Production and Operation Management
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Lecture 18
Different Variations in Basic EOQ Model

Welcome friends. In our last two sessions, we were discussing about inventory management models. We discussed about basic EOQ model in our last session, which is the foundation of inventory management models basic EOQ model, which actually tries to minimize the total inventory cost. And we also discussed that the formula for basic EOQ that is a very, very robust formula. So, if you are not able to apply the EOQ quantity because of some constraints, and if you are close to your EOQ quantity still you will have a very low level of total inventory cost. So, this formula is of a lot of practical significance. And therefore, in this particular session, we are going to discuss some important variations of that basic EOQ model. Because in real life, there will be different type of situations which are possible. So, how that basic EOQ model is applicable to those different situations.

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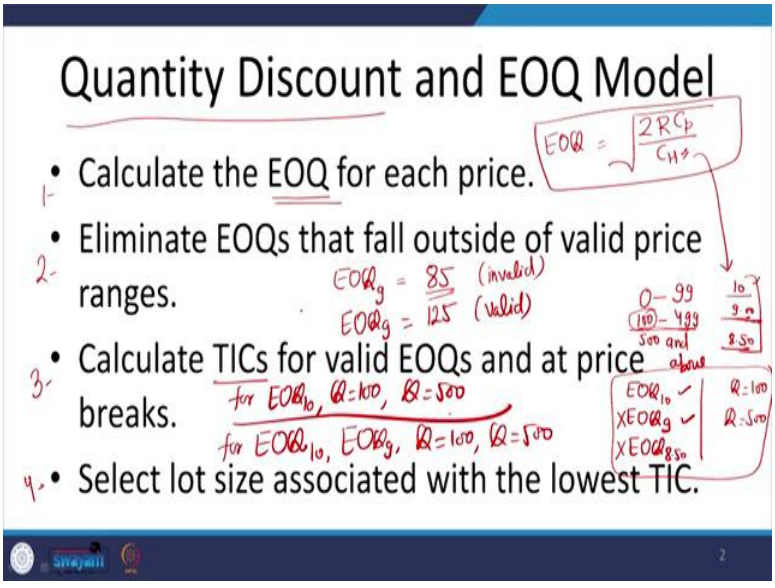

So, let us see a very simple case which is going to give you the variation of our basic EOQ model and this case is of Quantity Discount. You will have situations when some vendor will come to you and vendor will give you different types of Quantity Discounts, that if you are purchasing in large quantities, the price may be slightly less and if you purchase even larger quantities price will further be reduced. So, that type of quantity discounts we normally see.

You in your daily life can go to a vegetable shop and if you are purchasing potatoes, so, if you are purchasing 1 kg potato rates may be different, but if you say that I want 5 kg, rates may be different and if you say I want 20 kgs rates may be further different. So, this is a simple example of quantity discount. In our basic EOQ model we have already discussed that the formula was under root 2 RCP upon CH this is the formula of economic order quantity.

Now, in this formula, you are considering two cost parameters, one cost parameter is the cost of ordering, another cost parameter is the cost of holding the inventory. So, directly we are not involving the cost of material in this formula, but some time the discount given by the supplier is a very-very attractive discount. So, you want to take the benefit of that discount also. And on the other side, this EOQ formula is also there. So, now the question is how to incorporate that quantity discount in this EOQ model. And that is very much possible by slightly enhancing our calculation. So, this is a four stage process and what are the different stages, let me explain you.

First we will calculate EOQ for each price. So, let us say you have for 0 to 99 one price is there. For 100 to 499 another price is there, 500 and above some other price is there. So for each price, this is 10 rupees, this is 9 rupees; this is a 8 rupees 50 paisa. So, there are three different prizes given to us. And for each of these price, we will calculate economic order quantity.

This calculation of prize respective EOQ is only possible when your CH is depending on price. If CH is given in the absolute comes then this price dependent EOQ calculation will not be possible. when CH is given as a percentage of cost prize, then only this prize dependent EOQ will be calculated. The second thing is eliminate us that fall outside of valid price ranges.

