Production and Operation Management
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Lecture 17
Basic Inventory Model

Welcome friends, in our last session, we started discussions about Inventory Management and Inventory Control. We discussed that what are the symptoms of mismanaged inventory in an organization. If your organization is small or maybe medium sized lot of intuitive decisions are possible with respect to inventory management. But as the size of organization grows, it will not be a good idea to have intuitive decision making. Then you need to have scientific management of inventory and when we talk of scientific management of inventory, the two important questions which we discussed in our last session, that how much to order and when to order, we need to know.

If we have a proper should you have our order quantity and the time of ordering, then we can have an inventory management system, which will give us minimum cost of inventory and it will also help us in avoiding interruptions in our production cycle. It will avoid over stocking of the inventory material. It will avoid the stockouts of our material. So, the objectives which are, conflicting objective will simultaneously be satisfied if we have a proper scientific inventory management system. To start the development of scientific management of inventory we have a basic inventory model and we started discussions on that basic inventory model in our previous session by the name of EOQ model. Now, we will go into the details of that basic EOQ model.

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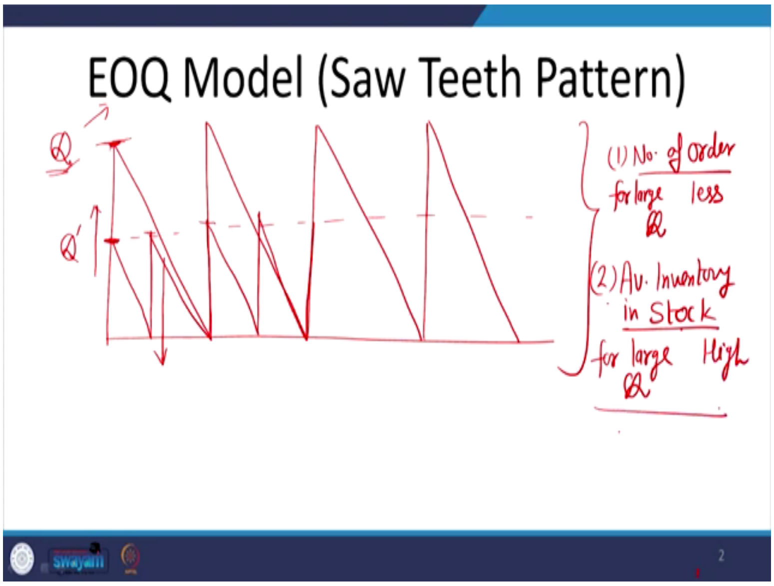

If you remember, in our last session, we discussed that we have a saw teeth pattern of inventory consumption and inventory replenishment. So, we get inventories, this is the replenishment of inventory and this is the consumption of inventory. Again, replenishment will take place and then you are going to consume this inventory. And this is a saw teeth pattern which will emerge for the inventory system in your organization.

So, if you have let us say hundred items in your stock, so you will have hundred different types of saw teeth pattern for those items. So for each item there will be a unique kind of saw teeth pattern. Why I am saying this unique kind of saw teeth pattern? Because the height of these teeth, height of these teeth is determined as Q that is the order quantity, this Q is the order quantity which is nothing but this economic order quantity.

Now, if I reduce the value of Q, the height of our teeth will also reduce. For an example, if I have a different type of Q value, which is let us say Q dash. Now if this Q dash is the inventory quantity which I am ordering, maybe this will be new pattern which will emerge. So, this is, now you see, the peak has reduced, earlier Q is there. So, this was the peak. Now Q dash is the order quantity, so, this is the peak. Now this will directly affect. This will directly affect two important things, one is it is going to affect the number of order.

If you have higher value of Q, your number of order will be less. The second important thing is average inventory in your stock. Now when you have larger value of Q, for large Q the average inventory will also be high. So, you see, it is a very conflicting case, when I increase my Q my number of orders will reduce, but the average inventory will increase. When I reduce Q, my number of order will increase, but the average inventory will reduce. So, that is we want to balance in our basic EOQ model. Now, when we are applying this basic EOQ model, there are certain assumptions which we need to know. So, what are these assumptions which are very much necessary for application of this basic EOQ model?

