Production and Operation Management
Professor Rajat Agrawal
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 22 - Single Period Inventory Model- II (Numerical)

Welcome friends! In our last session, we were discussing about single period inventory models. We discussed the concept with the help of one very popular example that is Newsboy problem and here we discussed that this newsboy has only single opportunity to procure newspapers in the morning. And whatever is available, he has to fulfil the demand in that particular day, and there is no second opportunity given to him for purchasing the additional quantities.

And whatever is leftover will not be carried forward to the next day, and this is a very common example in variety of situations where you have only single opportunity to procure and with that quantity you have to fulfil your entire period's demand. Now, when we were developing this model, we came across the concept of expected profit and we discussed that we want to procure additional units as long as our expected profit is increasing. We want to maximise our expected profit and expected profit is directly dependant on the quantities which you are stocking.

So, we discussed two types of cost: one is the excess cost, another is the shortage cost. Cost of overstocking and cost of understocking and we discussed with the help of a diagram that we want to create a balance between these two types of cost. Cost of excess and cost of shortage or cost of overstocking and cost of understocking. So, you can use any of these terminologies. Now, there are two types of situations which are possible in the single period inventory model. One situation where your demand is varying continuously, your demand discrete in nature.

So, these are the two situations which are possible. In the last of the last session, we had one example and we will like to complete that example by discussions of the solution and by the concept of discrete and continuous inventory management.

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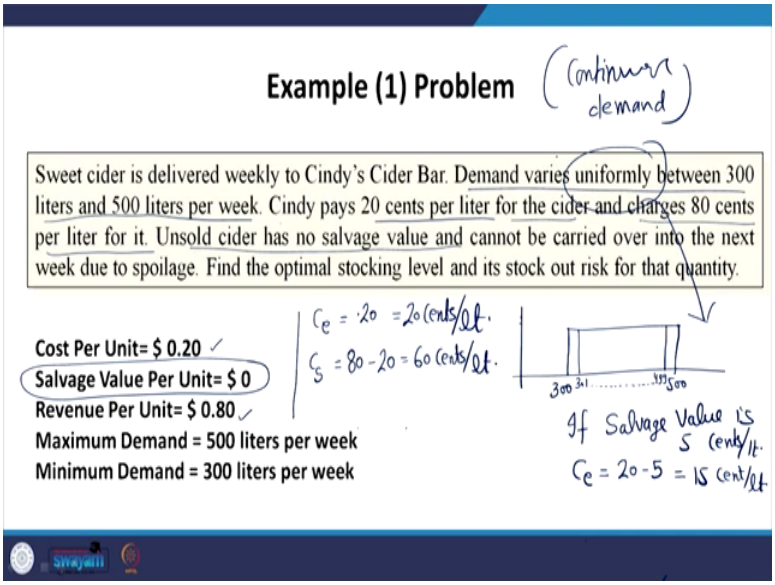

Now, in this particular example, we see that demand is varying uniformly between 300 liters to 500 liters per week. Now, for handling this continuous type of situation we discussed the concept of service level you remember in our last session. Now here the demand is varying between 300 liters to 500 liters per week.

So, this is not as simple as the case of newsboy problem, that also we will discuss but right now we are discussing the example of continuous demand. So, this problem is dealing with continuous demand and here this demand is varying uniformly between 300 liters to 500 liters. So, the graph of varying the demand is like this, this is 300 and this is 500. So, from 300 liters to 500 liters demand is varying uniformly. All levels of demand maybe it is 301 or it is 499, all these levels of demand have equal probability, the meaning of this statement is that all these levels of demand have equal probability.

In some other situation, in some other case it can be given to you that, demand varies normally, demand varies normally between 300 liters to and 500 liters per week. In that case, the curve will be like this, 300 liters to 500 liters demand varies normally. So, depending upon what type of language is given, what type of your experience is there, you can have a distribution curve of the demand in case of continuous demand.

Here, it is uniformly so this particular situation is applicable that demand is having the, all levels of demand have same level of probability. If it is normal distribution of demand, all levels will not have the same level of probability. Some levels will have higher probability and some levels will have lesser probabilities. Now, some cost parameters let us see. So, here let me remove this particular case where we have the normal distribution curve. It may create

some kind of confusion, so let us not have this normal distribution curve and we will like to go with our uniform colour.