Now for prize rupees 9, for prize rupees 9 I am calculating EOQ and the value of EOQ comes, EOQ for 9 rupees. Now, if for this calculation, EOQ comes let us say 85 units. Now for 85 units. If your order quantity is 85 then the supplier will charge you 10 rupees. So, this is a EOQ which is outside the valid price range. The valid price range for 9 rupees is 100 to 499. So, 85 is not falling in this range of 100 to 499, so, this becomes invalid.

If for this calculation of EOQ 9 my quantity comes 125 then it is a valid. So, all those EOQs which are falling outside of valid price range that I will eliminate. The third stage of this calculation is, calculate total inventory cost for all valid EOQs and at price breaks. Now, it is

possible that these three different price ranges are there 10 rupees, 9 rupees, 8 rupees 50 paisa. Now, out of these three EOQs, so now total calculations are you have EOQ for 10 rupees, EOQ for 9 rupees, EOQ for 8.50 these are the three EOQs.

And then you have two price breaks, one price break is coming when you change your order quantity from 99 to 100. So, one quantity is 100 and another quantity is when you are changing your order from 499 to 500 then another price break comes. So, as soon as you come to 500 level, you have another advantage offer price offered by your supplier. So, these are five quantity levels. EOQ for 10 rupees, EOQ for 9 rupees, EOQ for 8 rupees 50 paisa, quantity of 100 and quantity of 500, you will calculate your TICs for valid EOQ and at price break.

Now if your EOQs are invalid for these two ranges, 9 rupees and 8 rupees 50 paisa. So, you will calculate TIC for EOQ 10, Q equals to 100 and Q equals to 500. If your EOQs are valid for 10 and 9 in that case you will calculate your total inventory costs for 10, for 9, Q 100 and Q 500. So, it is depending that how many valid EOQs you are having So, those many EOQs and the price breaks, these are the number of points for which you will calculate total inventory cost. And then you will compare these total inventory cost that is the fourth stage and you will select the Lot side which is associated with the minimum total cost of inventory. Out of these total costs which you will be calculating you will select the cost which is giving you minimum total inventory costs.

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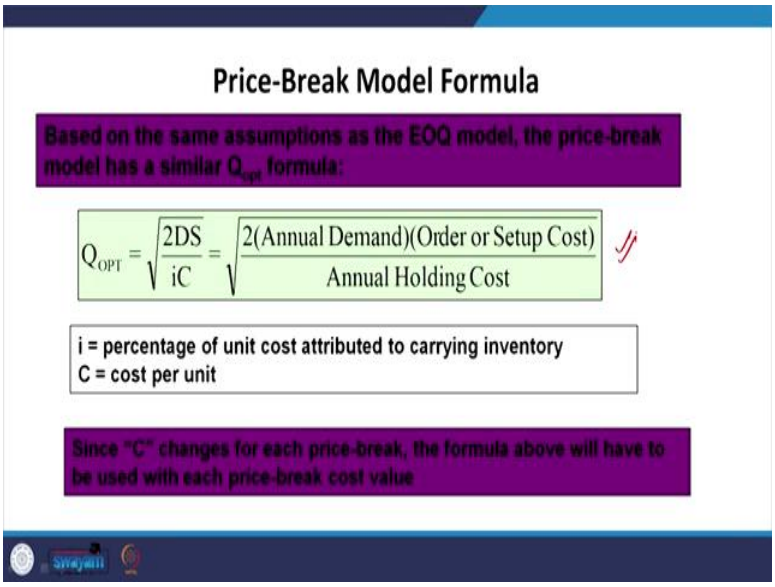

Now, we will see the practical application of these four stages with the help of an numerical. So, now, this is the same formula which we have already discussed under our basic EOQ model. So, the same formula we are going to use for the calculation of our EOQ.