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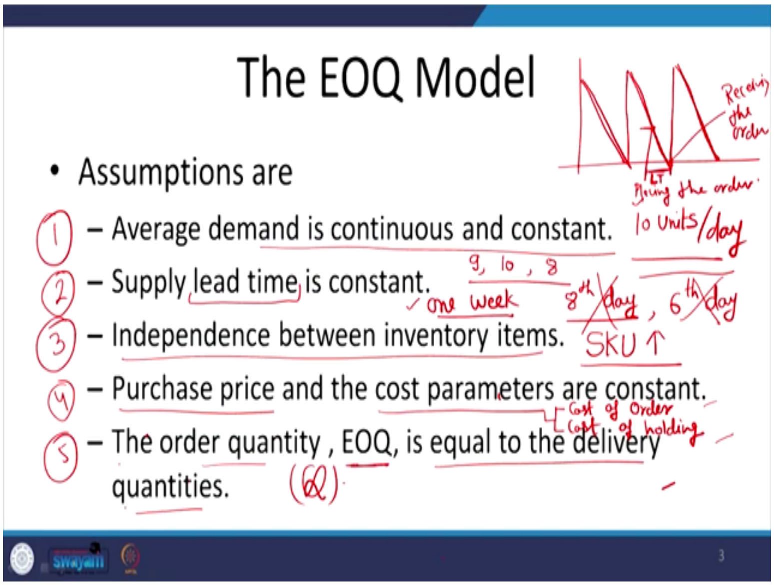

Now, again let me have this saw teeth pattern here and this will help us in understanding these assumptions. Now average demand is continuous and constant this is assumption number 1. So, what I am trying to say that the slope of these lines, slope of these consumption lines is constant and it is continuous. So, you have the same rate of consumption for the entire period, let us say for what I want to say 10 units per day. So, this is my rate of consumption. So this is constant, and it is continuous.

So, that is the first assumption that there are no fluctuation in by consumption rate. In practice it is possible that on one particular day, you are consuming 9 items, another day you are consuming 10 items, then another day you are consuming only 8 items. So there may be some fluctuations in real life. But for this model purpose, I am assuming these straight lines and that is the constant value of consumption rate.

The second is supply lead time is constant. Now what is lead time? Lead time is the time between when you are placing an order and you are receiving the supply. So, when I am at this level of inventory, when I am at this level of inventory I am placing the order and when I am reaching the 0 level, I am receiving the order. So, this time, this much time is my lead time, this much time is lead time. So, lead time is constant.

In practice, it may have some kind of fluctuations, but here we are assuming that lead time is constant. If lead time is not constant, if sometime let us say lead time is one week for one item, lead time is one week means seven days. Now, because of fluctuations if order comes on 8^{th} day. So, for one day I will be running out of stock, there will not be any material with me for one day it will disturb my entire production process or if inventory comes on 6^{th} day because of early supply, so, this will increase my average inventory level. So, both these cases are not there, I am assuming. So, the lead time is constant.

Third is independence between inventory items. We are actually having large number of SKUs. Number of SKUs are continuously increasing. And all the items which I am giving for my stock, they have no interdependence, they all are having independent supply. So, supply of one item is not dependent on supply of other item. So, that is the independence between inventory items. So, there is no interdependence, all items are having exclusive supplies, then fourth is purchase price and the cost parameters are constant.

The price of the product which I am purchasing and all other costs parameters and these costs parameters are particularly with respect to two things, one is cost of ordering and another is cost of holding. So, these are the two important cost parameters we consider in our basic EOQ model. So, these two cost parameters are constant. So, each time whenever I am giving order, so, if the cost of order is each time hundred rupees, so, it will be hundred rupees again and again throughout the year.

If cost of holding one unit for one particular ear is two rupee, so, it is constant throughout the year, so, that is the cost parameters and the purchase price that is the material price that is also constant. Then, another important assumption is that the order quantity that is the EOQ, the order quantity that is EOQ is equal to the delivery quantities. That means, there is no short supply, whatever you are ordering whatever you are ordering, you are ordering EOQ quantity or Q quantities. So, you receive the supply of Q quantities otherwise, it will again disturb your EOQ model. So, these are the important assumptions which we are considering for developing of the EOQ model.

