Production and Operation Management

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Lecture 13

Time Series Forecasting: Working Example of Exponential Smoothing 3

Welcome friends, now we are moving into the thirteenth session of the course of Production Operations Management. In last couple of session, we were practicing some numerical problems based on exponential smoothing which is part of forecasting. So we started forecasting, we discuss the means of time series analysis, which is based on historical data. And then we started doing different types of forecasting models of time series.

We started with the very simple model which is simple moving average method. We moved to weighted moving average method and then we studied discussions on exponential smoothing methods. In exponential smoothing methods, we started with basic exponential smoothing method and then we had two types of examples in our previous section. One, were with basic data, we also had some kind of trend in our historical data and in another case with basic data, we also have seasonality included in that data and we saw how to handle those types of cases.

In this particular section, we will like to focus on a case where all types of components are available, the historical data has trend as well as seasonality. So, you will be having a very complex kind of historical data where we will require all types of smoothing concepts. So, to start this session we will have a quick problem to just refresh our, what we have done in the previous session.

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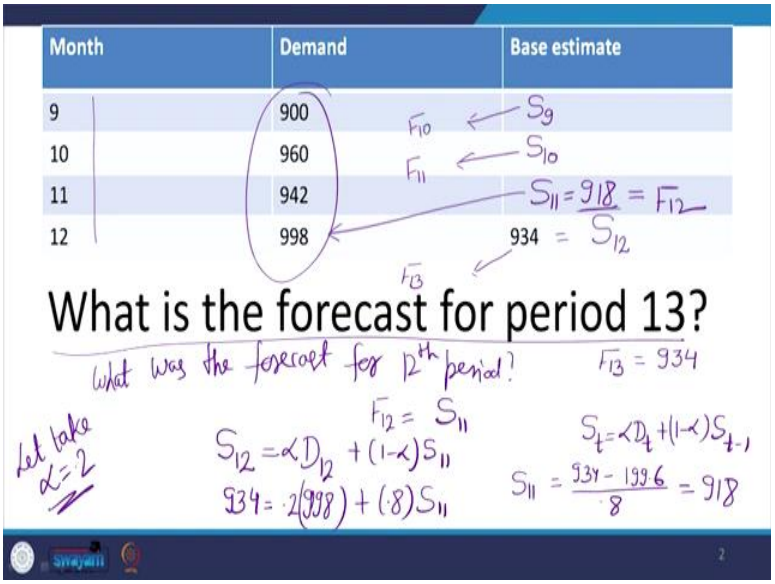

We have some data available with us just for the month of ninth, tenth and eleventh and twelfth. And you can say that this data is from piece of a bigger report and other part of the report is now not available. It is lost you have only this much part of the report where you have very limited information. You have the forecast for you do not have any forecast. You have the demand data for four periods. Ninth month, tenth month, eleventh month and twelfth month.

And you have the base estimate for the twelfth month S12. Now when you have the base estimate for 12 month. So, this is actually if I am asking you what is the forecast for period 13. So, in fact this S12, this S12 is nothing but it is going to be the forecast for thirteenth period. So, directly the answer for this question becomes that the base estimate for the twelfth period the updated base estimate for twelfth period becomes the forecast for the thirteenth period.

Now just to explore more, if somebody wants to know, that what was the forecast for twelfth period. For the twelfth period, your demand was 998. What was the forecast for twelfth period? So, now the F12 is actually going to be S11 and S11 is not available. So, for that purpose we can do a backward calculation, we can do a backward calculation to determine S11 and that is S12 and if you remember this formula St equal to alpha Dt Plus 1 minus alpha St minus 1.

That formula we have discussed in last two sections. Now here if St is S12, alpha Dt means alpha D12 Plus 1 minus alpha St minus 1 that is S11 and S12 is known to us that is 934. Let us say, value of alpha, let take alpha equal to 0.2. So it becomes 0.2 D12 that is known to us, that is 998 plus 0.8 S11.

And when we try to solve it, it becomes 934 998 multiplied by 0.2 that becomes 199.6. So, S11 will be 934 minus 199.6 divided by 0.8. So, it is going to be 934 minus 199.6. That is coming to be 734 divided by 0.8. So, it is 918, so the S11 is equals to the 918 and this is nothing but F12. So, you see with very limited information we are able to get whatever is desired. You and see this value of alpha 0.2 is giving you forecast of 918. But actually the demand was 998.

