Production and Operation Management
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Lecture 19
Safety Stock and Fixed Time Inventory Model

Welcome Friends, we are discussing about the inventory management models and in last few sessions we discussed about those inventory models where order quantity remains fixed and we are giving this order quantity because we are assumed that our consumption rate during the period of replenishment is going to be constant. But, we all understand that in practical situations it may not be possible to have a very constant rate of consumption, there may be some fluctuations.

And we have seen in our discussions of forecasting, that demand is always fluctuating around a base value and that is exactly going to applicable in inventory management models also. you cannot see a very constant rate of consumption, if rate of consumption is 10 units per day, it is not going to remain 10 units per day. On some particular day, it may raise to 11 then it may be 12 also, then it may go down to 8 also. So, more or less what I am trying to say that there will be fluctuations in your consumption rate.

For the sake of development of mathematical model, we assumed that the rate of consumption will be constant. But now, if we want to make these models more practically applicable then we have to introduce a concept which is known as safety stock. So, now in this particular session, we are going to discuss about the concept of safety stock in our inventory models.

And we are also going to introduce another type of inventory management models, which are fixed time interval models, which are also known as P type of inventory models. So, fixed period inventory model that is the another type of inventory models earlier we have discussed Q type of inventory model. So, these are the two very popular names in inventory management systems, P type of inventory and Q type of inventory.

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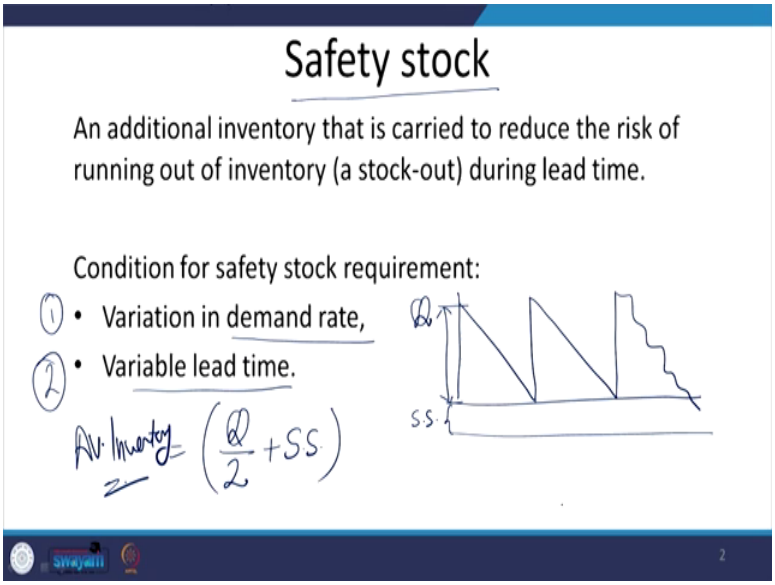

So, today in this session we will be discussing about safety stock issues and we will also introduce the concept of P type of inventory management systems. Now safety stock, as we all understand that there will be possibly variations in your demand rate, as I said that consumption rate will never be constant, we in our saw teeth model, if you remember, we had this type of very straight line for the consumption, then this straight vertical line for the replenishment.

Again the straight line for the consumption, but in reality, it will not be so, in reality, the replenishment will have the vertical straight line, but the consumption right will have this kind of fluctuations and therefore, you have requirement of this safety stock, you need some cushion, so, that you can handle this variability. And then the second important reason is lead time variation.

If you recall in our basic EOQ model, this was one of the assumption that lead time is constant. If my supplier has guaranteed me, that he will supply these products in one week in 10 days in 15 days, so, he will supply that product on that particular day, which he has promised, but due to lot of practical complications, maybes those reasons, which are beyond my control, we may have some kind of variations in lead time also.

So, if your supplier has said that he will supply in 10 days, but because of some variation if it comes to be 11 days, then again you will be running out of stock for one day. And again for the compensation for that we need to have some kind of cushion with us. So, these are two very important reasons that force us, these two reasons force us for keeping the safety stock.

Now, safety stock is that stock which is in addition to our routine requirement, which will be used, if there are some variations if our demand is increasing beyond the normal demand. So, we will take some material from the safety stock. You may be driving scooters, motorcycles, different types of vehicles in that also we have some kind of indicator.