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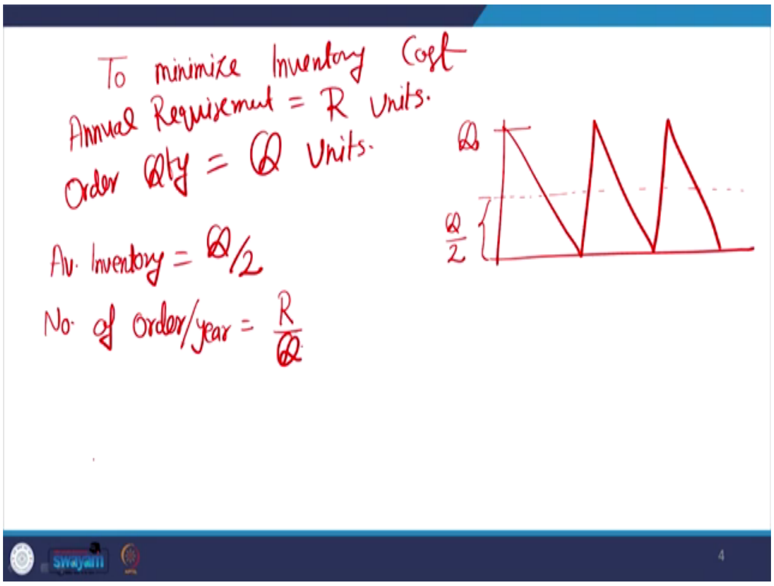

Now, based on these assumptions, and the basic idea is to minimize our inventory cost. The idea is to minimize the inventory cost. Now, if my annual requirement is R units let us say the order quantity is Q units. So, I have to determine two important things. One is when I am having the order quantity of Q, so this is, Q. Now, you can understand that this Q by 2. So, every inventory in my stock is Q by 2 and number of orders which I need to make in an year will be R by Q. So, every year I will make R by Q number of orders and my average inventory will be Q by2.

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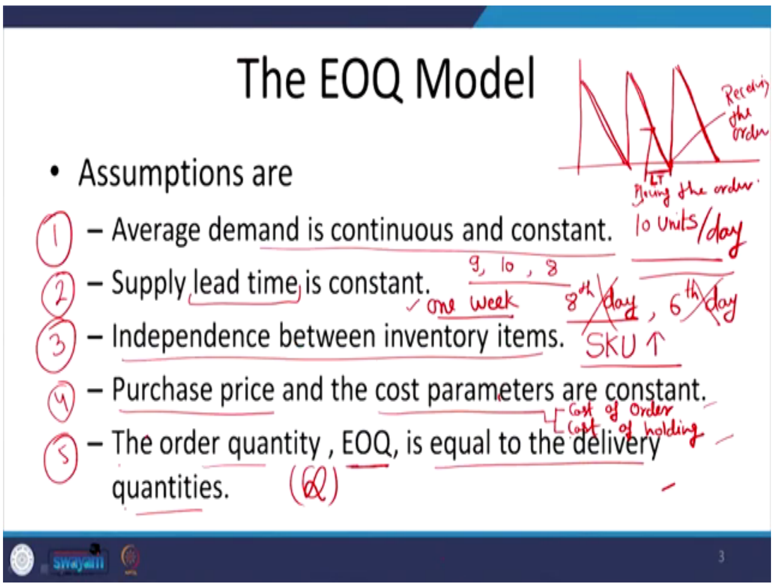

Now, I am giving you two cost parameters. In our previous slide we discussed when we were discussing the assumptions that cost parameters are constant.

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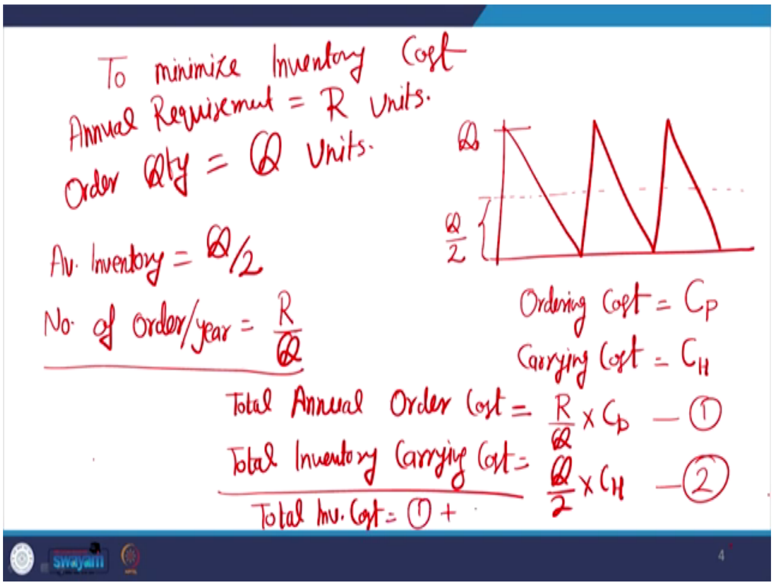

