you are not able to fulfil the demand of first of period up to third period then you have to pay 10 dollar for that unfulfilled demand, if you are fulfilling the demand of first period in the second period then you have to pay only 5 dollar per skateboard, so that is the different type of cost parameters available to you.

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- They now want to evaluate a plan that calls for a steady rate of regular-time output, mainly using inventory to absorb the uneven demand but allowing some backlog. Level St
- Overtime and subcontracting are not used because they want steady output. They intend to start with zero inventory on hand in the first period. Prepare an aggregate plan and determine its cost using the preceding information.
- Assume a level output rate of 300 units (skateboards) per period with regular time (i.e., $1,800/6 = 300$). Note that the planned ending inventory is zero. There are 15 workers, and each can produce 20 skateboards per period. $15 \times 20 = 300$

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Now, we can use different type of strategies, let us see. Now, what we can do. They now want to evaluate a plant that calls for a study rate of regular time output mainly using inventory to absorb the uneven demand but allowing some backlog. Now, we want to have a level strategy, this is study rate regular time output so this is level strategy, that we will maintain a constant rate of output and because of constant rate of output sometime you will have some inventory and sometime you will have some backorders also.

Overtime and sub-contracting are not used because they want steady output, so obviously in this particular case when we are going for a steady rate regular time output. So, already this word means that you are not going to use any overtime right now. They intend to start with 0 inventory on hand in the first period and we have to prepare an aggregate plan and determine its cost using the given information.

Now, what should be the level, now the question is what should be the level of steady output? What should be the level of steady output? Now, if you see the table here 200 200 300 400 500 200 total 1800 units are required in 6 periods. So, obviously it is very simple to understand that we have to make 1800 units in 6 periods. So, per period we will make 300 units per period we will make 300 units.

So, total demand for all the period divided by number of periods that will give you the steady rate of output. So, I will make 300 units per period. Now, for making a particular number of skateboards, for making a particular skateboard how much time is required, how much time is required?

Now, that will determine the number of employees I need to keep for making 300 units in regular time, now that the plan ending inventory is 0 there are 15 workers and each can produce 20 skateboards per period, it says that 15 workers and each can produce 20 skateboards per period, 15 into 20 that makes 300.

So, one worker one worker is making 20 skateboards in a period. So, if you remember our assumptions that period to period there may be different number of holidays there may be different reasons of relaxations but we consider for the sake of simplification of calculation that the rate of output is constant per worker. So, one worker in a particular period is going to produce 20 skateboards and we are also not considering, we are also not considering that how many hours are there in a particular day.

Whether a particular period is one day period, whether a particular period is one week period, whether a particular period is one month period. So, we are simply talking in terms of period and in one period, one particular worker is going to produce 20 skateboards and therefore there are 15 workers.

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Example solution (1)							
Period	1	2	3	4	5	6	Total
Forecast	200	200	300	400	500	200	1,800
Output							
Regular	300	300	300	300	300	300	1,800
Overtime	-	-	-	-	-	-	
Subcontract	-	-	-	-	-	-	
Output-forecast	100	100	0	(100)	(200)	100	0
Inventory							() → Back order
Beginning	→ 0	→ 100	→ 200	→ 200	→ 100	→ 0	
Ending	→ 100	→ 200	→ 200	→ 100	→ 0	→ 0	
Average Inventory	50	150	200	150	50	0	600
Backlog	0	0	0	0	100	0	100

Now, if you see that 200 200 300 400 500 200, so total requirement is of 1800. We have set our system in such a way that 15 employees each 20 board per period, therefore every period you have 300 units available with you, so that you can understand now that the final demand is being fulfilled, but and obviously we are using only the regular time so in this all overtime periods we have put the dash in all subcontracting cases we have put the dash.

Now, your output is 300 units the forecast was of 200 units. So, you have the 100 units which are left with you in this period. Here again the regular output is 300 unit and the forecast was of 200 units. So, you have 100 units left with you in this third period your demand is of 300 units and you have produce 300 so there is no leftover in this particular period.

In the fourth period you have a forecast of 400 units, you produce 300 units. So, you have a backorder of 100 units. So, this sign it represents backorder. Similarly in the fifth period you have backorder of 200 units because the forecast was of 500 units and you produce 300 units. So, there is a backorder of 200 units.

In the sixth period demand is of 200 units you produce 300 units so again you have an inventory of 100 units now. Going with this data the beginning inventory is 0, beginning inventory is 0 and end inventory because these output minus forecast is available to you at the end of the period. So, it becomes 100 units.

Now, this 100 unit is available in the beginning of the second period, these 100 units are available to you as a beginning inventory for the second period. And these 100 additional are available to you during this second period. So, in the second period this 100 plus this 100 now it becomes 200 unit, this 100 and this is added into 100 so it becomes 200 unit.

