Production and Operation Management
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Lecture 21 - Single Period Inventory Model- I (Theory)

Welcome friends, in last couple of lectures we were discussing different types of inventory models and we discussed that, how we have two very important type of inventory models: Q models and P models. Q models, we discussed have the fixed order quantity and P models are those models where we have fixed interval of order and each time the order quantity may vary. So, these are two primarily different type of inventory models and either you will have a system where you will regularly order same quantity.

But, depending upon your reorder point, the frequency with which you are ordering means, the time interval between two orders may vary. On the other hand, in our P models, we discussed the case where each time your quantity may be different, but the time interval between successive orders will remain same. So, if you are giving order on each month on the first of the month, then every month you will give order on first of month. But, in first month you may order for 200 units, the next month you may order for 180 units. Then next month you can order for 210 units.

So, that is our P-type of model. In Q-type of model, you will each time order for 200 units, but it may be on first of month then it may be on 21th of month, then it will be on 10^{th} of next month. So, the date on which you will be ordering may change because of your consumption pattern. Now, we are going to discuss in this session a very special type of inventory model which is known as single period inventory model. Now, there are variety of situations where you have only single opportunity to procure. You do not have multiple opportunities to procure, you have only opportunity to procure.

And maybe it is because of some seasonal factor, maybe because of some natural reasons or you want to purchase cotton and cotton is available only in once in a year. Now, with that availability of cotton once in a year, I have to take a decision, how much cotton to procure so that I will be able to fulfil by requirement for entire year. So, there are various such cases where you have only single opportunity to procure, single opportunity to purchase the raw material. And based on that decision you have to continue for rest of the period. Now, a very popular example is known as Newsboy Problem.

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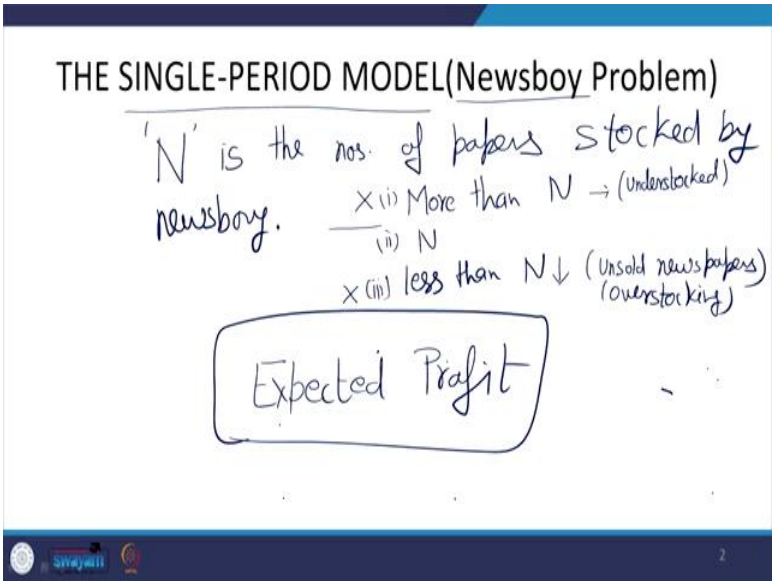

This Newsboy problem is the classical name for the single period inventory model cases. Newsboy problem is that problem, where there is a newsboy; newsboy means the news hawker, newspaper hawker who is on a bus-stand. Now, he has to decide that how many newspapers he should keep with himself so that he can maximise his profit. How many newspapers in the morning he should keep with himself so that he can maximise his profit? Now, in this particular case the issues are that, if he keeps let say N number of newspaper. N is the number of papers stocked by newsboy.

Now, there are three scenarios which are possible. On that particular day when this boy is stocking N items, demand can be either more than N or N or less than N. So, these three scenarios are possible that on that particular day demand is more than N, demand is N or demand is less than N. Now, when demand is more than N, chances of higher profits are there because you could have sold more papers to the customers. But, you are having less number of papers so you are deprived of that additional profit. The demand is exactly N, so whatever you have stocked you are able to sell that much and that is your optimum level of profit.