So, one of the cost parameter is the ordering cost and the second is carrying Cost. Now cost of placing an order is determined as CP and it is known as CH. So, my overall cost of order total annual order cost will be simply number of orders per year multiplied by order cost per order. So, this will be R by Q into CP and the total inventory carrying cost will be Q by 2 into carrying cost that is CH. So, that becomes my two important costs, one is R by Q CP, R by Q into CP and the second one is Q by 2 into CH. Now my total inventory cost, total inventory cost will be actually sum of 1 and 2 total inventory cost will be sum of 1 and 2.

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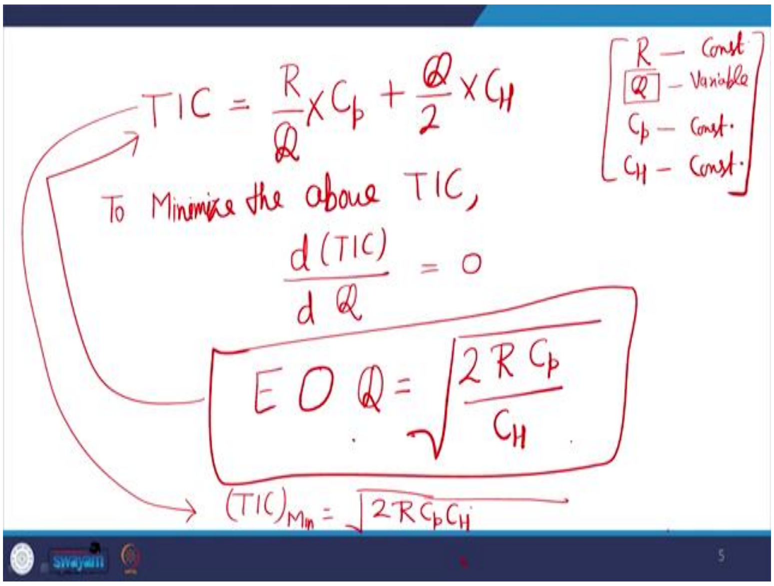

And therefore, I can write TIC equals to R by Q into CP plus Q by 2 into CH this is my total inventory cost. And I want to actually minimize this total inventory cost. To minimize the above TIC, we need to apply some calculus and here you see out of various terms, which I have written in this above equation Q, R, CP and CH. there are four terms R, Q, CP and CH these are the four terms this is a constant value, this is a constant value and this is a constant value. So, only thing which is variable is this Q. So, we have to differentiate this expression of total inventory costs with respect to Q and put it equals to 0 and by doing that simple calculus, we will find that Q equals to under root 2R CP upon CH.

So, this is the formula for economic order quantity and this becomes our base formula for rest of the inventory management discussion, economic order quantity is equals to underfoot of have 2R CP upon CH and when I put this formula in this expression, then the value of TIC will be minimum, when I put this value of Q, what I am trying to say when I put this value of Q in this expression, the value of Q, value of TIC will be the minimum and by putting the value of Q equals to under root 2R CP upon CH here, the value of TIC will be under root 2R CP CH that is the minimum inventory cost you are going to have. So, this is the development of this formula of, now, you see how it is going to help us.