In motorcycles in other two wheelers we have that reserve kind of switch that when your fuel level decreases to a particular level, you have to switch to the reserve category. That means, now it is the time to replenish your stocks. So, that is an indicator and whatever is there in the reserve category that is like safety stock available to you. You should ideally replenish your stock before it touches that reserved category, but because you could not do that, now, it is an alarming situation that you must immediately replenish your stock.

Same thing is applicable in this particular case of inventory management that many a times whatever is available to you up to the getting the supply from the supplier, you may not be able to have enough material and at that time you require the additional material in the form of safety stock. So, safety stock is a very-very important and in our inventory management discussion, we need safety stock, but as we have already discussed in previous sessions, that we want to minimize the total cost of inventory and safety stock in addition to what we are already keeping.

So, the value of safety stock will come below this line. So, whatever you have a safety stock and then your EOQ will come. So, safety stock is going to increase your value for average inventory, your average inventory earlier was Q by 2, we discussed that average inventory is the half of whatever is the peak inventory. Now, the safety stock in addition to that average inventory throughout the year. So, this safety stock is increasing the average inventory and average inventory is directly affecting your cost of carrying the inventory.

So, if you have more safety stock, it is going to increase the total cost of inventory. So, there is a very-very important serious decision we need to take, we want safety stock, so that we can counter these two variations, but at the same time, we do not want to increase our carrying costs too much so that it becomes an extra cost to the organization. So, how to handle that, that is a very important decision, we are going to discuss.

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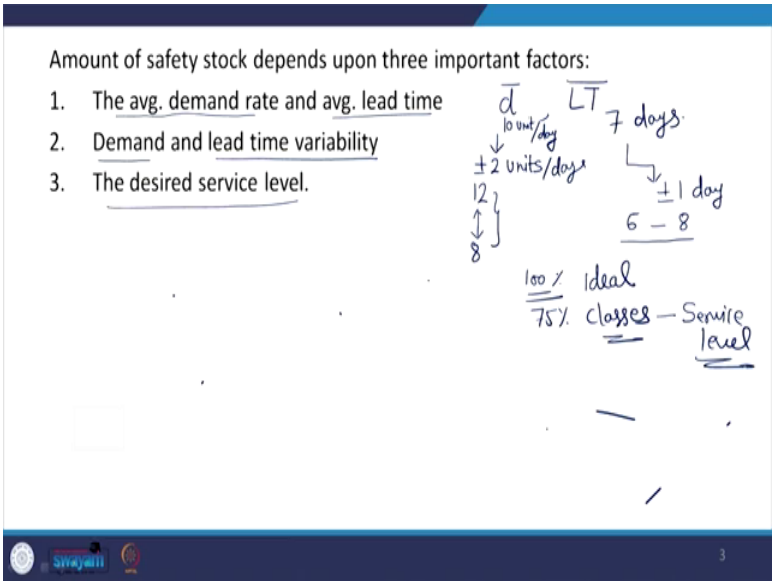

Now for safety stock calculation, how to determine the safety stock, there are some very important factors and let us see what are these important factors. The average demand rate and average lead time this is one important thing that what is your average demand and average lead time. So, these are two important things on which our calculation of the safety stock will depend.

Then, how much variations are there? How much variations in value of demand and how much variations in lead time is variations, maybe your average demand is 10 units per day and lead time is 7 days. So, in the second point, my meaning is that your demand may vary by plus minus 2 units per day. So, on any particular day, demand can go either up to 12 units or it may go up to 8 units. So, demand is varying from 8 units per day to 12 units per day, that is the meaning of variability in the demand.

Similarly, lead time is 7 days. So, lead time variability can be plus minus 1 day. This is just for our understanding purpose. Now, when I am saying that lead time is plus minus 1day, so lead time can be either 6 days or it can go up to 8 days. So, lead time can vary from 6 days to 8 days. So, that is the, if variability is more, if variability is more obviously, you need to have bigger safety stock, if variability is less, you can have a smaller values of safety stock. So, this variability directly affect the calculation of safety stock.