But, it is also possible that demand is less than N; if demand is less than N, it means you have unsold newspapers. At the end of the day, some newspapers are with you which are unsold and here you have understocked, this case is of overstocking. So, three possibilities are there; either you have understocked the newspapers or you have exactly stocked that much which is going to be the demand. And third possibility is that you have overstocked. We do not want understocked situations, we do not want overstocked situations. If you are left with some inventory at the end of the period, so that is going to be losses.

If you are understocking, so you are deprived of additional profit which is possible, so both these scenarios are not desirable. So, you have to make a balance between these scenarios that how you do some kind of stocking, some kind of stocking, that by stocking that number of items, that number of newspapers you should be able to maximise your profit. So, that is a very classical case in our single period model because this newspaper boy has only one opportunity to procure to stock newspapers early in the morning from the press, from the vehicle which is coming from the press.

So, you can say that I want to stock 100 newspaper today and if demand is of 110 newspapers on that particular day, so you can earn some additional profit if you could have 110 newspapers on that particular day. But, now since you have only 100 newspapers, you are deprived of profit from that additional sale of 10 newspapers. Similarly, on that particular day if the demand is only of 80 newspapers, so at the end of the day you have 20 unsold newspapers with you. And that is also going to be additional cost for you.

So, what we are saying that both this cost of overstocking and cost of understocking are non-desirable. We want to decide a level of stocking which should maximise our expected profit, which should maximise our expected profit. I do not know what is my exact profit. But, I will like to maximise my expected profit and now in this single period inventory management model, we will develop a formula.

We will develop a mathematical relationship which will help us in maximising our expected profit. Now, let us see what are some of the important assumptions? When we are talking of development of this single period inventory management model, let us see what are the important assumptions in this particular case.

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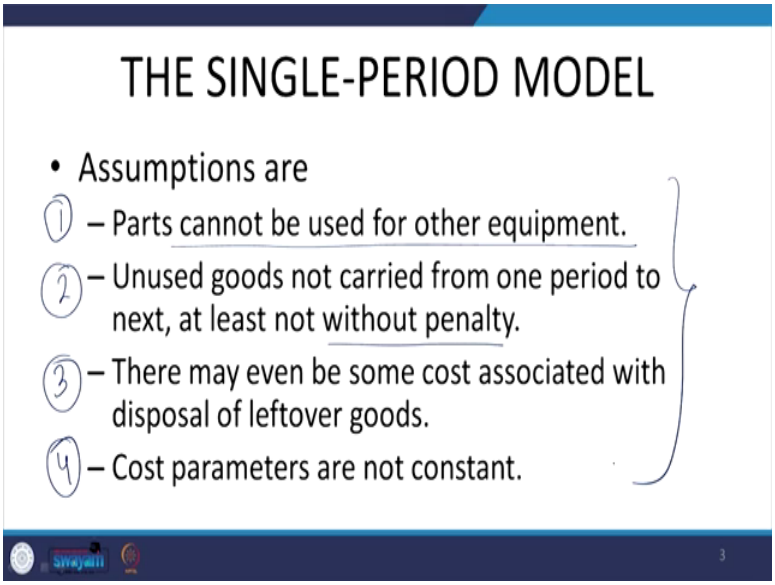

That assumption number one: Parts cannot be used for other equipment. So, whatever part you are going to have that is going to be used for that particular equipment. So, interchangeability is not possible, so that is kind of a restriction for the development of this single period model.

The second is unused goods not carried from one period to next, at least not without penalty. So, like in the case of newspaper we know that, every day you are getting a new newspaper. So, it is not possible that the newspaper of yesterday you are able to sell today. So, the unsold newspapers are waste or if you want to use that newspaper for some other purpose then there will be very little salvage value. So, that is a kind of penalty you can say, so unused goods not carried from one period to next, you have only one opportunity to use those goods, so it is another very important thing.