Now, here Cindy pays 20 cents per liter for this product and charges 80 cents per liter for it. So, the cost price is 20 cents and the revenue, the selling price is 80 cent. Unsold products has no salvage value, so this is low salvage value. If there is right now you understand with this we can calculate the excess cost and shortage cost. Now, the excess cost is the cost per unit, here it is 20 pounds, 20 dollar or 0.2 dollar or 20 cents. Since salvage value is not given, so it is only 20 cents.

For an example, some salvage value if let us say salvage value is 5 cents per liter. In that case C e will be 20 cents minus 5 cents that will be 15 cents per liter. But, since salvage value in this is specifically written that, unsold product has no salvage value and cannot be carried over into the next week due to spoilage, so here the C e is our 20 cents per liter that is the excess cost. The cost of shortage, the profit which you are not able to earn, that will be revenue minus the cost price. 80 is the 80 cents is the revenue and the 20 cent is the cost price.

So, your 60 cents per liter is the cost of short supply, shortage, cost of understocking you can say. So, these are the cost parameters. Maximum demand is 500 liters per week, minimum demand is 300 liters per week. Now, let us see how are we going to solve it.

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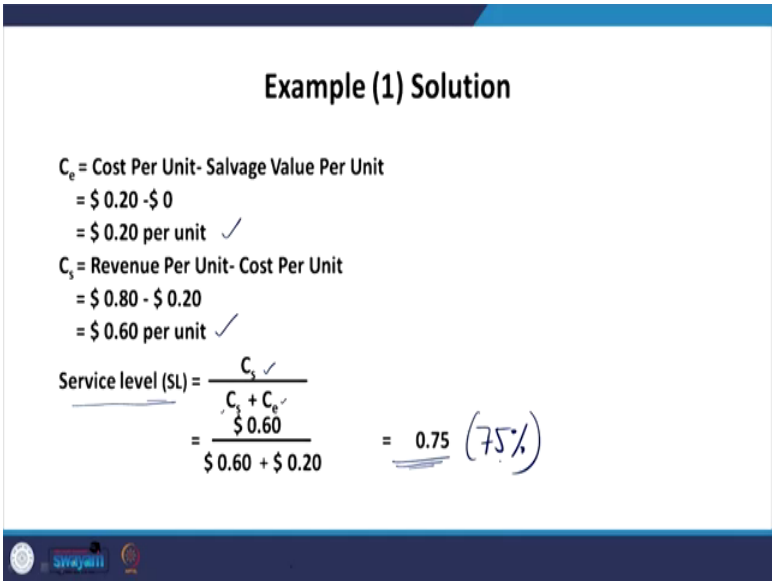

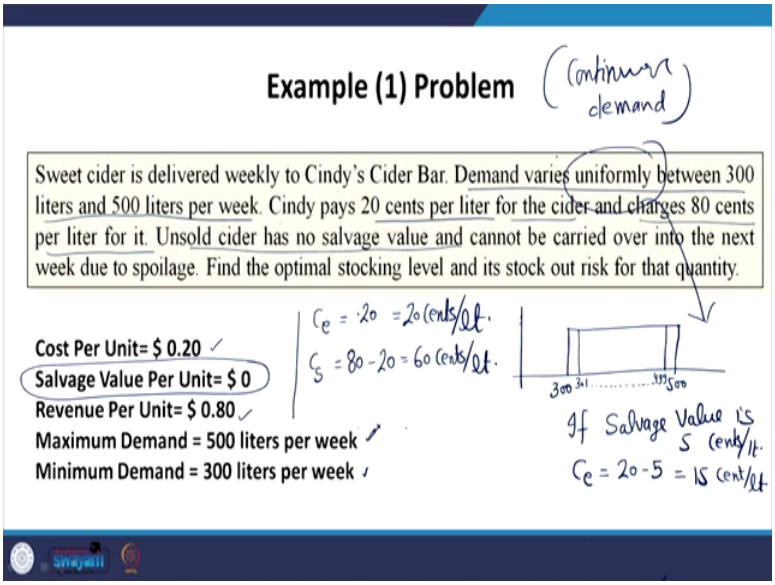

So, we have these parameters, already we have calculated these parameters. Now, the service level is, if you remember we have already discussed the formula of service level in our previous session. So, that is cost of short supply divided by cost of short supply plus cost of excess supply and this is we have already calculated these values. 60 and divided by 80 so this service level required in this particular case is 75 percent. 0.75 means or you can write it as 75 percent level. If you operate at a 75 percent service level, you will have maximum expected profit.