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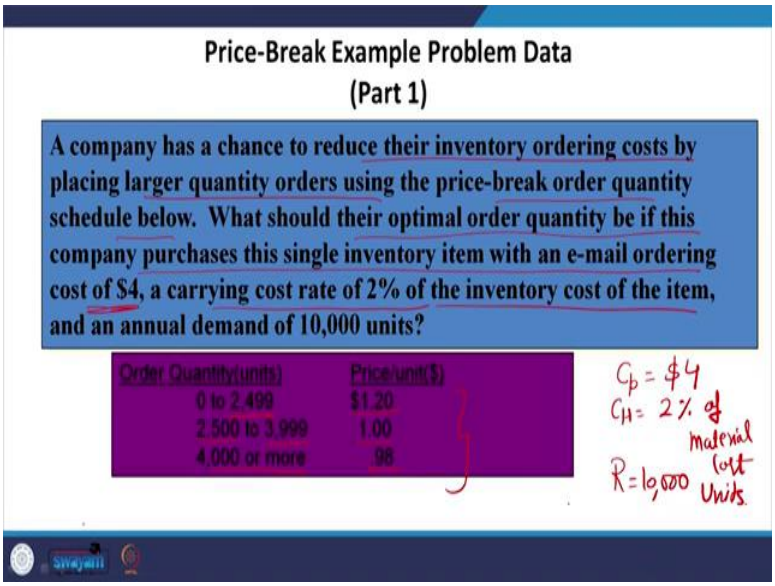

Now, we have this example and with the help of this example, we will see the application of this entire calculation of four stages. Now, company has a chance to reduce their inventory ordering costs by placing larger quantity orders using the price break order quantity scheduled below. What should their optimal order quantity be if this company purchases this single inventory item with an email ordering cost of, so, your cost of ordering that is CP is 4 dollar, carrying cost rate is 2 percent of the inventory cost of the item. Now, CH is given as 2 percent of the inventory cost of the item. So, the C 2 percent of material cost and the annual demand is of 10,000 units, the R is 10,000 units.

The schedule of quantity discounts is like this, if you are giving order of units up to 2499, if you are ordering up to this level the price will be 1 dollar 20 cents. If your order quantity is between 2500 to 3999 the vendor is ready to give you a discount of 20 cents, then you will be charged 1 dollar per unit and if your order quantity is 4000 or more, then you will further get a discount of 2 cents and you will be charged 98 cents per unit. So now this type of schedule is available to you and you want to take the benefit of this scheduled as well as you also want to reduce your inventory cost. Let us see how do we go ahead.

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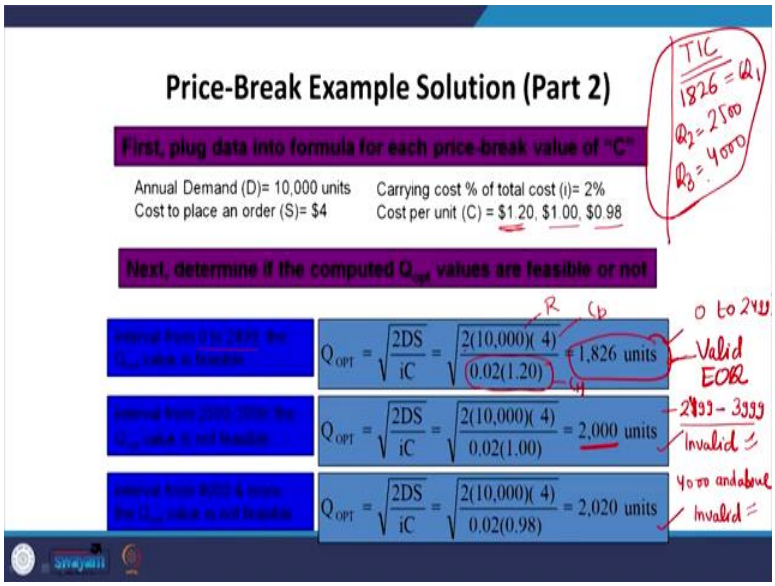

So, all these parameters we have written here. So, this is how we will calculate three inventory values. Now first is under root 2 RCP upon CH that formula we are using. So, this is the value of R, this is the value of CP and this is the value of CH. So, first is your cost price is 1 rupees 20 paisa and accordingly the EOQ is coming 1826 units. The second is, price reduces to 1 dollar the quantity coming 2000 units and in the third is 98 cents and quantity coming is 2020.