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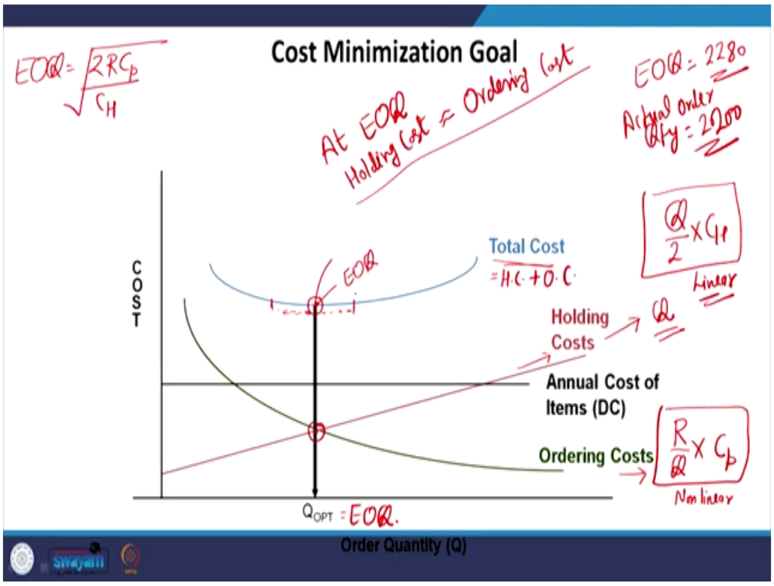

In this slide, we are able to see the practical implication of this formula. Now, here you see on our y axis we have taken the cost on x axis we have taken the order quantities. Now as your hold order quantities are increasing, as the value of Q is increasing, your holding cost is increasing, it is increasing with Q. So, as your Q is increasing your holding cost is increasing.

As your Q is increasing, because number of orders per year is decreasing. So, this curve is showing the decrease, the value of ordering cost is decreasing with increased value of Q. So, holding cost is increasing and ordering cost is decreasing. Now, you will see that the expression of holding cost was Q by 2 into CH. So, this is a linear relationship, linear. So, therefore, this is a straight line, the holding cost curve is a straight line because this is a linear relationship.

The ordering cost relationship if you remember is R by Q into CP. Now here Q is coming in denominator, here Q is coming in denominator. So, this is a nonlinear relationship and therefore, you see this curve of ordering cost is not a straight line, it is a slightly curved, shape of curve. Now, based on this holding cost and the ordering cost when I combine these two things, I get this total cost curve. This is, actually the sum of holding cost plus ordering cost. So, this is a curve which is combining both these costs.

Now, you have seen that this ordering cost and holding costs are intersecting at this particular point. And this total cost curve is also coming at its minimum value at the point of this intersection of these two cost curves. So, it gives you a very interesting insight. It gives you a very interesting insight. That insight is that at EOQ, at EOQ, your holding cost is almost equal to ordering cost. Theoretically it is equal to, but in practically there may be some difference of paisa or cents but at EOQ holding cost is equal to ordering cost. That is one very important thing, which we come to know that both these costs almost become equal at the point of EOQ and that is giving you the total minimum cost of inventory that is one interesting thing.

Second important thing, which is going to be there in this formula, we have developed this formula that EOQ equals to under root 2RCP upon CH. Now in this formula, if you see the value of EOQ coming here, this is the value of EOQ which is written as Q optimal or EOQ. Now you see that this total cost curve, this total cost curve around this value of EOQ is relatively flat. This total cost curve is relatively flat, that means that this formula is a very robust formula. It is not a sensitive formula.

Even if you are not going to have the value you have EOQ, what I am trying to say that for an example, if by doing some calculation in a particular situation, the EOQ value is coming, let say 2280. And for some practical reasons, the actual order quantity is 2200. EOQ is 2280 the actual order quantity for some reason is 2200, which is close to EOQ but because maybe your truck size is not that much or your warehouse capacity is not that much, so your order quantity is 2200.

So, the formula is so robust that it is not going to drastically change your inventory cost. You see that up to, from this point to this point, there is not much change in the total inventory cost value. So, this is a very useful practically applicable formula because of its robustness, it is not sensitive. In some time, there are some cases where formulae are very sensitive, immediately as soon as you shift from that particular optimum value, the difference or the change will be drastic.

So, you have to be very particular about the optimum value, but here the formula is very robust. The curve itself shows you that sensitivity of this particular case that even if you are not at the optimal level, but because the it is very very straight line kind of curve around the EOQ value. So, it is not going to affect much of your inventory cost even if you are not ordering EOQ. So, now, we have understood the theoretical part of this EOQ formula development.