And this 200 is the starting inventory for the third period and in the third period there is no left over units available to you, so whatever is the beginning inventory that remains your ending inventory also and this ending inventory of the third period becomes the opening inventory of the fourth period.

But in the fourth period you are incurring a backorder of 100 you have a shortage of 100 units, so therefore you will fulfill the demand of these additional 100 units from this available inventory. So, 200 were the additional supply available to you and there was a demand of additional 100 units.

So, out of 200, 100 you will consume to fulfill the extra demand, so the final inventory available to you at the end of the period is 100 units, these 100 units will be the beginning inventory for the next period, now in the next period you have 200 units as the additional demand and 100 units are available to you to fulfill the demand of additional units.

So, still even if you have some beginning inventory you will not be able to fulfill the entire additional demand. So, finally you will end up with a backlog of 100 units, you will create a backlog of 100 units because out of this 100 you will fulfill the demand of 100 units 100 units which are extra required but still 100 units cannot be fulfilled so they will go to the backlog.

For all this previous periods there is no backlog because you are able to fulfill the demand either from the current period or whatever is available from the previous periods from that stock you are able to fulfill the extra demand. But in the fifth period you finally have some backlogs, then since this is the 0 inventory left with you.

So, this 0 will go to beginning inventory level at the sixth period and in the sixth period you have a demand of 300 units, 300 is the regular production, 200 was the demand. So, you have a stock available of 100 units in the sixth period and already the backlog of 100 units were there from the fifth period.

So, from this production of 300 units you will fulfill the demand of current period and you will also fulfill the backlog of 100 units of the previous period, so finally you will have 0 ending inventory at the end of the sixth period and this is also a kind of litmus test for our level strategy that the end of the period you should have 0 inventory, that is the idea of a level strategy that from your level of production, the constant rate of production you should not have any closing inventory at the end of your aggregate plan.

So, that is what we are able to see that if there is some available inventory in this particular cell that means we have not done it properly, some units may be 5, 10, 20 units may remain because of some rounding issues some computational issues some that may be possible but it is not be significant number of units which can be in the sixth periods ending inventory.

And then you have this all backlog is also fulfilled, so this backlog is also 0. Now, another important calculation which we have done for all these 6 periods that this is the beginning inventory, this is the ending inventory. So, the average of these two beginning and ending inventory is the average inventory.

So, this is beginning plus ending divided by 2. So, 100 plus 0 divided by 2 that makes 50, 100 plus 200, 300 divided by 2 150, 200 plus 200 400 divided by 2 200, 200 plus 100 300 divided by 2 150, 100 plus 0 equals to 100 divided by 2 50, 0 plus 0 divided by 2 it is 0. So, that is

my average inventory level for the six periods, so you can have a total of that 50 plus 150 200, 400 550 and 50 that is 600.

So, this is the average inventory level at total of average inventory level for all the periods and the total backlog for different period is 0, 0, 0, 0 plus 100 and 0. So, total backlog is 100 units. So, this is the calculation of the entire table. Now, let us go to the cost parameters for these different values.

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Example problem (1)

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Planners for a company that makes several models of skateboards are about to prepare the aggregate plan that will cover six periods. They have assembled the following information-

Period	1	2	3	4	5	6	Total
Forecast	200	200	300	400	500	200	1,800

Cost

Output	
Regular time	= \$2 per skateboard
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Back orders	= \$5 per skateboard per period

Example solution (1)

Period	1	2	3	4	5	6	Total
Forecast	200	200	300	400	500	200	1,800
Output							
Regular	300	300	300	300	300	300	1,800
Overtime	-	-	-	-	-	-	
Subcontract	-	-	-	-	-	-	
Output-forecast	100	100	0	(100)	(200)	100	0
Inventory							
Beginning	0	100	200	200	100	0	
Ending	100	200	200	100	0	0	
Average Inventory	50	150	200	150	50	0	600
Backlog	0	0	0	0	100	0	100

Period	1	2	3	4	5	6	Total
Costs							
Output							
Regular Time Production Cost	\$600	\$600	\$600	\$600	\$600	\$600	\$3600
Overtime	-	-	-	-	-	-	
Subcontract	-	-	-	-	-	-	
Hire/Lay off	-	-	-	-	-	-	
Inventory Holding Cost	\$50	\$150	\$200	\$150	\$50	\$0	\$600
Back orders Cost (Penalty)	\$0	\$0	\$0	\$0	\$500	\$0	\$500
Total	\$650	\$750	\$800	\$750	\$1150	\$600	\$4700

So, when we are calculating the cost of all these things if you remember the cost of our regular output is 2 dollar per board that is the cost of our regular output. Now, in our regular period we are producing 300 boards per period. So, that is same 600, 600, 600 for all the 6 periods, so that is the cost of our regular output that is constant for all the 6 periods.