And the third important thing is how far you want to address this variability. You cannot address the entire variability. In our classes we may have a issue of that all the time, we want 100 percent attendance for all the students. But it is possible that 100 percent attendance is

not practically possible. This is ideal but we give permission to all those who are able to attend 75 percent classes.

So, that is practically feasible that if somebody has attended 75 percent classes, so, he is permitted to appear in the final exams. So, meaning is that you have to decide a particular feasible service level you ideally will like to have 100 percent service level, but that 100 percent service level practically may not be possible. So, you may decide a particular service level. So, with this three different type of factors, we will decide the calculation of our safety stock.

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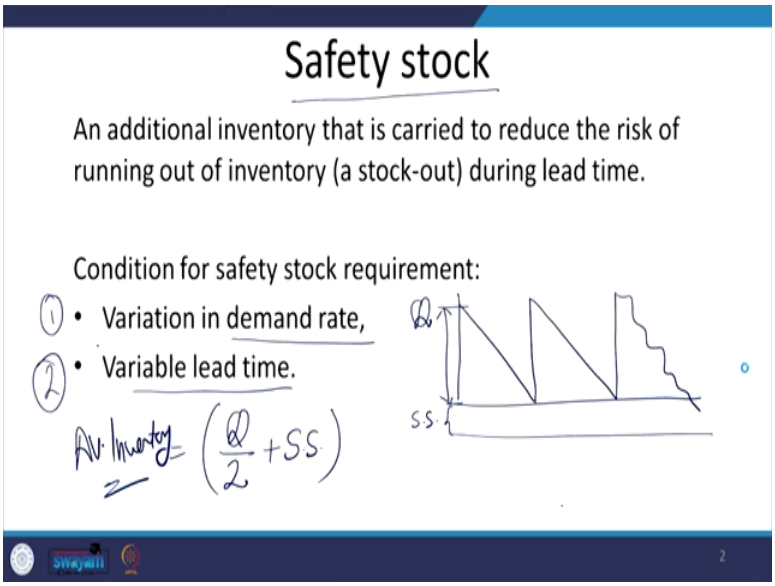

Another important thing in this calculation of safety stock is that, the safety stock means during this period of when fluctuations are there and you have a variability. So, we have assumed that the demand during this period of variability is going to behave in the normal distribution manner. Demand during this period of variability is going to behave indeed normal distribution manner. And that is going to help us in developing this formula offer safety stock.

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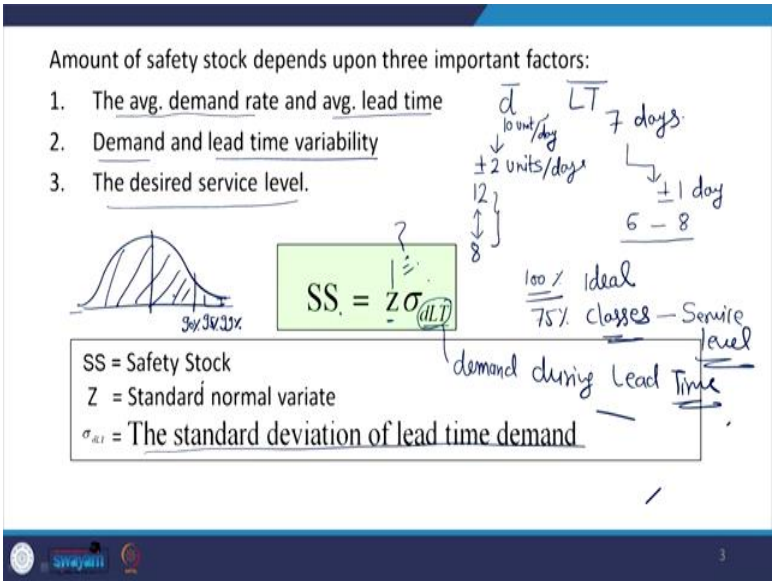

That formula is a Z into sigma dLT. This dLT is demand during lead time. The dLT is stands for demand during lead time and the Sigma value of demand during lead time is the standard deviation of lead time demand. So, dLT stands for lead time demand and the Sigma stands for standard deviation of that lead time demand and this Z is the standard normal variation. Now a standard normal (variat) we will find from our normal distribution table, but the idea is that the demand during lead time period will vary normally.