So, there is a huge difference between the demand and the forecast. So that is going to be the matter of our discussion in the coming classes, that how to measure the accuracy of your forecast. Similarly I request you to calculate the values of S10 and S9 and the S10 will be the forecast for eleventh period, S9 will be the forecast for the tenth period.

So, this way you can do backward calculation and you can determine the forecast for the respective periods, even the report is lost. But whatever information is available with that information itself you could find out the forecast for the thirteenth period and you can now also able to find the forecast for previous periods. So, that is just for the practice purpose. We did this question.

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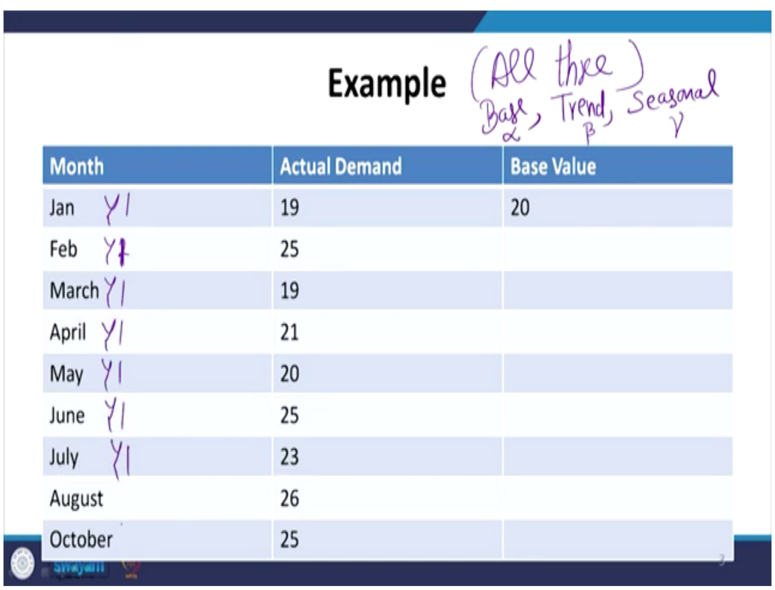

Now we are going to discuss a typical case, where all three means the base fluctuations are there, trend fluctuations are also there, and seasonal fluctuations are also there and for maintaining the smoothness of these three types of fluctuations you require alpha, beta and gamma. You require alpha for smoothening the fluctuations of base value.

You require beta for smoothing the fluctuations of trend and you require gamma for smoothing the fluctuations of seasonal component. So, we are taking the saved old data for this purpose and with this help of this data, now we see that, how three components are there. So the data is available with us.

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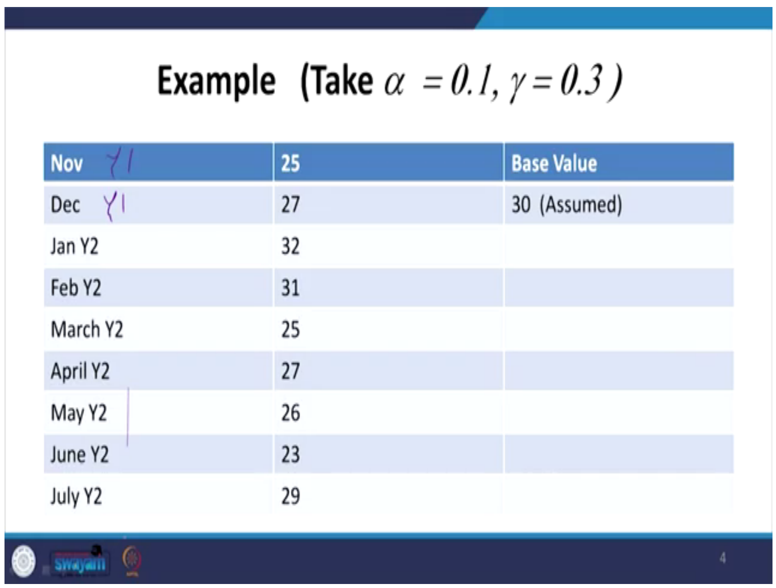
Example (Take $\alpha = 0.1, \gamma = 0.3$)

Nov Y1	25	Base Value
Dec Y1	27	30 (Assumed)
Jan Y2	32	
Feb Y2	31	
March Y2	25	
April Y2	27	
May Y2	26	
June Y2	23	
July Y2	29	

This is for year one and then it is for year two. Now in this case we have some data for around 1 year 7 months and for this data we have already discussed in the previous section. One example where no trend, no seasonality was there. Then we took a case where seasonality is included and we used alpha and gamma. Now we are trying to include trend also in this data and then we see that how to handle such kind of cases.