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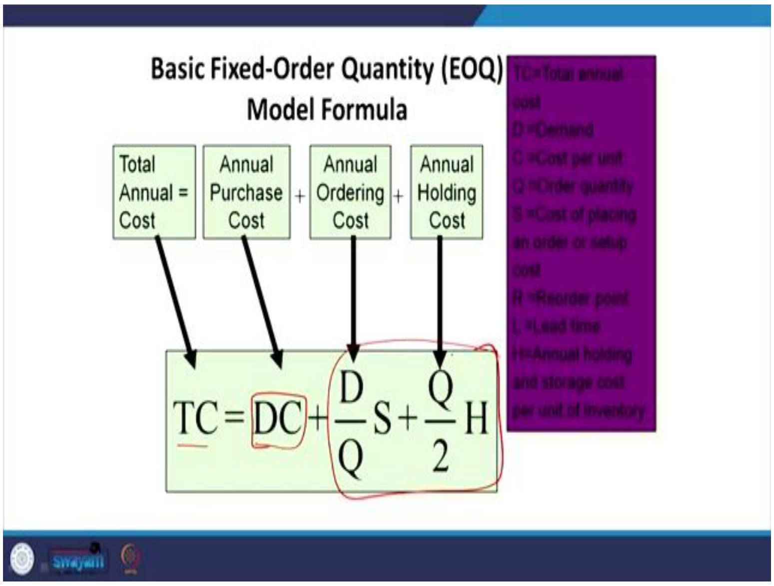

Now, we can see that how this formula was developed. So, this is the total cost, in this total cost, this particular part is the part of annual purchase cost, which is you see, this formula, which we have just developed has nothing to do with this purchase costs, we have only considered these two particular parameters. So, we will see that in our numerical examples, that if some values are given to us for the price of the product that price of the product may affect the calculations of the holding cost, this is the CH parameter, so, this CH parameter values may be affected.

Sometime, you will see in the numerical examples, values of CH are given as part of percentage of the material price and sometimes the values of CH are given in terms of absolute units. So, both the cases we will do and that will help us in understanding whether the purchase price has some meaning in the EOQ model or not. So, this is a we have discussed.

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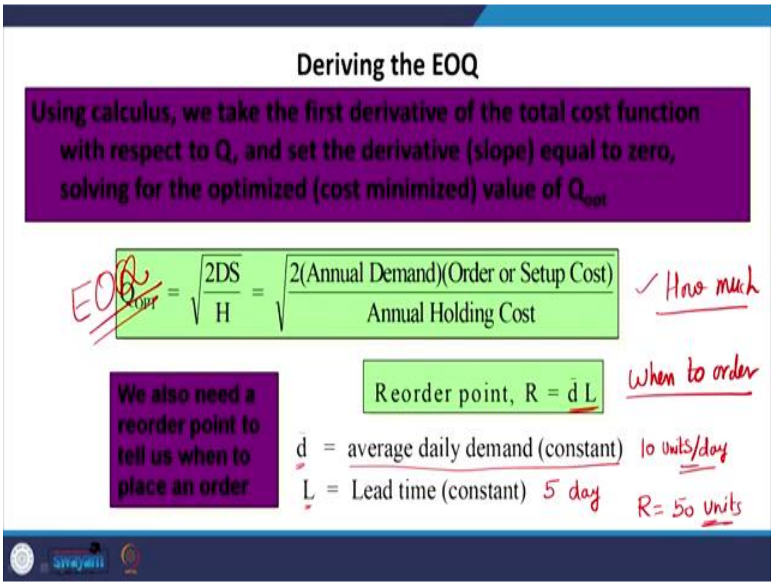

And one important thing that at the same time when we have calculated this formula of EOQ, we also need to have when to order because we have discussed in the beginning of this session that we require two important things one is how much. So, this how much has come in the form of EOQ. This is our EOQ under 2R CP upon CH. so, this is our EOQ calculation. Now, when to order? Our average consumption rate is or our average daily demand is given as d bar that is the average daily consumption which we are having and lead time is L. So, our reorder point when could we order that will be during lead time, how much you are going to consume.

So, let us say, if my average daily demand is 10 units and lead time is 5 day. So my reorder point will be 10 into 5, 50 units. The meaning is that as soon as, as soon as my stock comes to 50 units, I have consumed my stock and now only 50 units are remaining in my stock, I need to trigger a new order. So that is when to order. So I have answered both these questions. How much to order and when to order.