So, we will see that after understanding this normal distribution curve that we want to address how much of, so, depending upon whether you want to fulfill 90 percent, 95 percent or 99 percent how much of the variability you will like to address and depending upon that, the value of Z will be selected. So, you will go to normal distribution curve before that you will decide a particular service level and based on that service level value, the value of Z can be selected directly from the normal distribution curve. So, this is the reason of the calculation of safety stock and this is the procedure of calculation of the safety stock.

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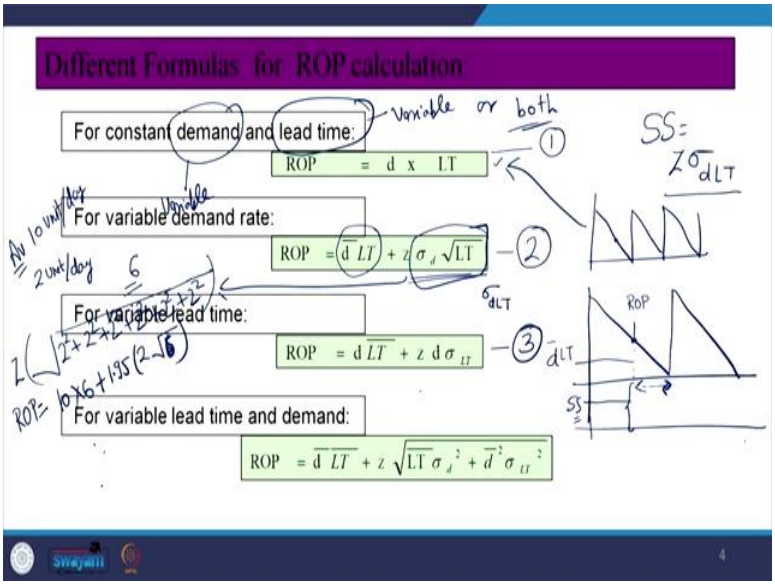

Now, we see that based on this understanding of safety stock calculation, that safety stock is equals to Z into sigma of dLT. Now, there is a very important thing with respect to this calculation of Z sigma dLT. And for that purpose we have developed different types of formulas for the calculation of our reorder point. Now, these are the four different formulas. And here we see that in the first case, we have a old situation when we have no variability, the demand as well as lead time both are constant.

So, in this particular case of a reorder point will be d into LT because there is no variability there is no requirement of safety stock and you have the exact saw teeth pattern. So, this particular case is applicable to this d into LT. Now, then there are different type of variations which are possible either your demand can be variable or lead time can be variable or both can be variable. And based on that, we have these three other formula.

Now in the first case this is the formula one. Now, the second formula. In the second formula, we have variable demand and lead time is constant, variable demand and lead time is constant. Now, in this particular case, you have two components because this is that component which is required, because whatever you are ordering you see in this particular bigger diagram so, let us say you are clicking your new order at this particular point. Now, for this particular point, this is my ROP for this particular point, whatever you are consuming during this period.

So, because of the constant demand that is coming to be d bar LT and then this is the safety stock. And this safety stock because only variable demand is there, so, here, this safety stock

that each day, each day demand is going to vary by let us say how we are going to use it, we say that each day there is a variation of 2 units per day and lead time is 6 days, lead time is 6 days. So, you have to see the application of this formula in such way that you are going to do 12345 plus 6. So, this calculation of and then you are going to apply some Z value depending upon the service level.

So, that is how you will use this formula for calculation of variable demand rate, because your lead time is fixed, lead time is 6 days. So, there is no variation in that lead time, but daily demand can fluctuate by 2 units. So, and the average demand, average demand, mean demand is let us say 10 units per day. Mean demand is 10 units per day. So, your ROP calculation will be in this particular case, mean demand 10, lead time is 6 plus let us say for some service level, my value of Z is 1.95. And then this is a calculation of sigma d under root LT.