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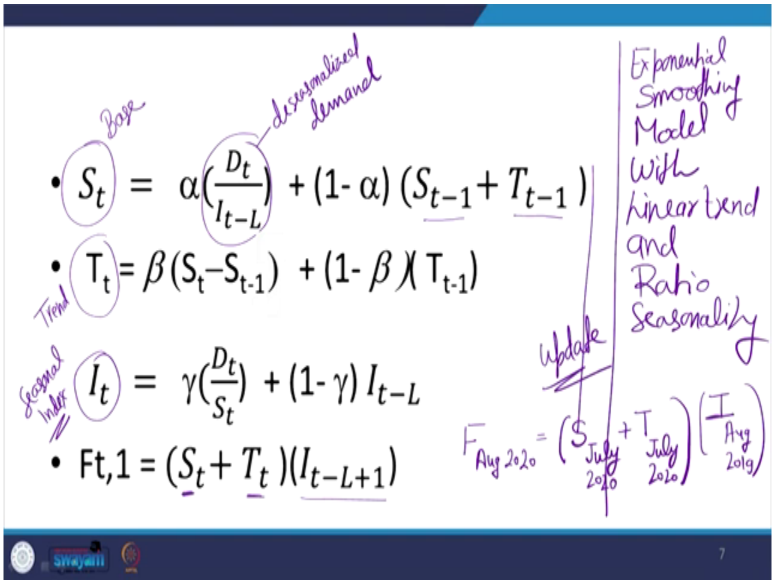

Now for such kind of cases to solve, first let us understand the theory part and once we understand the theory part, then we go back to the table and we will start using this formula for developing the values for respective periods. Now as three components are there, one is your base value. Another is your trend value, this is base, this is trend and this is seasonal index.

So, these three values are there and with the help of these three values we will actually determine the forecast for the next period. Now here the seasonal index is in the form of ratio. If the model is actually, this is exponential smoothing model with linear trend and ratio seasonality. So, this is the complete name of this model.

Now what we are doing? We are smoothing the fluctuation of base value. Now please remember in this term here, first we are deseasonalizing the demand of the current period. We have divided the current periods demand Dt by seasonal index of that period. So, this becomes the deseasonalized demand.

This is the deseasonalized demand, and this is the base plus trend of the previous period. So, this becomes the updated base for the current period. Second is the calculation of your updated trend for the current period. So, this becomes because this is a linear trend, so the linear trends definition is the difference between current base and the previous base.

So, this is the calculation of your updated trend and then this is the calculation for the updation of your seasonal index. So seasonal index is this Dt upon St is the current value of seasonal index for this period and this It minus L, this is the seasonal index of the same time in the previous period. Previous period of the seasonality, what was value of the seasonal index.

So, that is how you will update all these three values you will update base, you will update trend and you will update seasonal index. Then using these three things, though these updated seasonal index is not used immediately. For the next periods forecast this is Ft,1 that is the from the current period the next period. What you are going to have? The updated base value of the current period, updated trend value of the current period and the seasonal index which is to be multiplied to this summation that is of the period of the previous seasonal period.

For an example, if I am going to forecast for August 2020. If I am going to forecast for August 2020, so from this expression it will be S of July 2020 plus trend of July 2020 and this will be multiplied by seasonal index that is I of August 2019. That is the expression we will have for calculation of forecast for August 2020. So, this is the popular way, there can be one more variation of this formula.