The meaning is that, you should not try to fulfil, the meaning is that our demand is varying from 300 liters to 500 liters. Demand is varying from 300 liters to 500 liters, so there are equal probabilities as I told you from this graph for different levels of demand. 300 liter will also have the same probability and 500 liter will also have the same probability. Now, we want to fulfil for an example, what is the meaning of service level. If I want to have 100 percent service level, if I want to have 100 percent service level that means, I am always ready, I am always ready to fulfil 100 percent demand. And 100 percent demand is 500 liters. So, every week I will stock 500 liters of products.

But, this particular calculation says that you should not try for 100 percent service level. The 75 percent service level is the most optimal level of service level for your organisation. So, you should have some stocking which is less than 500. And this is 75 percent of the amount of the maximum stocking level that will give you the full-fledged you can say profit or the maximum expected profit.

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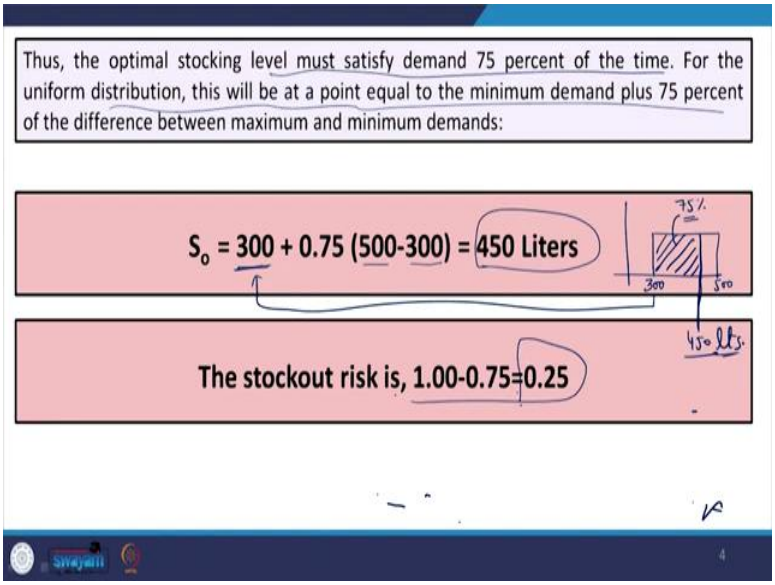

So, the answer for this question is thus the optimal stocking level must satisfy demand 75 percent of the time. You should not try to fulfil 100 percent time of the demand. You should be satisfied with 75 percent of the fulfilment of the demand because it is going to give you maximum expected profit. And for the uniform distribution we will have equal demand plus 75 percent of the difference between maximum and minimum. So, the minimum demand is the 300 and out of the distribution from 500 to 300, we are taking 75 percent of that. And again going back that curve, this is your 300 level, this is your 500 level.

So, you are fulfilling this 300 and out of this, this is the 75 percent. So, this is entire area from here to here it is 200 and 75 percent of 200 is 150. So, 300 plus 150 that makes 450 liter. So, you are going to stock this quantity, that is 450 liters and this is going to maximise your expected profit. But, you are not stocking 100 percent, you are stocking only 75 percent service level you are fulfilling. So, there are chances that 25 percent of the time you may run stock outs.

25 percent of the times you may not be able to fulfil customer's requirements. So, there is no problem as long as you are able to achieve higher expected profit. Even if you are not able to fulfil customer requirement for 25 percent of the time, do not worry. But, this calculation helps us in deciding more scientifically, otherwise intuitively we feel that, we should be able to fulfil our customer's requirement for most of the time. So, that will push, that will encourage us to stock as much as we can.

We will like to stock 480 or maybe sometime 500 also. But, that will not help us in improving our performance with respect to expected profit. So, this scientific method helps

us even though some demand will be unfilled. But it is not going to benefit in our increase in the expected profit. So, that is the case of uniform distribution of demand and uniform and continuous distribution of the demand. Now, let us see the case of discrete distribution, let us see the case of discrete distribution.