So two times 2 square so, we are actually calculating the variance, we are calculating the variance of daily demand for 6 days and then we are doing the summation of that variance and then taking the under root, so that we can determine the value of sigma dLT this calculation is actually sigma dLT. This calculation in this particular case is sigma DLT so, that is going to become your this value of 2 into under root 6 that is how you will calculate the ROP. The third possible case is here that is going to for variable lead time. Now, in the earlier case the demand was varying.

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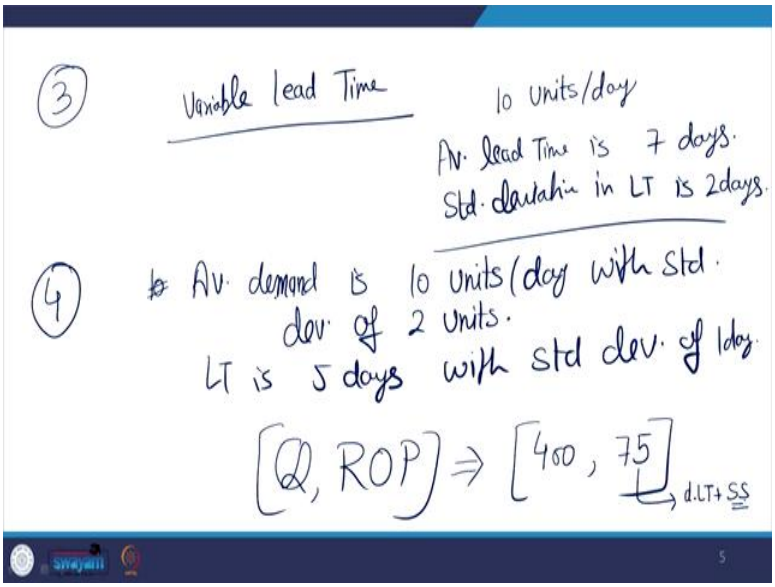

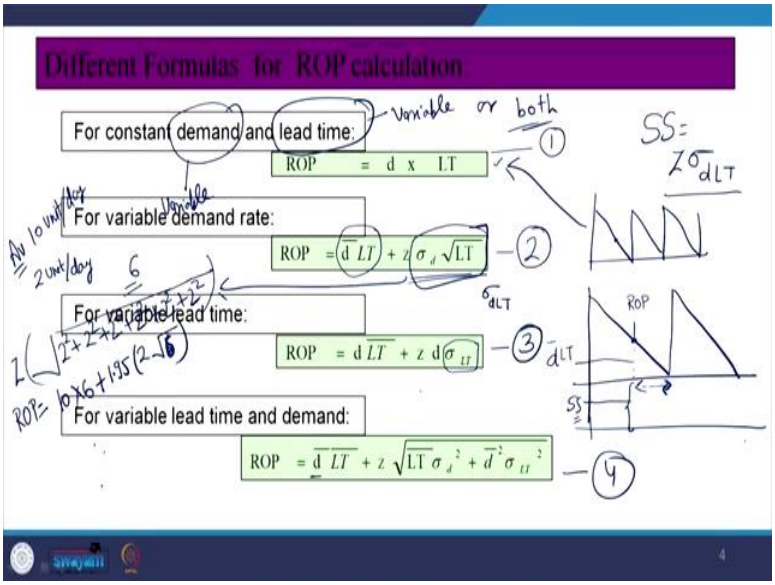

Now, in the next case when we are having the variable lead time what is going to happen, that demand is 10 units per day, average lead time is 7 days and standard deviation in lead time is 2 days. So, here we are having the daily demand as fixed that there is no variation in the daily demand that is going to be constant 10 units per day. But the lead time may vary from 5 days to 9 days. And therefore, if lead time is varying if lead time becomes more because you are not able to get supply on the 7th day, you are getting supply on the 8th day. So, even though daily demand is constant, you require more number of items to take care of this extra lead time.

And for that purpose this formula the third variation of the formula that d into LT plus Zd sigma LT that is the standard deviation in the lead time. d is the daily demand, average demand, Z is the service level and that is the value of a standard deviation in the lead time. So, this is another variation of this particular situation. Then 4th possibility is that is the most complex one, you have variation in both lead time as well as in demand. You have variation in both lead time as well as in demand.