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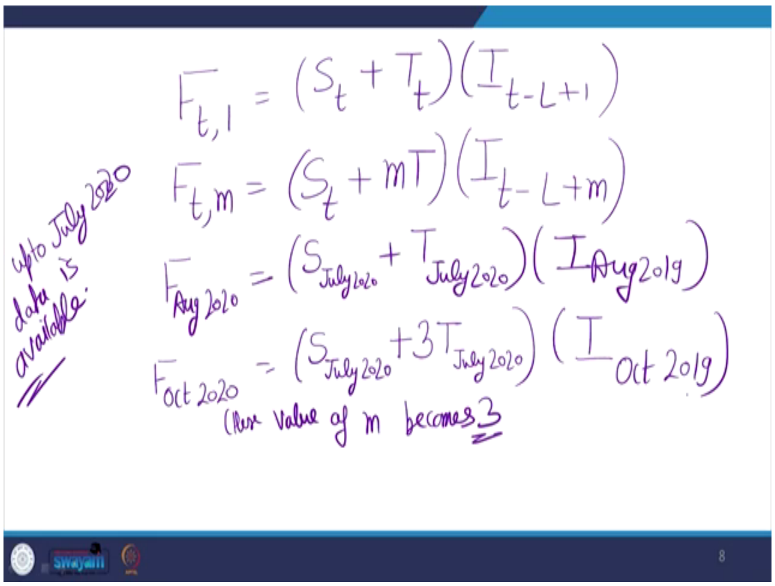

This we are calculating for Ft,1 that is St plus DT into It minus L plus 1. But sometimes it is also possible that we can use this formula for m period ahead. So, in that case we do two things. One the current trend, whatever is the value of trend in the current period that we assume as constant. So, we will multiply this with mT and here also for the same corresponding period you will take the seasonal index.

The point which is again so that it becomes easy to understand, like we have data upto July. So, for August 2020 we just saw that if we want to forecast this becomes S July 20 plus T July 20 into I August 2019. This is the formula for calculation of forecast of August 2020. If we have updated information up to July 2020.

But somebody says I want to forecast for October 2020 with this information. I have information up to July 2020 and now I want to forecast for October 2020, how will it happen. For that purpose, since I have the most updated base. The most updated base which I have that is of July 2020. This is the most updated base I have.

Now the current period is July, so July plus one is August, July plus 2 is September and July plus 3 is October. So I am forecasting for three periods ahead. So, here it will come 3 into trend of July 2020. Here it will come 3, so the value of m here value of m becomes 3. So, therefore this three into trend of July 2020 and then this I will be taken for October 2019.

So this will become the formula for calculation of forecast for October 2020. So this way, from the current period you can forecast not only for the immediately next period but 2,3,4 periods ahead also. So that is though the quality of forecast will not be of very good nature. But even then a possible forecast is there and that is how we can update our forecast regularly.

Now going back to this particular example, which we were discussing. Now we have understood that we have to do three smoothings. We have to do smoothing of base, we have to do smoothing of trend and we have to do smoothing of our seasonal factors. Now as we did in the case of seasonal factor, we will do same thing here also.

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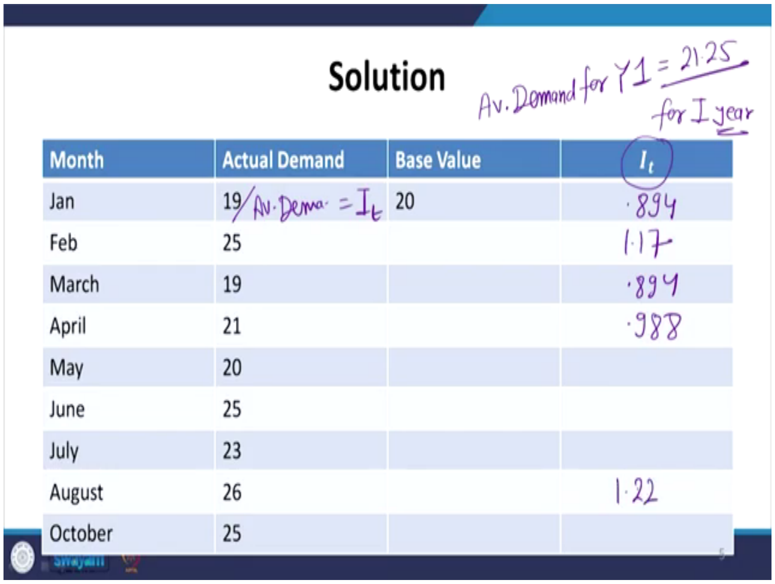

Month	Actual Demand	Base Value	I_t
Jan	19/Av. Dema. = I_t	20	.894
Feb	25		1.17
March	19		.894
April	21		.988
May	20		
June	25		
July	23		
August	26		1.22
October	25		

First we will calculate the seasonal index, calculate the seasonal index for first year and that seasonal index you remember, we calculated by dividing the average value to all the actual demands. Actual demand divided by average demand that will give me my It. So the, because we have taken this data from our previous example, so average demand for real one was 21.25 and therefore we can immediately calculate some of the values of It, that is 19 divided 21.25. It becomes 0.894, 25 divided by 21.25 it becomes 1.17, 19 divided by 21.25 it becomes 0.894, 21 divided by 21.25 it becomes 0.988 etc.