You have only one opportunity to procure and only one opportunity to use them. Then another important assumption is that, there may even be some cost associated with disposal of leftover goods. So, whatever goods are available to you at the end of the period; in some cases it may be possible to incur extra cost to dispose those unsold goods which are leftover goods. If you want to dispose them, maybe you have to transport them from your place to place of disposal. So, that is the additional cost you need to incur in case of these unsold goods.

So, this will further decrease your profit because you are incurring extra cost for those leftover goods. Then cost parameters are not constant. In our earlier models, we considered that cost parameters are constant. But, here in this particular, cost parameters may not be

constant. So, that is also an assumption because we will see that how the variable cost parameters are handled in this particular case. Because as you go for higher quantities your cost parameters may change because of quantity discounts offered by the suppliers. So, these are the assumptions which are there for the development of our single period inventory management model.

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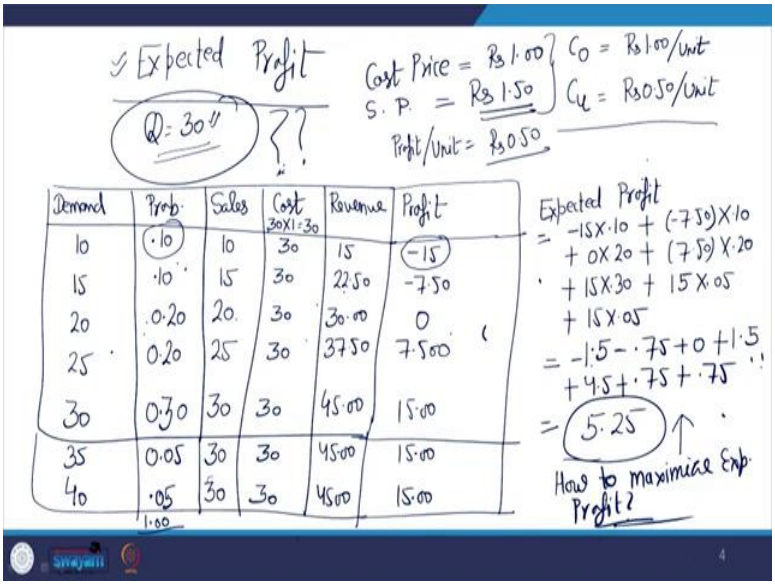

Now, to understand the concept of single period inventory management model, let us first understand what is the meaning of this expected profit. Now going back to this example of newsboy problem, now this newsboy on some intuition is stocking 30 newspapers, this newsboy is stocking 30 newspapers. Now, based on his past experience based on his past experience, this newsboy can assign different level of probabilities to different level of demand. So, I am giving you a table that is demand and the second is probability.

So, demand can be 10, 15, 20, 25 and 30. These are the different demand or let say we can add more values to it 35, 40. These are the different levels of demand which are possible and the probabilities for the different level of demand based on my past experience I will assign. So, let us say for demand of 10 newspaper on a particular day, the probability is 0.10 or 10 percent. Demand is of 15 newspapers again the probability is 10 percent. Demand is of 20 newspaper, the probability is 20 percent. Demand is of 25 newspaper, probability is 20 percent.

Demand is of 30 newspaper, probability is 40 percent. 4, 5, 6, 7, 8, 9, 10 okay, so let us change some data because then it is already crossing the…So, let me take some smaller values it is 30. Then the probabilities are 5 percent each, so the total of this 10, 10, 20, 30, 40,

60, 90 and 100. So, it has to be 1 only, it cannot be more than 1. Now, these are the probabilities I have assigned based on my past experience that on a particular day, the demand can be either 10. So, the probability is 10 percent, demand can be 15, probability again can be of 10 percent and so on.

Demand can be 40 also on a particular day, but the probability is very small that is only 5 percent. The highest probability is of 30 newspaper that is 30 percent and based on this, I am deciding to keep 30 newspapers on any particular day. Now, the cost price of the newspaper is let say rupees 1. The selling price of the newspaper is rupees 1.50. So, you can understand with this cost price and selling price what is going to happen. We are going to define two types of cost; the cost of overstocking and cost of understocking. Now, what are these two cost? If profit per unit if you see this data, so profit per unit is 50 paisa.