And here you will have the situation this is the 3rd particular situation. And now, we are discussing the 4th situation where we have the variations in both the cases. So, it is that way average demand is 10 units per day with a standard deviation of 2 units, lead time is 5 days with a standard deviation of 1 day. Now, you have possibility that everyday demand is also fluctuating. Maybe on any particular day demand is either 8 units or 12 units and lead time can also vary from 4 days to 6 days.

So, you take extreme situation when your demand is also 12 days and lead time also becomes 6. So, that is going to be the situation where you will be having huge demand and accordingly your safety stock calculation takes care the fluctuation in both these things, it is average demand, average lead time. And then the calculation of this entire expression, this entire expression is a calculation of sigma dLT that is taking care of variations in lead time as well as variations in a daily demand.

So, we will do some numerical examples also in our coming session. But here we have just explained you four different types of formula for the calculation of reorder points and all these particular calculations are related to Q type of models, where we are only changing the reorder point depending upon the type of model, but our order quantity Q remains same. So, here in generic sales, you can also call these models as QROP models. We, if you remember, in the basic EOQ model class, we said that, we need answer of two questions, how much to order and when to order.

So, this Q is how much and ROP is when. So, the answers may come like this way that each time you will order 400 quantities whenever you reach 75 units. So this type of (order) answer will come for our questions that whenever quantities are 75 in your stock, you will trigger order of 400 units. out of 75, out of 75 maybe 10 units maybe 15 units, there is a component of d into LT plus safety stock. So, some units are there, because of safety stock and some units are there, which you are going to actually consume during the period of lead time. So, that is about the calculations related to safety stock.

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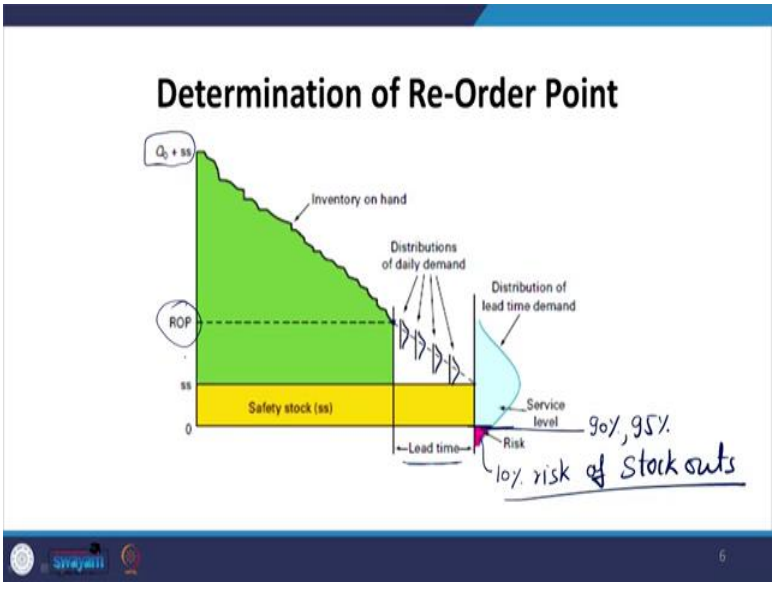

Now, after understanding this whole game of safety stock and reorder point, to summarize this discussion, we can have this neat and clean diagram and with the help of this neat and clean diagram, you will see that you are consuming some product in a fluctuating manner and when you are consuming this inventory in a fluctuating manner, here is coming that point where you are triggering a new order.

So, this is your ROP point and things which are shown in yellow color, these are your safety stock and the calculation of safety stock we have just discussed. Now from reorder point to the point of receiving the supply, this is known as lead time and distribution of daily demand as these curves are showing you, these are normally distributed, these curves are distribution of daily demand. So, these curves are normally distribution, so, you can see a bigger curve. And here in this bigger curve, we have two components. One component is in this pink color and one is light blue color.

So, this is deciding the service level, how much service level you want? Whether you want 90 percent, you want 95 percent, so, on the basis of that, you have this light blue color and this pink color is the risk area. So, if 90 percent service level you are having so, you are ready to take 10 percent risk of stockouts. So, risk is of stock out, there are chances that 10 percent of the time you may go out of stock. So, this is in a neat and clean diagram now, we have understood the concept of reorder point and safety stock.