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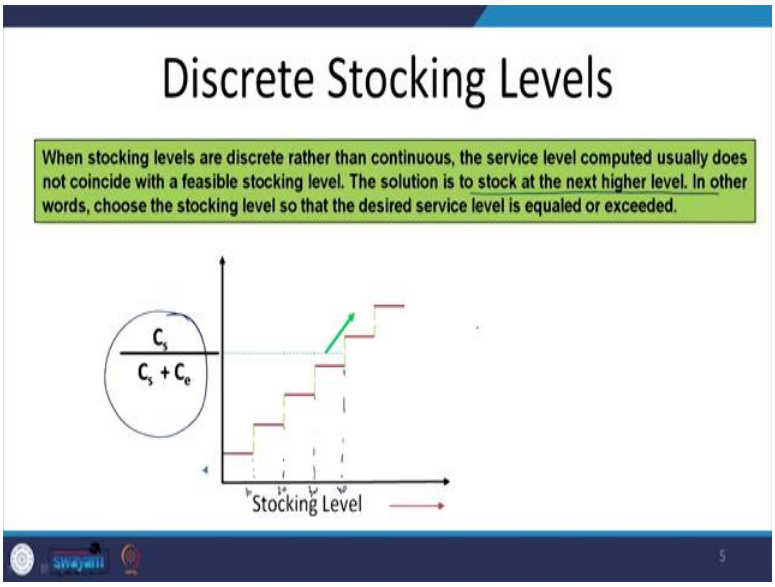

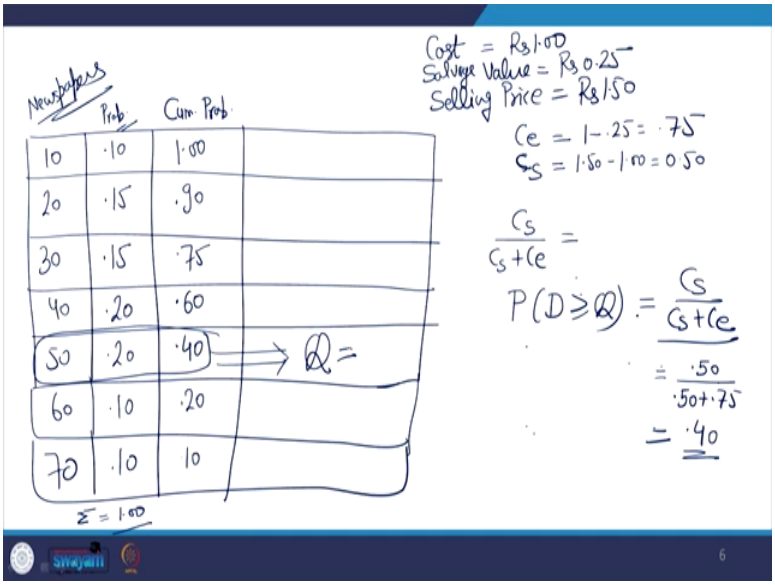

Here you have different levels of stocking, here you have different levels of stocking and here you are seeing that we have step by step levels of stocking. Now, here the question is to stock at the next higher level so that we can maximise our expected profit. So, now let us see how to use this formula in that particular case. Here we take an example and it is similar to that newsboy problem which we have already discussed in our previous class. We have let us say different levels of demand 10, 20, 30, 40, 50, 60, 70.

And these are some product, again you take it is newspapers because model popular by newsboy name. I will assign different levels of demand, different levels of you can say probability. 10, 20, 30, 40, 50, 60, 70 are different levels of demand. So, if you see this diagram, so these are those different levels of demand where you are going to decide that this is let us say 10, this is 20, this is 30, this is 40 and so on.

So, these are the different stocking levels and I have given the probabilities to these different stocking levels based on my previous experience. The only condition is that sigma of this probabilities should be 1. So, let us say I am assigning 0.10, 0.15, 0.15, it is coming some additional points here. So, let me do the corrections. So, it is also let us say 0.15 so this is 30, 40 then 0.20, 60, 70, 80, 90 and 10. So, it is 30, 40, 50, 60, 70, 80, 90 and 100. So, this is the total probability of my demand levels.