So, when you are overstocking newspapers and at the end of the day some newspapers remains unsold, so you are incurring a cost of rupees 1 per unit. So, cost of overstocking becomes rupees 1 per unit and cost of understocking is the profit which you could not earn. Because demand was more but you had less newspapers so you are deprived of some profit, so that is cost of understocking. Now, cost of understocking will be the profit which is not earned by you, so profit per unit is 50 paisa. So, that is your cost of understocking in this particular case.

So, we got the meaning of cost of overstocking and cost of understocking. Now, let us go for the calculation of what is expected profit. Now, to understand the concept of expected profit, let me do the calculation in this table itself. Now, the demand is given, the purchase quantity is available to you. So, how much you are able to sell? Your sales are because 10 is the demand, you are able to sell 10 newspaper. Demand is of 15, you are able to sell 15 newspapers. Demand is of 20, you are able to sell 20 newspapers. Demand is of 25, you are able to sell 25 newspapers. Demand is of 30, you are able to sell 30 newspapers.

But, when demand increases to 35, you will be able to sell only 30 because you have stocked only 30 newspapers. Demand is of 40 but again you will be able to sell only 30 newspapers as you have stocked only 30 newspapers. So, you understand that if demand is less, you are selling only that much. But, if demand is more, you are not able to sell more because you have not stocked that many number of newspapers. Now, the cost because you have stocked 30 newspapers, so cost is all these days is constant. 30 multiplied by 1. 30 is the number of newspapers and cost price of one newspaper is 1 rupee.

So, the total cost is 30. So, each day you are incurring a cost of 30 rupees. So, that is how we complete this column. Now, the revenue, on this particular day you are selling 10 newspapers, so 10 into 1.5 that is the selling price. So, your revenue on this particular day becomes 15 rupees. 15 is the selling quantity, selling price is 1.5 rupees, 22 rupees 50 paisa. 20 is the selling quantity, 1.5 is the selling price so it becomes 30. 25 you are able to sell and 1.5 is the price.

So, 37.5 then 30 you are selling, so your revenue is of 45 rupees again it is of 45 rupees, again it is of 45 rupees. So, now your profit is revenue minus cost, so on the first particular way it is minus 15, minus 7.50. Then it is 0, then it is 7.50, then it is 15, then it is 15, then it is 15. So, your expected profit is your expected profit is, the probability of happening this profit of minus 15 is 10 percent.

The probability of happening of this profit is 10 percent. So, the expected profit will be the, minus 15 into 0.10. The probability of happening of profit of minus 7.5 rupees is again 10 percent, so plus 7.50 into 0.10. The probability of happening the profit of 0 rupees is 20 percent plus 0 into 0.20. The probability of happening the profit of 7.50 is again 20 percent. 7.50 into 0.20. The probability of happening of this profit of 15 rupees is 30 percent.

15 into 0.30 plus the profit's probability of 15 is again 0.05. So, 15 into 0.05 plus 15 into 0.05, so this is your expected profit calculation. So, if I do this calculation so this becomes, minus 15 into (one point) 0.10, that is minus 1.5 then again it is minus 0.75 plus 0 plus 7.5 into 0.20. 7.5 into 0.2 that becomes 1.5 plus 15 into 0.30 that is 4.5 plus 15 into 0.05. That becomes 0.75 plus 0.75.

So, in all, the value comes this is 0.75 plus 0.75 plus 4.5 plus 1.5 equals to 7.5 minus 1.5 minus 0.75. So, it is coming to be 5.25. So, this 5.25 is the expected profit if I am stocking 30 newspapers. If I am stocking 30 newspapers, my expected profit is 5 rupees 25 paisa. Now, in the single period inventory management model, my objective is to increase this. How to maximise expected profit, that is my objective and by deciding the value of Q, what should be the value of Q which can maximise my expected profit that is my objective.