Now, if you remember the question the order quantity between 0 to 2499 in that case this value becomes feasible and 1826 is between 0 to 2499. So, this is a valid EOQ, valid or feasible EOQ. Now, second order quantity is 2000 units. Now this price of 1 dollar is applicable when your order quantity is between 2499 to 3999. Now 2000 is outside of this limit, so, this becomes invalid, this becomes invalid.

And similarly, this price of 98 cents is applicable for order quantity 4000 and above, the order quantity is 2020. So, 2020 is less than 4000, so, this is also invalid. So, out of three EOQs which we calculated for 1 dollar 20 cents, 1 dollar and 98 cents, only one EOQ is valid, that is for 1 dollar 20 cents. So, these two EOQs are invalid. So, please see that these are invalid EOQ.

So, now, we will go to next stage and in the next stage we will see that, we will determine total inventory cost, we will determine total inventory costs, we will see the formula also for that, for one valid EOQ that is 1826. So, that is one quantity and two other quantities for

which we will calculate the total inventory cost, one is that your price is applicable of 1 dollar from 2500 to 3999. So, another level is 2500 and third level is 4000. So, for these three values of Q, you will calculate your total inventory cost out of which, whichever is minimum three TICs will be available to us. So, whichever TIC will be minimum out of these three that corresponding Q will be our order quantity.

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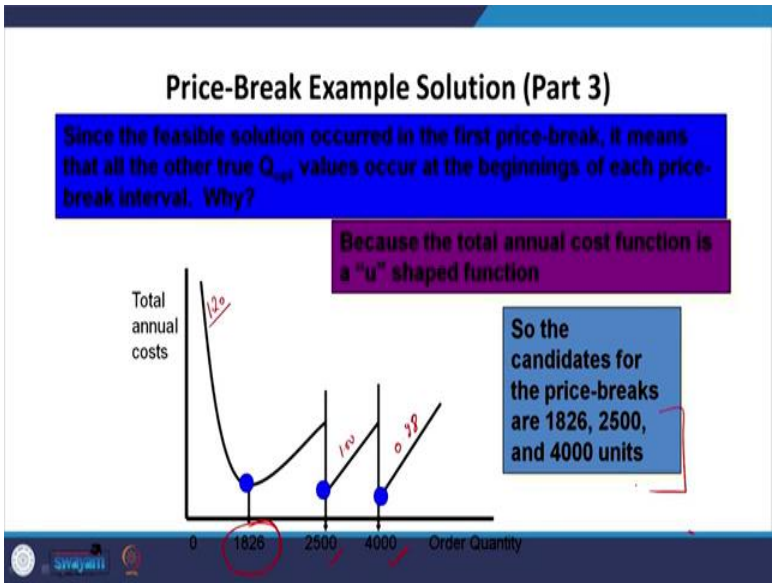

So, let us see that also. Now here you see, we have three different types of cost curves. Now, this is the first cost curve, which is corresponding to 1.20, this is the second, which is corresponding to one and this is third, which is corresponding to 0.98. And now, for these two curves, the starting point will be the order quantities which will take. So, I as suggested that for this particular curve, this will be the order quantity which we will select for calculation of total inventory costs and that for others it will be 2500 and 4000. So, that is what we will consider that three price breaks at three different levels.