Now, the cost price as we considered in the last case is rupees 1. The salvage value let us say introduced this time is 25 paisa. The selling price is rupees 1.50. So, now let us calculate the cost of overstocking or cost of excess and cost of shortage. So, cost of excess will be the cost price minus salvage value, so 1 minus 0.25 that is your cost of excess or cost of overstocking. Cost of understocking will be the profit which you are going to have per unit. So, selling price is 1 rupees 50 paisa minus the cost price 1 rupee.

That is 50 paisa per unit is your cost of shortage. Now, in this particular case, we determined if you see this particular value. C s upon C s plus C e so this is a very important calculation C s upon C s plus C e. Now, what is this? We want to select the stocking level, we want to select this stocking level on the basis of this particular calculation. In our continuous demand case, this was giving the optimal service level. Now, here it is going to give us optimal stocking level, it is going to give us optimal stocking level in the discrete particular case.

And this formula is being derived on the basis of that by stocking additional units by stocking additional unit, the probability of demand is more than Q. Probability that demand is more than Q will be determined by this particular formula. So, the idea is probability that demand is more than our stocking quantity is, C s upon C s plus C e. So, whatever will come from this particular calculation that is going to maximise my expected profit. So, now for using this formula we need to do a bit more calculation here and that is we need to calculate the cumulative probability.

We need to calculate the cumulative probability in this particular table and here we will calculate cumulative probability in such a fashion that will give us the concept of cumulative

probability. So, for that purpose, we will not calculate the cumulative probability from top to bottom as we do normally in all the cases. Here we will calculate cumulative probability from bottom to up because in all cases whether demand is 10 or 20 or 30 or 40, 50, 60, 70, you will be able to sell 10 newspapers.

The concept is that the demand is of only 10 papers, you will not be able to sell more than 10 is 10 percent. But, when you are selling 30 newspapers, when you are selling 40 newspapers, you have already sold 10 newspapers. Then only you have selling 20, 30 or any higher level. So, by understanding that concept of cumulative probability, we will calculate cumulative probability in this particular case from bottom to up. And that is a very important point we need to keep in mind.

So, my cumulative probability column is 0.10, 0.20, 0.40, 0.60, 0.75, 0.90 and 1.000. Now, I will calculate this value, C s upon C s plus C e. So, that will be coming to be 0.50 divided by 0.50 plus 0.75 and that is going to be this calculation is 0.50 divided by 125 is coming 0.4. This calculation is coming 0.4 and here this 0.4, we will select that what is the immediately equal or higher number than 0.4.

So, here this number is coming here, I have started seeing 0.10, 0.20 and this is 0.40. So, the level of 50 is that level of Q, it is that level of Q which is going to maximise my expected profit, which is going to maximise my expected profit and now I can do this calculation if I repeat this data on my next slide.

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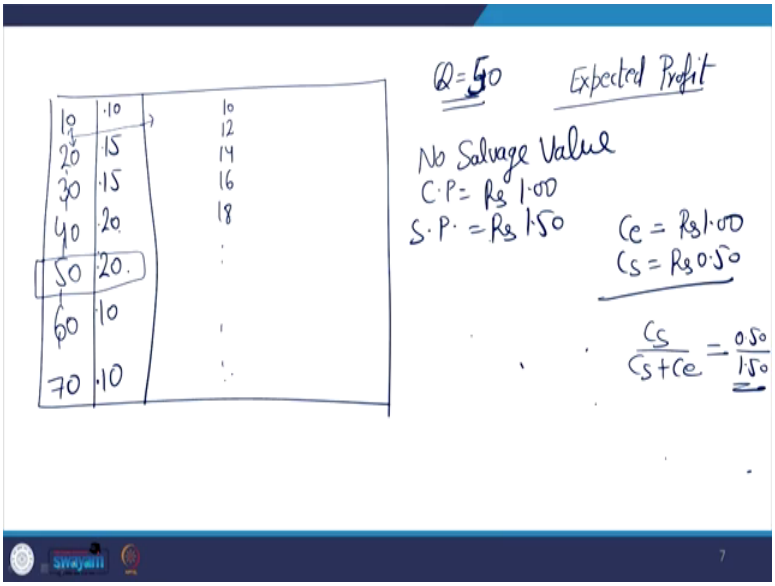