So, this is how I and because we have to calculate for August, so let me calculate the value of August also. 26 divided by 21.25 it becomes 1.22. So, this way we calculated the values of various seasonal index for previous year. Now when we need to start the calculation for our forecast using this three types of smoothings.

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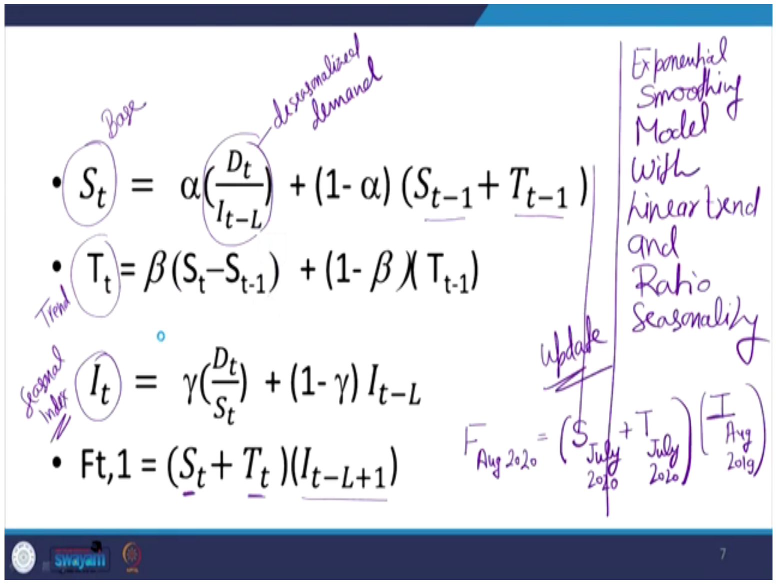

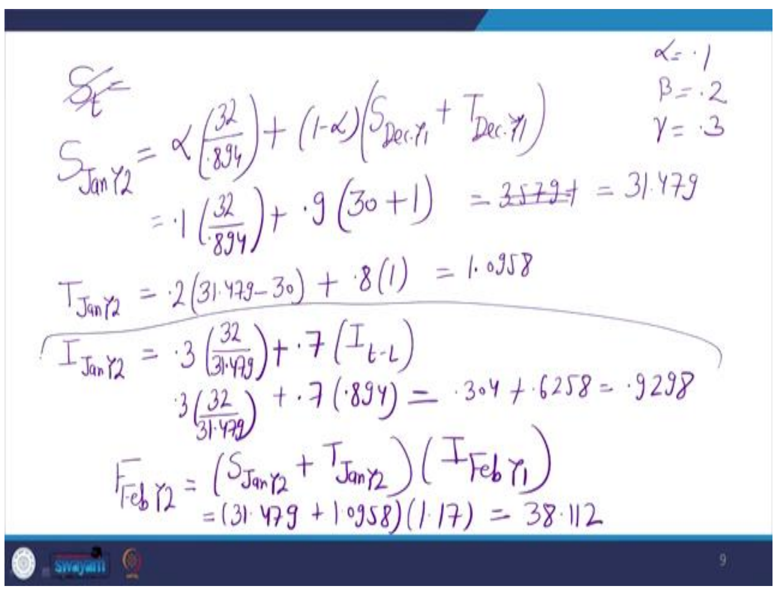

Example (Take $\alpha = 0.1, \gamma = 0.3$)

Nov	25	Seasonal Index	Base Value	Trend
Dec Y1	27	1.25 (Ass.)	30 (Assumed)	1 (Assumed)
Jan Y2	32			
Feb Y2	31			
March Y2	25			
April Y2	27			
May Y2	26			
June Y2	23			
July Y2	29			

Solution

Av. Demand for Y1 = 21.25 for I year

Month	Actual Demand	Base Value	I_t
Jan	19/Av. Dema. = I_t	20	.894
Feb	25		1.17
March	19		.894
April	21		.988
May	20		
June	25		
July	23		
August	26		1.22
October	25		

For that purpose first is you need to calculate the updated value of St. Now if we go in the detail calculation, so what we have assumed that for the period of December in the month, in the first year we have assumed a value of 30 as our base value and this base value may help us in doing the calculation a bit faster. Because that we need not to go right from the January of year 1. Otherwise we can do that also.