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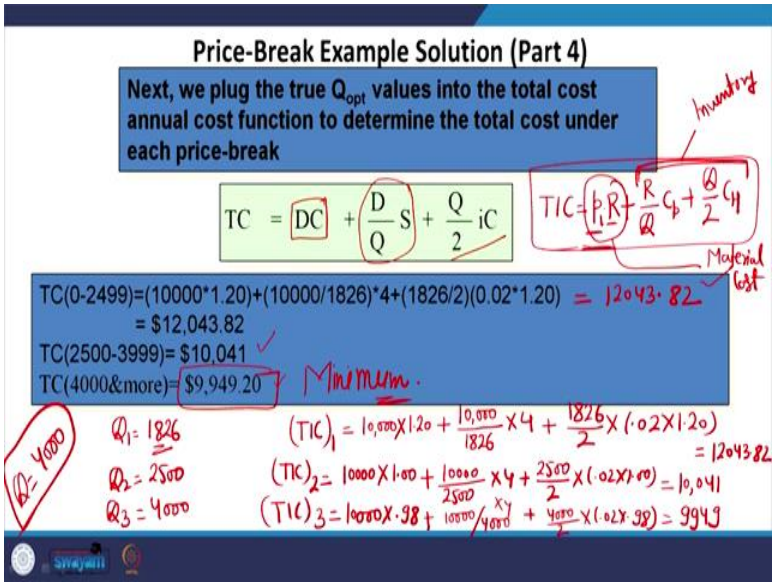

Now, the total cost which we are going to calculate. Now this will have the component of cost of the material also. So, this total inventory cost TIC will be having the cost of material Pi into R plus that is the function of your ordering cost. So R by Q into CP and this is the function of your holding cost that is Q by 2 into CH. So, you will see that these three calculations which we have done, so, TIC 1 which is corresponding to Q1 equals to 1826 and I am writing it for the benefit of you.

Our total requirement is 10,000 units. And Pi is the corresponding price. Now, since Q1 is 1826, so, the price which will be charged by the vendor will be 1 dollar 20 cents. So, this is giving you the effect of material price. Annual requirement is 10,000 divided by Q that is 1826 into CP. So, you are incurring 4 dollar per order Q is 1826 divided by 2 into the holding cost that is 2 percent of the material cost like this. So, this is the first equation and the value is coming 12043.82. So, the value of this is coming 12043.82.

Similarly, you will calculate TIC2 that is for Q2 equals to 2500 and this is 10,000. Now, for 2500 you will be charged 1 dollar per unit plus 10,000 divided by 2500 into 4 plus 2500 divided by 2 into 0.02 into 1.00. So, I have done already this calculation and this is coming 10,041 and then you will do the third calculation Q3 for 4000 and this is TIC3 and this will be 10,000 into now you will be charged 98 cents plus 10,000 divided by 4000 into 4 plus 4000 divided by two into 0.02 into 0.98 and again the calculation is already done, it is going to come 9949.

Now, these three TICs are there 12,000, 10,000 something and 9949 and when you compare these three different TICs you see that this is the minimum. So, therefore, when this is the minimum TIC your order quantity becomes 4000, you will order 4000 units to take the benefit of this cost advantage offered because of you are ordering in large quantities and at the same time, it will keep your inventory cost minimum. So, that has this use of a bigger formula.

Use of a bigger formula, these two terms in this formula, these two terms are taking care of my inventory and this term the first term, this is taking care of material. So, by combining material cost and inventory cost, we have now come up with a situation where we are trying to take the advantage of both the things and as a result 4000 becomes my order quantity. So, this is a very practical situation many a times you will see these things in case, in case the you do not have the variable kind of holding cost.

If holding costs becomes constant. So you will not have multiple EOQs, you will not have multiple EOQs you will have only one EOQ and that you will fall in any of the price range. So, you will calculate in that particular case your total inventory cost for that particular EOQ and for other price breaks. So, there is no need because if EOQ is not relative that means, if the holding cost is not relative multiple EOQs will not be there. So, only single EOQ will be there. So, single EOQ plus price breaks that will be the different levels of calculation of total inventory cost. And with this case now we have understood that how my basic EOQ model can incorporate this type of variation, where you have the quantity discounts also.