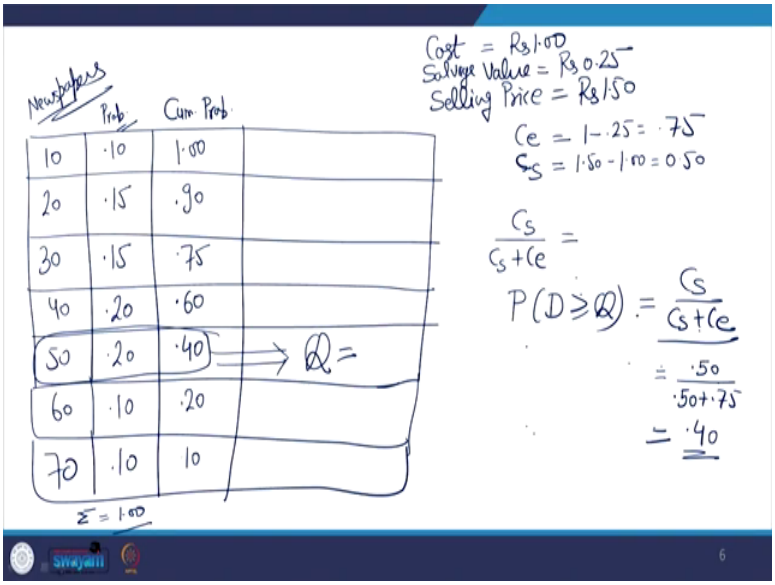

That we have determined that by keeping these different levels from 10 to 70, and different probability levels were there 0.10, 0.15, 0.15, 0.20, 0.20 and then we have 0.10 and 0.10. So, these are the different levels for demand 10, 20, 30, 40, 50, 60 and 70. Now, our answer came that we should stock Q equals to 40 that is going to, Q equals to 50 sorry. Q equals to 50 that is the answer we got. Now, for this particular level, you can now determine, you can now calculate the expected profit. This is a kind of you can say exercise we will like to do that what is the expected profit when Q equals to 50. Now, let us slightly change the data.

In this particular case my cost price was 1 rupee, I was having a salvage value of 25 paisa, selling price was 1 rupees 50 paisa. Let us see there is no salvage value, if no salvage value the cost price was rupees 1, selling price is rupees 1.50. So, your excess cost is rupees 1 only now. Shortage cost will be 50 paisa, so you can calculate your new probability level and using that formula C s upon C s plus C e. C s upon C s plus C e, you will have a new value 0.50 divided by 1.50.

And this will give you, this will give you 150 upon 0.33, 0.5 divided by 1.5, that is 0.33. So, again, almost you will have the similar level of stocking units because of the arrangement of these probabilities in such way that again you will stock 50 number of units. So, point which is there, point which is important to understand that we need to develop, we need to develop the levels at a close interval. If you want to have a better inventory model in this particular case, you need to have like in this particular case, I have inventory levels, stocking levels at a gap of 10 units.

If I have more closed inventory levels, if I have 10, 12, 14, 16, 18 and so on, I will be able to take a better decision with respect to my stocking quantity because that will give me closed

answers. Here I will be taking an answer at an interval of 10 units, so which can drastically differentiate between my stocking values. The other important thing is that, how I am having the salvage value, whether the salvage value is there or not so, that is also going to affect my decision.

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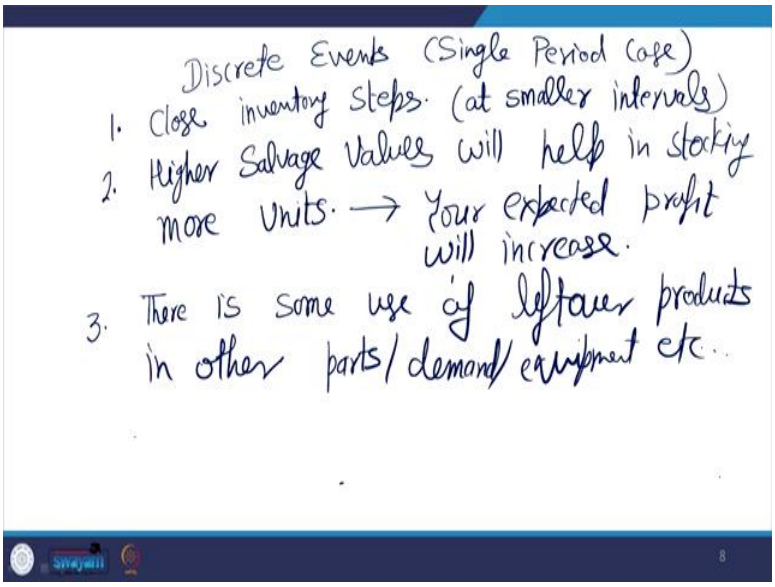