Similarly, we can assume the initial trend also. We can assume the initial trend also. Let us say we assume the initial trend of 1 unit here. We are assuming a initial trend of 1 unit here and let me write assume and we can also assume though it is not very appropriate, this seasonal index, this we are just assuming for the sake of doing the calculations. So, I am assuming the seasonal index of 1.25 here. So, these values I am assuming and with the help of these values, let me see that how we proceed further. Now for this purpose, this updated value of January's base. First is we need to calculate what is the updated value of January's base.

So, first I want to calculate S January Y2. Now you remember it will be alpha Dt upon I minus tL I into tL. So, this is the deseasonalized demand of January Year 2 and for that purpose if you see the demand of January is 32 and the original It for the January one is 0.894. So, 32 divided by find 0.894 becomes the first expression plus 1 minus alpha.

Now here, you remember the formulas is St minus 1 plus Tt minus 1. So, St minus 1 plus Tt minus one. St minus 1 is this 30 and Tt minus 1 is this 1. So this Is S December Y1 Plus T December Y1. So, here it becomes and let us assume the value of alpha equals to 0.1. So 0.1 into 32 divided by 0.894 plus 0.9 and S December Y1, we have assumed 30 T December Y1 we assume 1. So, this becomes your S January.

The value can be calculated as this is 32 divided by 0.894 multiplied by 0.1. So, this becomes 3.579 plus 31 into 9 plus 3.579. So, this actually totally it becomes 31.479. So, that is the value of S January 2. Similarly, now we will calculate the value of T January Y2.

Now, if you see the formula for T January Y2, this will beta St minus St minus 1. So, beta let us assume is 0.2. St minus St minus 1. So, S January Y2 that is 31.479 minus St minus 1, that is S December, that was 30 plus 1 minus beta, that is 0.8 into T minus 1. That is T December, T December was 1. So it becomes this and therefore this is 22.22 plus 0.8 this becomes 1.0958.

Then we can also calculate the value of I January Y2. Now if you see the formula, this is gamma Dt upon St. Let us say gamma is 0.3. Gamma Dt upon St. So, Dt for January, Dt for January was 32 upon St. St is 31.479 plus 1 minus gamma, that becomes 0.7 into It minus L. That means the January Y1 and therefore it will be 0.7 into January Y1 was 0.894. So, this value becomes 32 divided by 31.479. 1.0 into 0.3, so this becomes 0.304 plus 0.894 into 0.7 0.6258. So plus 0.304. So it becomes 0.9298.

Now all these data is available and now if I want to forecast for February Y2, the formula will become SjanY2 plus TjanY2 multiplied by IfebY1 and using this formula I will say that, SjanY2 is 31.479 plus TjanY2 is 1.0958 and IfebY1 I need to see the table is 1.17, this is 1.17.

So, this becomes the forecast and the calculation will say that this is going to be 1.0958 into 1.17 so this becomes 38.112. So this is going to be the forecast and here we saw that this value of IjY2, I January Y2 is not used immediately. It will be used for Y3 when we will be calculating the forecast for January Y3 then this value of Y2 January I will be used.

Right now for calculation of February year two's forecast, we used February Y1's updated value of seasonal index. So this way, now we understood that how all three types of fluctuations can be smoothed and how we need to put together, so that we can get updated value of the forecast. So, with this we saw the use of alpha we saw the use of beta and we saw the uses of gamma and these are the values of alpha, beta, gamma. We took arbitrarily, you can take any value of alpha beta gamma. As long as these values of alpha beta gamma minimize your forecasting errors.

That is the bottom line the selection of alpha beta gamma should be in such a way that your forecasting error should be minimum or you can say the forecasting accuracy should be as high as possible. So with this, we come to end of this section. Thank you very much.