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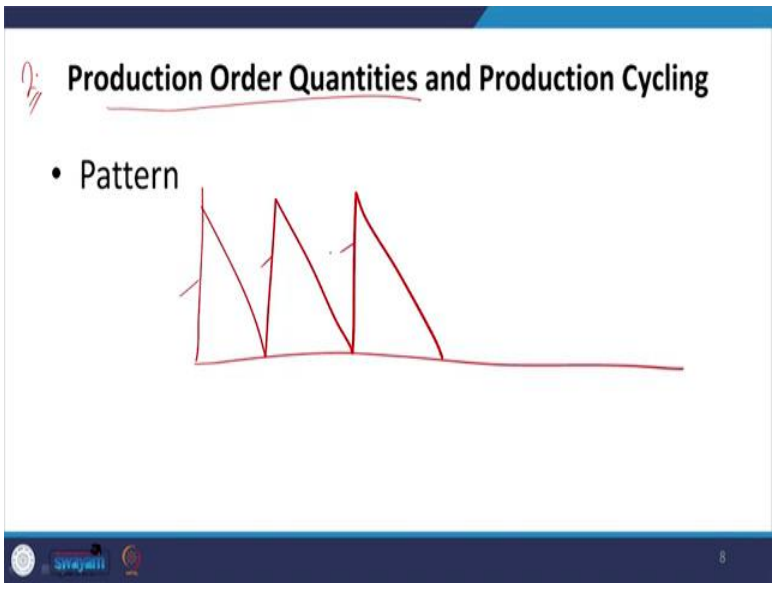

Now another variation of this particular basic EOQ model, the second type of variation we are going to discuss after this quantity discount that this can also help you in deciding the economic production quantities. Now economic production quantity, the model is slightly different than the basic EOQ model. In basic EOQ model, our pattern was like this. The replenishment was a straight line, replenishment was all these replenishment lines are the straight lines and the consumption lines were having some kind of slope.

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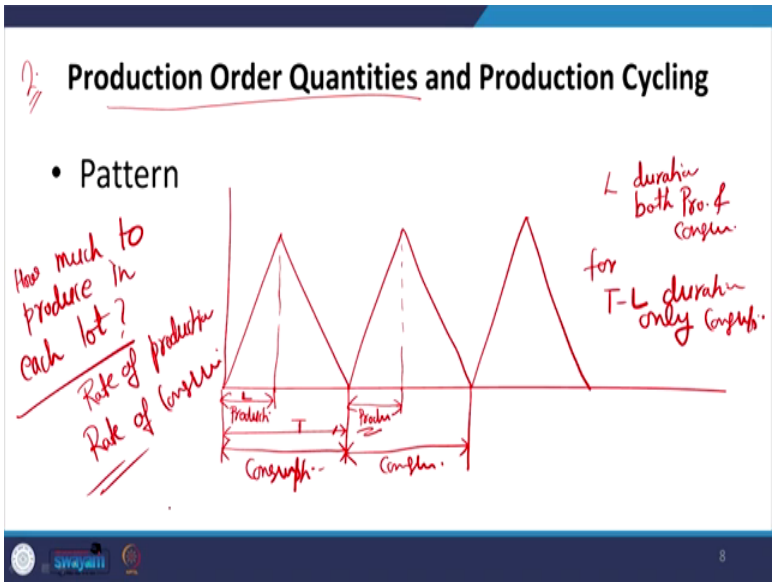

But in case of this case you will see that we will not have the straight replenishment lines here we are producing and we are consuming also. So, we have two important things, one is how much to produce in each lot because when we are producing we are consuming also simultaneously. So, it is not that you produce all the items at a time and then you start consuming them.

The only thing is that the rate of production and rate of consumption and now you can understand easily that rate of production is higher than rate of consumption, rate of production is higher than a rate of consumption. Therefore, you will not produce for entire period you will produce for a limited amount of time. So, you are producing from here to here. So here to here, you see, from here to here, you have consumption and from here to here, you have production.