Overtime we are not producing anything sub-contracting we are not doing anything hiring and laying off we are not doing anything. So, for all these thing we have the zeros. And then inventory, the inventory cost if you remember we incur at the rate of 1 dollar per board per period on average inventory.

So, if you see the inventory average inventory in the first period was 50 then 150 then 200, 50 150, 200 so the average inventory cost is 1 dollar per period on average inventory. So, this is 50, 150, 200, 150, 50 and 0. So, that is my inventory holding cost so you can write that inventory holding cost, this is the cost of regular time production cost and this becomes that.

Then this is the backorder cost, or penalty. Now, only one back order is there that is in period number 5 that is of 100 units and for all other periods there is no backorder, so for period 1, 2, 3, 4 the backorder cost is 0 and in period 6 also the backorder cost is 0, in the period 5 you have a backorder of 100 units and now if you go to the original data the backorder cost is 5 dollar per board per period.

So, you have 500 that is 100 into 5 that is the 500 dollars that is the backorder for the fifth period. Now, you can do the total of cost for different periods. 600 plus 50 plus 0 650, 750,

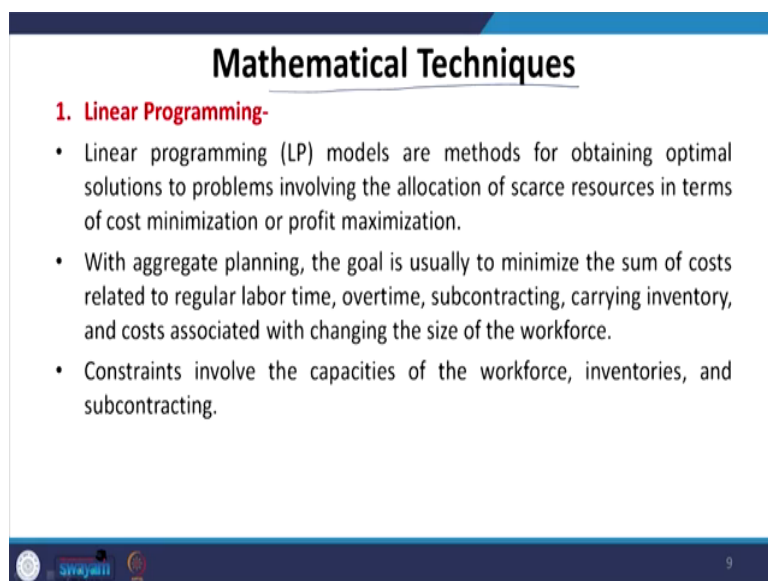
800, 750 this is 1150 the 0 is coming to the lower side this is 1150 and this is 600. So, the total cost of this plan is coming to be 4700 dollars.

So, this is how you will make the calculation of the entire plan. Similarly, in this particular case we have not touched the overtime, we have not touch the sub-contracting, we have not touch the hiring and laying off, and we followed a very case of level strategy that we have only 15 workers each of them is producing 20 boards per period and therefore this calculation was a very simple calculation and we understood how to do the calculation.

Now, moving ahead you can also do the advance calculations, because in this case I on my own intuitions considered that there is no overtime there is no sub-contracting, there is no hiring and laying off, but I can also think of that I can only produce instead of 600 units I will produce in the regular time only 100 units.

And the remaining units I may either go by a sub-contracting or by overtime, now I have to additionally calculate the cost of overtime and sub-contracting and then I will determine weather that is a profitable decision, profitable will only happen when the cost will be lower than this value. So, that is how I will generate I will create more alternative plans and more alternative plans means more such computation and then I have to compare the total cost of different types of alternatives.

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Mathematical Techniques

1. Linear Programming-

- Linear programming (LP) models are methods for obtaining optimal solutions to problems involving the allocation of scarce resources in terms of cost minimization or profit maximization.
- With aggregate planning, the goal is usually to minimize the sum of costs related to regular labor time, overtime, subcontracting, carrying inventory, and costs associated with changing the size of the workforce.
- Constraints involve the capacities of the workforce, inventories, and subcontracting.

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Now, the other this was a trial and error method. The other method is mathematical techniques. In mathematical techniques it is quite possible that we may develop a more

accurate solution, accurate means the optimal that means the best solution for the particular situation because here we will take some limited number of inputs and we will compare only those limited number of inputs.

But our mathematical techniques, particularly LP Linear Programming, these algorithms are capable to compare infinite number of alternatives and therefore it is possible to get more accurate (30:51) to discuss an example where we will use these mathematical techniques particularly linear programming to solve such cases of aggregate plant. That how you can model this situation to solve it more accurately, thank you very much.