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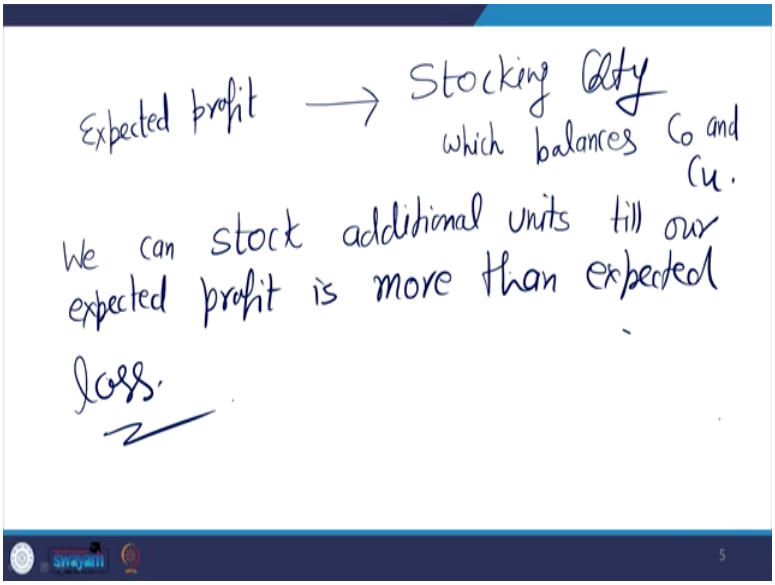

So, that is what we are going to do in this particular case that expected profit is dependent on stocking quantity. So, I have to determine that level of stocking that level of stocking which balances my Co and Cu. And here the important concept is that, we can stock additional units till our expected profit is more than expected loss. So, as long as by stocking additional units, we have Q units, 30 units in our stock.

And by stocking 30 units my expected profit in this example is coming 5 rupees 25 paisa. Now, if I decide should I stock 31 units, now by stocking 31 units, if my expected profit is more, increase in expected profit is more than the expected loss, I will like to stock 30 first unit. But, if my stocking additional units my expected profit is less, it decreases then I will not like to stock additional units. So, this becomes my basic premise for development of this model.

And for that purpose, we have a very simple formula which will help us in suggesting that how to select that particular level of quantity, that particular level of stocking which will maximise my expected profit.

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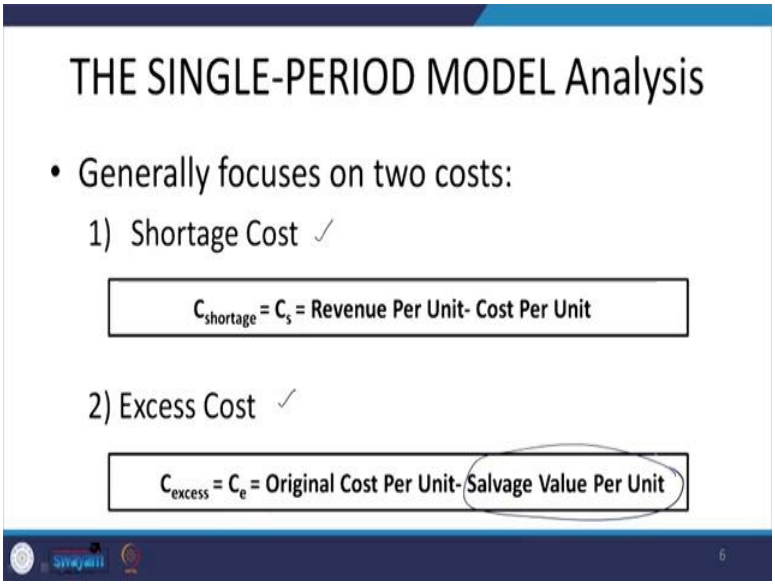

So, we have just defined these two cost that is the shortage cost that is the cost of understocking and the excess cost is also known. We are giving different types of names so that whatever book you refer, you should be able to understand our terminology. So, the shortage cost is also known as cost of understocking which I am repeatedly using.