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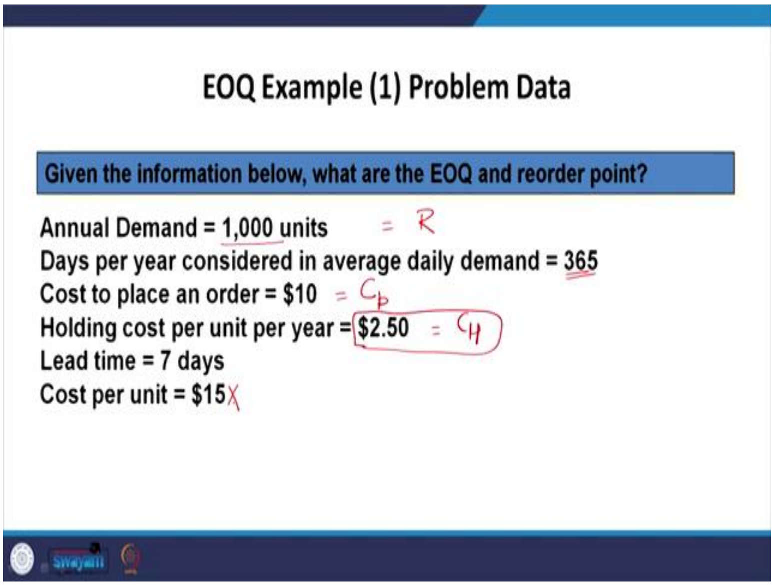

Now, let us see some numerical example and with the help of numerical example, we will see the practical application of this formula. The annual demand is 1000 units. So, this is my R in a year, we are considering 365 days, cost of placing an order that is CP that is 10 dollars, holding costs per unit per year that is CH. Lead time is 7 days and cost per unit is 15 dollars. Now here in this example, you see that CH is given in absolute terms, CH is given in absolute terms. So, in this particular example, there will not be much use of this cost per unit, there will not be much use of this cost per unit.

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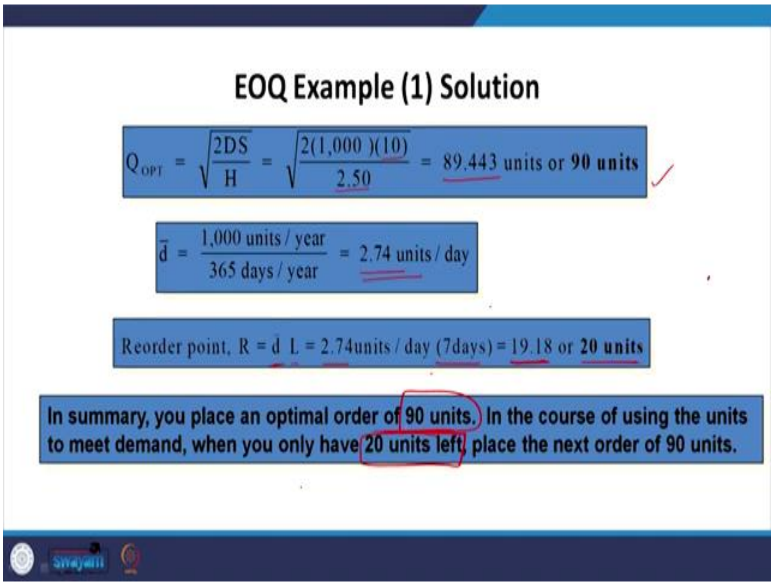

Now, we go for the calculation so, UC under root 2 RCP upon CH that we have calculated the 1000 unit is the annual demand, the cost of placing an order is 10 rupees, the holding cost is 2.5 rupees. So, our calculation gives us that 89.443 or approximately 90 units that is our EOQ. Now, average demand, 1000 units are the annual demand. 365 days are considered to be in a year. So, 2.74 units is our daily demand. So, when we multiply this with the lead time d bar into L 2.74 into 7 days, so, around 19.18 units or 20 units when our stock will decrease to 20 units, we will trigger an order of 90 units. So, our final answer becomes, so, we will place an order of 90 units whenever our 20 units are left in the stock. So, whenever 20 units are left in the stock, you will place an order of 90 units that is the solution of this particular simple case.