Similarly, in the second cycle also from this point to this point, you have consumption and from here to here you have production. So, for some duration, if this total cycle is of T period, so, out of T, let us say this is L. So, for L duration, both production and consumption

is going on and for T minus L duration only consumption is happening. So, you need to produce in your L period, so much that you can complete the consumption of T minus L period also. So, how much to produce in that particular period so that you can complete the cycle of T period. So our Basic EOQ model can very well help in this type of situation.

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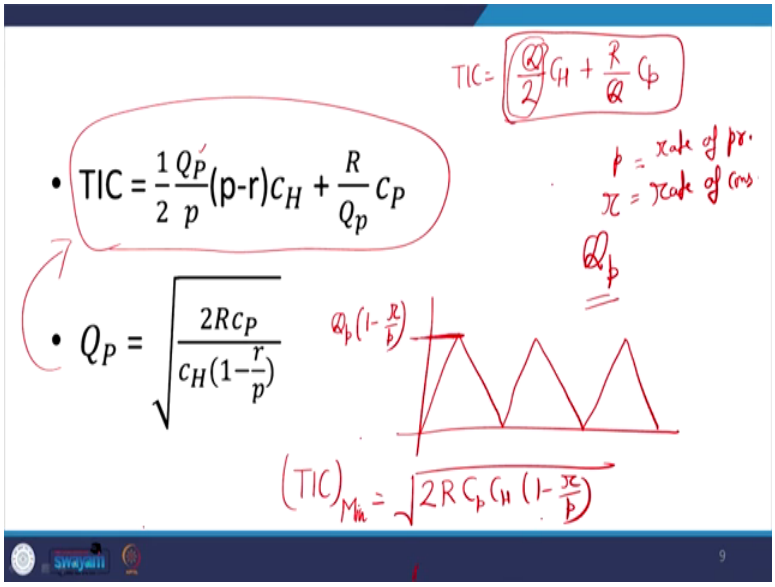

And here our total inventory cost, if I see the original formula, which was there, when we developed the basic EOQ model that was Q by 2 into CH plus R by Q into Cp that was the original formula for total inventory cost. Now, here in this particular case, we have also introduced one term that is rate of consumption and rate of production. So, p is the rate of production and this r is rate of consumption.

So, and we have to decide Qp that is the economic production quantity, Qp is the economic production quantity and that economic production quantity Qp is represented here. So, this Qp by 2, Qp by 2 is the average value and then the level of inventory because you see our curve is slightly triangular now, there is a slope available to here. So, what is the maximum inventory which is going to be there in a particular cycle that is going to be decided by this rate of production and rate of consumption and based on that, this is the maximum inventory which is going to be there if in a particular cycle, you are going to produce Qp quantity.

So, Qp into 1 minus r by p that will be the maximum level of inventory you are going to have and half of that will be the average inventory level. So, that is the logic of this first term and r by Qp into Cp that how many times you have to produce that item and doing the same process which we did for the basic EOQ model, we calculate, we did the differentiation of

this particular equation with respect to Qp and now the value of Qp coming under root 2 RCp upon CH into 1 minus r by p.

And when we substitute this value of Qp into this TIC so you can have TIC minimum also. That will be under root 2RCp CH into one minus r by p. So, this is the variation of basic EOQ model for the production quantities. So, in our production plants, when we are using Lot sizing of production system, it is important to decide how much to produce in each lot and this calculation system is good because you have producing let us say for initial 10 days in a month and then the production of that 10 days is going to work you for the entire month.

So, how much you are going to produce in those 10 days, so, that it is going to fulfill the requirement of entire month that is the kind of situation in most of the organizations which are working on lot sizing principle. So, wherever you have a lot sizing principle, this variation of basic EOQ model will be applicable. So, we will do so, some more numerical examples for our quantity discount models and this economic production quantity models in our next session, and I request you to ask more questions with respect to quantity, discount, and economic production quantity models on our discussion forums. Thank you very much.