The excess cost is cost of overstocking, when you have stocked more units then it is excess cost. Now, in this formula of excess cost here I have used a word Salvage value. In our newspaper boy, we have said that whatever newspapers are unsold will be thrown, will be waste. But, many a times you may have some kind of recovery value from your leftover items, that recovery value is known as salvage value. So, this will reduce, this will reduce your cost of understocking. If you are stocking 30 newspapers and only 10 newspapers are sold on a particular day, so you are left with 20 unsold newspapers.

So, the cost of those 20 unsold newspapers is at the rate of 1 rupee per newspaper will be 20 rupees. But, it is quite possible that you may sell those unsold newspapers at the rate of 25 paisa, 50 paisa to some scrap vendor and that is known as salvage value. So, many a times you have this kind of arrangement that whatever is unsold will be taken back by the supplier. So, in that case this salvage value will be subtracted from the cost price of that particular item. So, if we have a salvage value of 50 paisa and the cost price was 1 rupee so, in that case the cost of overstocking will be only 50 paisa. Now, as I just mentioned that we will like to balance the two cost.

We will like to balance the two cost in this particular single period model and these two cost if you recall from our basic EOQ model, in that also we wanted to balance two cost; one is the cost of ordering and another is the cost of carrying.

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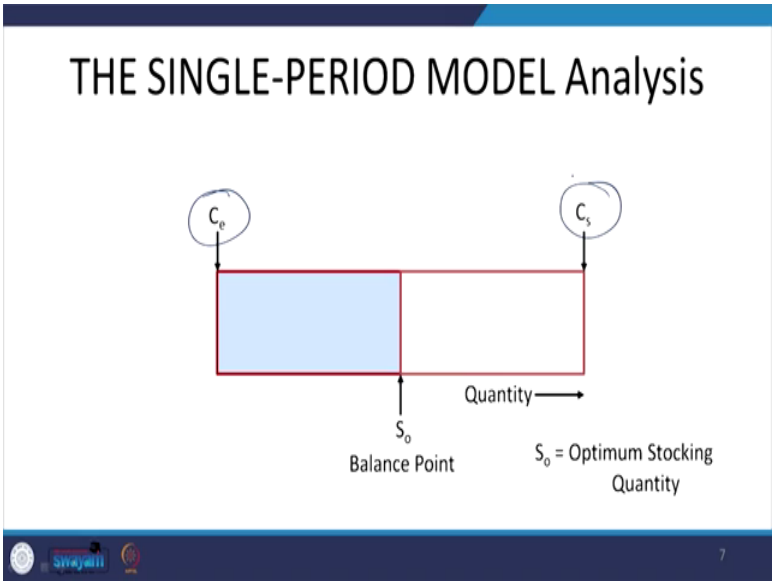