The third important thing, so now for discrete events particularly in single period case, we need to have close inventory levels, inventory steps, so that means at smaller intervals. If you have bigger intervals, you will not be able to take better decisions. Second thing is higher salvage values will help in stocking more units and therefore your expected profit may also increase. If you have no salvage value, so your risk of overstocking will increase and that risk of overstocking will reduce your potential of stocking additional units. So, that is also a problem.

The third is you need to see that there is some use of leftover products in other parts, demand, equipment et cetera, this will further increase your ability to stock more units. If you recall in our previous class, when we started development of this model for inventory management for single period, we discussed that there is no use of these items which we are procuring in single period for any other period or for any other equipment. But, if we can, because always we need to apply scientific model, scientific management systems and we also need to customise those things in our working style also.

So, if we are following this single system model where we have only single opportunity, we also need to see that how best we can use this model. So, more you stock, more expected

profit is possible in case your service level calculation allows. Now, how that service level calculation will allow it when your cost of overstocking is going down.

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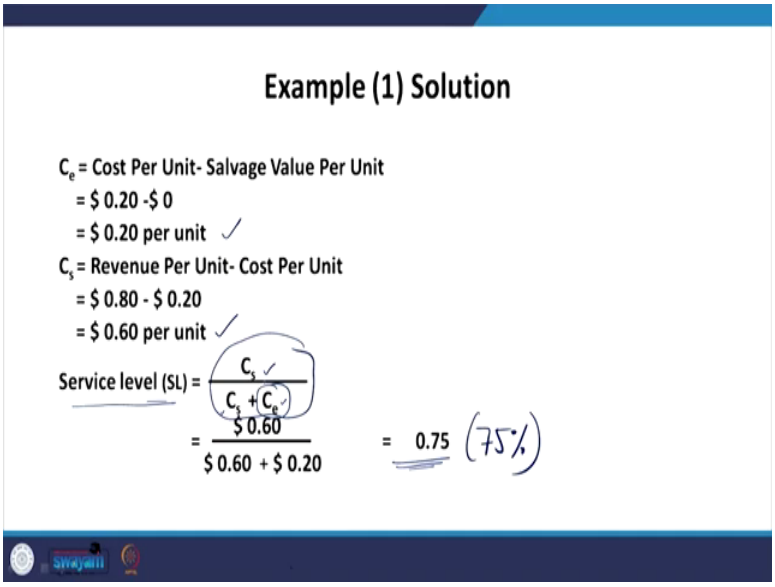

When your cost of overstocking is going down because if you see in this formula, if you see in the formula, everywhere this service level calculation, the cost of shortage is at the numerator and in the denominator we have this excess cost, cost of overstocking.

So, we can think of how to reduce the cost of excess and because you cannot increase this C s, this is not in your hand. So, because the revenue, the price which you are charging that is market determined, so you cannot charge extra price, you can only reduce your cost of purchase, so that may give you additional margin. But, the more important thing which can help you that is the cost of overstocking. If you can reduce your cost of overstocking by multiple ways, so it will be against the assumption of our single period model.

But, it can help you in reducing the cost of overstocking if you can find some alternative usage of leftover periods. And therefore, the concept which is not the part of this discussion, but the concept of circular economy is going to be very-very useful. Where whatever is leftover is going to be used as input for some other purpose and that way our salvage value is sufficiently high. And when salvage value is sufficiently high, we are having lower values of cost of overstocking or cost of excess.

And that will help us in improving our service levels, that will help us in stocking more units and by that we will be able to have higher expected profit. So, this is how the use of this single period model is not just limited to this mathematical calculation. But, this is also helpful in developing and designing the overall strategy, that how the leftover of one

particular thing will be used as input to some other process or some other equipment. So, we need to have a larger vision, considering the current scenario we need to have a larger vision for using this type of concepts in our industries so that the profitability, the competitiveness of our organisations can be increased. With this we come to end of this session. Thank you very much.