And here also you see at this top level your inventory level is because this much is the order quantity which you are giving that is Q naught and this is the safety stock. So, at this top you have Q naught plus SS the inventory which is the maximum inventory level available with you at any point of time that is Q naught that is the economic order quantity and the safety stock.

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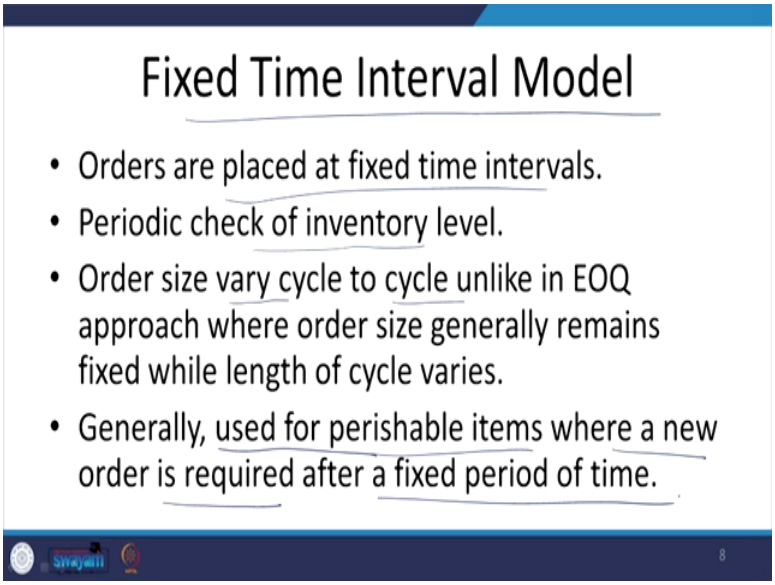

Now after going to these Q models, where we have fixed order quantities, the other type of models are P type of models. In the P type of models, which are Fixed Interval Models, here you will give order after fixed intervals, after some regular frequency that regular frequency can be decided like 15 days, 20 days, 30 days, 60 days. So, depending upon your choice, you can decide the fixed time interval for placing the order. So, what are the conditions are orders are placed at fixed time intervals, periodic check of inventory levels, order size vary cycle to cycle.

So, like we have again and again said that in our Q type of models order quantity remains fixed, but in P type of models order quantity may change from order to order. And these things are normally this type of inventory system is normally used for perishable items, where a new order is required after a fixed period of time like in case of our grocery items. So, a very common example is fixed time of inventory systems in vegetables etcetera. So, in these cases, we have this fixed time interval model.

Now, if you see the saw teeth pattern in the case of fixed type interval model, so, you see this type of diagram. Now, what is happening we have already assumed that demand is variable. If we have already assume that demand is variable. So, with this assumption, we are now going into this fixed time interval model, though initially we can eliminate the safety stock discussions and we can only discuss the fixed time interval development model. But since we have gone into the details of safety stock discussion. So, now we can directly go for the development of a advanced model in the case of fixed time interval.

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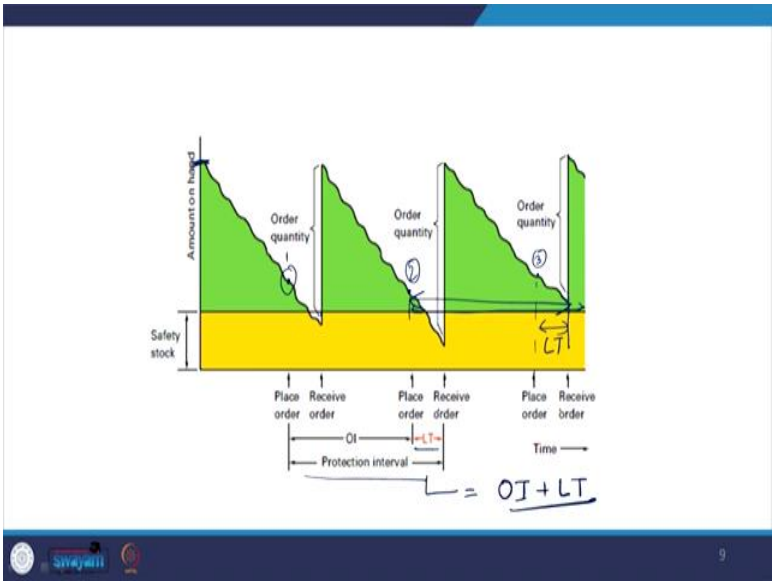