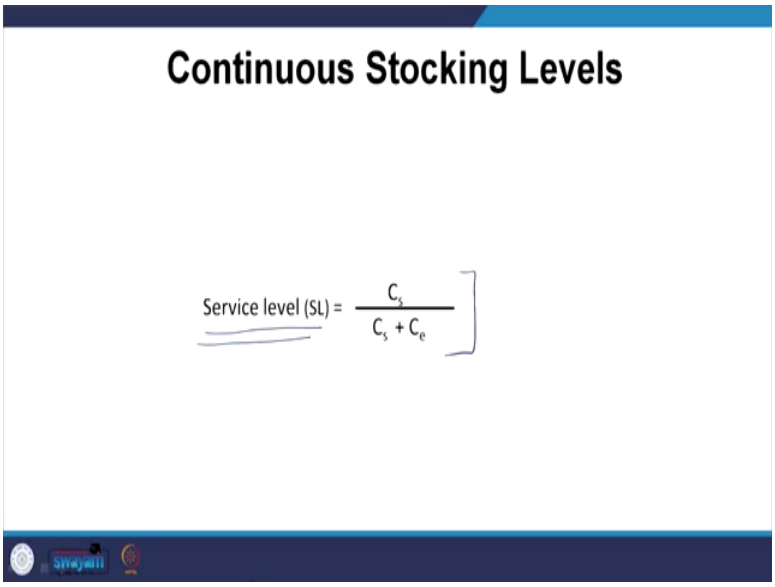

Similarly, in this particular case of single period model, we like to balance the cost of excess and cost of shortage. So, the cost of understocking and cost of overstocking need to be balanced in a single period model. So, in all inventory management problems, there are different types of cost and you are looking to have a balance between those different types of cost.

That is the basic philosophy of inventory models. Now, in this particular case of single period models we have two types of situations, where we have either continuous stocking levels or

we have discrete stocking levels. So, in the continuous stocking level we calculate the service level and the very simple formula at this stage we are not interested in the development of this formula. So, I am directly giving you the formula that the formula is this. That the cost of shortage divided by cost of shortage plus cost of excess inventory.

And with this you will determine the service level. Now, the concept of service level is that, we have already discussed this concept in earlier sessions also. But, just to refresh our knowledge that, how many times we are able to fulfil the requirement of the customer from our readily available stocks. So, you can decide, you can choose a service level for your organisation. The more service level you choose, the higher values of service level you choose, you need to keep more inventory to achieve that service level.

So, that is the simple logic all organisations want to have maximum service level, 100 percent service level. So, whenever a customer comes, customer should not go empty handed. I should be able to fulfil the 100 percent requirement of the customer. But, that will require huge inventories that will require huge inventories. So, you may choose 80 percent service level, 90 percent service level and depending upon that service level you can decide your inventory level. So, what should be the service level so that you can maximise your expected profit that is possible from this calculation.

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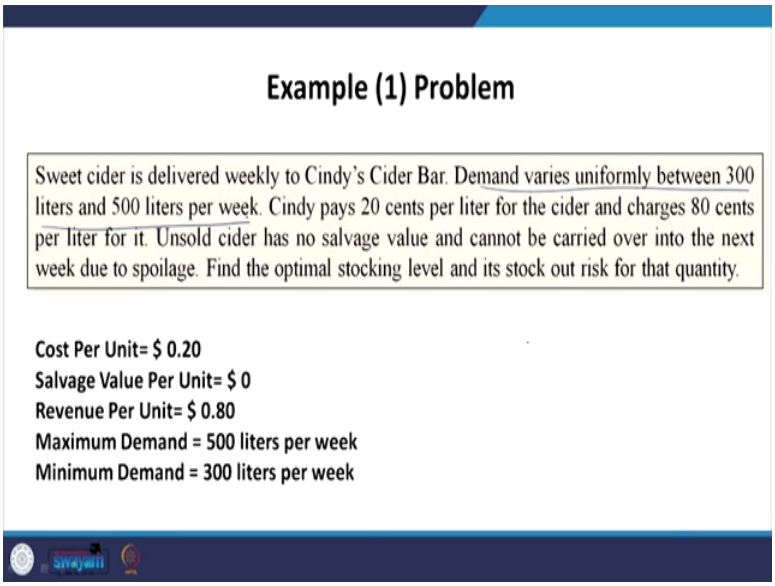

And this is going to be discussed with the help of this particular example, numerical values. So, here in this particular case, we have a demand which is varying uniformly between 300 liters to 500 liters per week. So, there are different levels of demand which are varying from 300 liters to 500 liters per week. Now, in the example of newsboy problem, if you recall we

had different levels of demand and those different levels of demand were arranged in the discrete manner. In this particular case, the different levels of demand are going to be on a continuous basis.

So, we will discuss the solution of these types of problems, where we have the continuous level of demand and where we have discrete level of demand in our coming session. So, with this we want to stop here that helped us in understanding the philosophy of single period model. And now we have understood that there are two possibilities, either you have continuous type of demand distribution or you have a discrete type of demand distribution. How to solve them that will be the agenda for our next session. Thank you very much.