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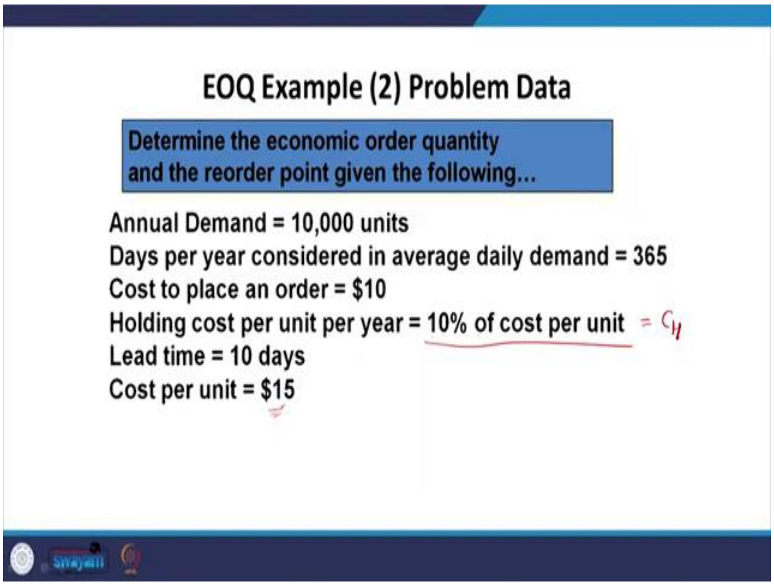

Now moving ahead, we take another example, almost of similar data, but here the only change is that this is CH. Now, here the CH is given as 10 percent of costs per unit. Here the CH is given as 10 percent of cost per unit. So, it is not the absolute value of CH. the CH is given in terms of price of the material.

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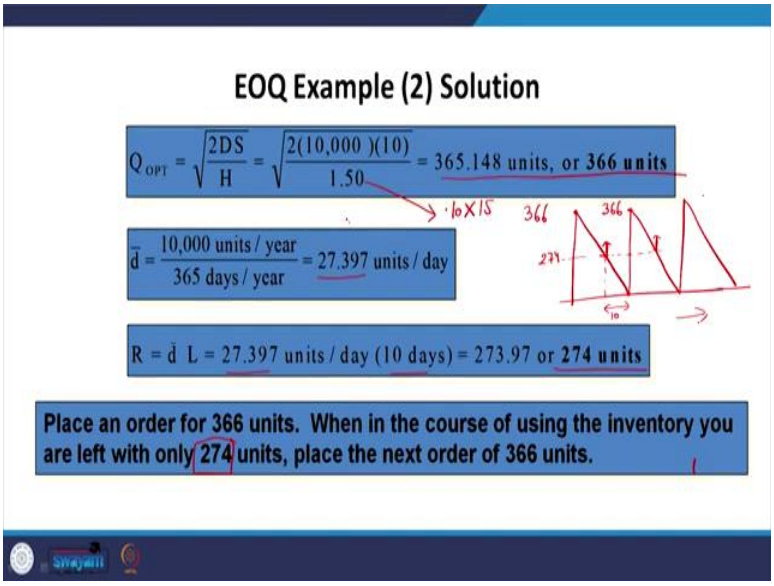

So, when we made the calculation so here this value of H is coming because of 10 percent that is 0.10 into the cost of material, the cost of material is given as a 15 dollar. So, here we have taken 0.10 into 15 and that is given as 1.50. And therefore, this calculation has resulted into 366 units. Earlier case, because it was a fixed value of 2.50 dollar per year. So, that was 90 units. Now, it is 366 units, drest whole issue will remain as it is.

So, here 10000 units divided by 365 our 27 units almost per day is the consumption and the lead time is given as 10 days. So, 27 into 10 days that becomes a 274 units. So, whenever our stock comes to 274 unit, wherever a stock drops to 274 units, it is like this way, whenever you are giving order, it is of 366 units, you have started using these units. So, now, somewhere here the stock will come to 274 units. So, you will trigger a new order.

You will trigger a new order and this new order will come, because it will take 10 days. So, this is 10 days period, which is the lead time and after that again your supply will reach to 366 because full supply will come to you and that way again when it reaches to 274 you will trigger in your order. So, that is how saw teeth pattern will go on. So, this is the application of basic EOQ model for the inventory management. In our coming lectures, we will see more variations of this basic EOQ model that how different types of models are available, which are based on this basic EOQ model for this lecture, thank you very much.