What is happening, if you see this diagram, in this diagram particularly, you have this quantity on hand at particular day, which is a combination of which you have given the order and the safety stock. Now, you are consuming these items. Now, by consuming these items, you have started consumption on the safety stock level also and you have consumed a part of safety stock also. Now when you are giving a new order, how you actually do in this particular case. So, here you are placing the order here, you are placing the order here, you are receiving the order here when the lead time period is over.

Now, the second order will be placed again on the same day when you have place the previous order. So, you have placed another order here. So, your this is OI that is the period of two successive orders. And here during these two successive order periods, you have the period of order interval and this lead time, when your second order will be delivered, will be placed, how much time it is going to take in receiving that order that is known as protection interval. So, your protection interval is equal to OI plus lead time that becomes your protection interval and you have to ensure, you have to ensure that whatever you are ordering here, whatever you are ordering at the first time this is able to take care this entire production interval.

So, this entire order interval plus lead time, your that is the period for which you should be able to take care by ordering when you are placing order at 1. When you are placing order at 2, similarly, you should be able to take care of this much period that when you will place the next order, third order, this third order somewhere here and this third order will come here. So, this is the LT. So, when you are placing an order at second you should be able to take

care for this entire duration. So, your period of production is much longer in case of fixed time interval models as compared to fixed quantity type of models.

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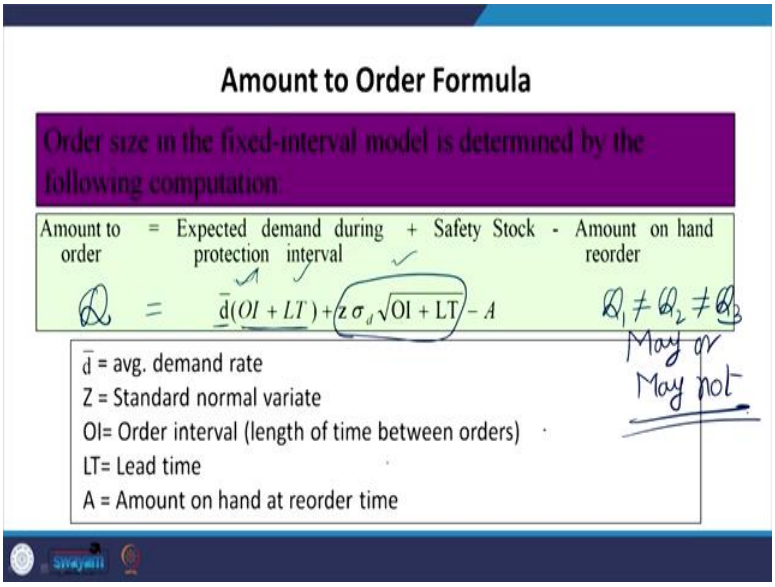

Now, how do we do the calculation in this particular case, let us see that. In these models, the formula is very simple that does not require complicated calculus to derive the formula, how much quantity you want to order, for that purpose the average demand and the entire period of production OI plus LT that is the entire period of protection for which you will calculate the normal demand plus the safety stock, plus the safety stock that what is required as a safety stock for this entire protection period.

And we have taken the generalized formula for this particular case minus inventory available on hand that A is inventory available on hand. That is the formula for calculation of how much your order quantity. Most of the time, these two things will remain constant, but inventory available on hand may vary because of variations in your consumption rate. So, inventory available on hand may vary and therefore, amount to order will also vary from order to order.

So, this quantity Q which you are going to order in this particular case will vary for different orders Q1 will not be equal to Q2 will not be equal to Q3 they may be equal also may or may not, there is no hard and fast rule that they will be equal or they will not be equal. So, both these things are possible and we will do some numerical examples to do calculations for different type of ROP calculations and calculations of order quantity in case of this fixed time

interval models in our next session. So we close this session with this formula development here only. Thank you very much.