Production and Operation Management
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Lecture 12
Times Series Forecasting: Working Example of Exponential Smoothing 2

Welcome friends, this is the twelfth session of this course of Production Operations Management. In our last session, we discussed an example where we used concepts of exponential smoothing. We take some data and on the data we perform some calculations. Now, if you remember, that data was of six months period, and in that we used two methods, one basic exponential smoothing method where no trend, no seasonality was present.

And then we used same data and we considered that trend may be present in that data and we used two smoothing constants alpha and beta because two types of fluctuations were possible, one in the base value and second in the trend value and therefore we have already understood how to handle these situations were two such characteristics are available.

Now, in this particular session, we will see the third type of characteristics which may be available in my historical data, and that third characteristic is of seasonality. We discussed the example of linear trend and here for the seasonality, we will discuss the example of ratio seasonality.

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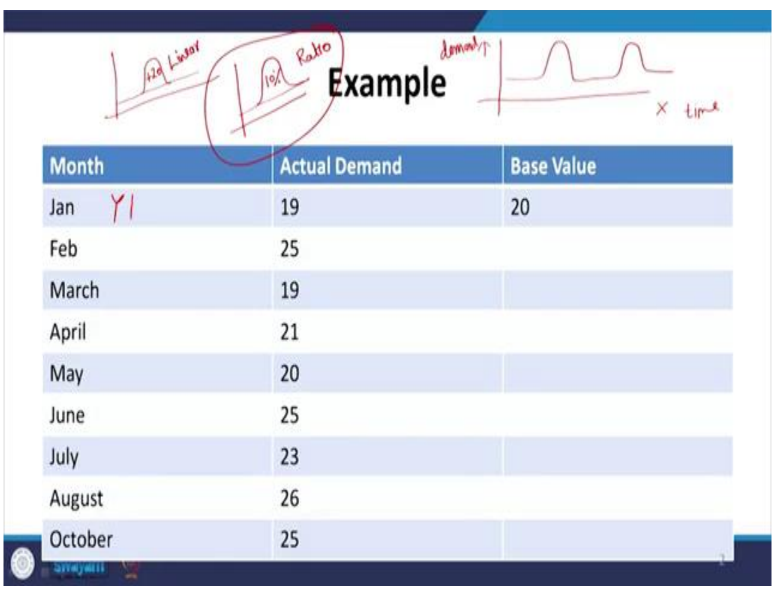

Month	Actual Demand	Base Value
Jan	19	20
Feb	25	
March	19	
April	21	
May	20	
June	25	
July	23	
August	26	
October	25	

Now, we all know what is the meaning of seasonality, the meaning of seasonality is that your data is remaining to a, this is your x axis, that is the time. This is the Y axis that is the demand value. Now in the seasonality case, what happens your demand is like this and all of a sudden the period of seasonality comes and demand increases to a high value.

And then again, demand comes to a low value and then again, seasonality comes and like that. So, you have in your demand data, some intermittent spikes and these intermittent spikes are the period of seasonality. So, various examples are there, products which are having demands because of some festivals, seasons, particularly weather like winter, summer, rainy seasons, these seasons have demand of some peculiar products.

If you are in winters, you require woolen products, you are in some kind of rainy season, you require umbrella. You are in summer season, you require A.C's. So, different types of products may have the effect of weather. So, all those products which are having fluctuations because of weather conditions are also seasonal products which are because of some festivals, again, these are the seasonal products.

So, good amount of products, good amount of examples are available for which demand is affected by seasonal factors. So, therefore, it is also very important component of our discussion that all those products where seasonality is available, how to do forecasting of those types of demand data. We have discussed the number tenth session that seasonality can also be of two types, the ratio seasonality and the linear seasonality.

In case of linear seasonality, your demand is high. This is the demand date, this is the average value and whenever seasonal season comes, demand increases by let us say 20 units from the average value. Demand increases by 20 units during the season period that is the linear seasonality. But in other case, it is also possible that this is the average and demand increases by 10 percent of the average value.

So, this is an example of ratio seasonality. This is an example of linear seasonality or additive seasonality. So, here today in this class, we are going to discuss the example of the ratio seasonality where the demand is a percentage of average value. So, we will use some demand data and since this requires a considerable amount of data, so we have taken the data for two years period.

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- $S_t = \alpha(\frac{D_t}{I_{t-L}}) + (1-\alpha)\, S_{t-1}$ → Updated Base

0 to 1 $\begin{cases} \alpha = \text{Base Value (Average)} \\ \gamma = \text{Seasonality} \end{cases}$

$I_t = \gamma(\frac{D_t}{S_t}) + (1-\gamma)\, I_{t-L}$ → Updated Seasonal Component

$S_{Jan} = \alpha\left(\frac{D_{Jan\,Y2}}{I_{Jan,Y1}}\right) + (1-\alpha)S_{t-1}$

$\cancel{F_{t,1} = S_t}$

$F_{t,1} = S_t \times$ Appropriate I

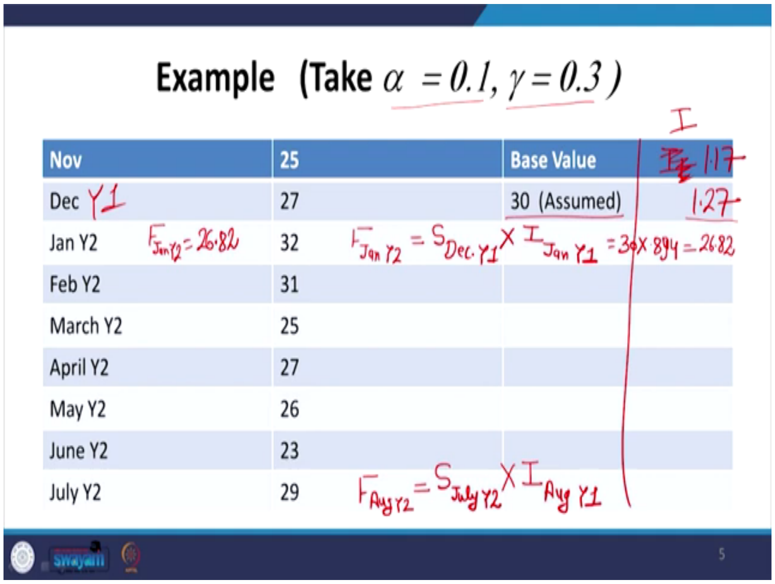
Appropriate $I \neq I_t$

$F_{t,1} = S_t \times I_{t-L+1}$

L = period of Seasonality

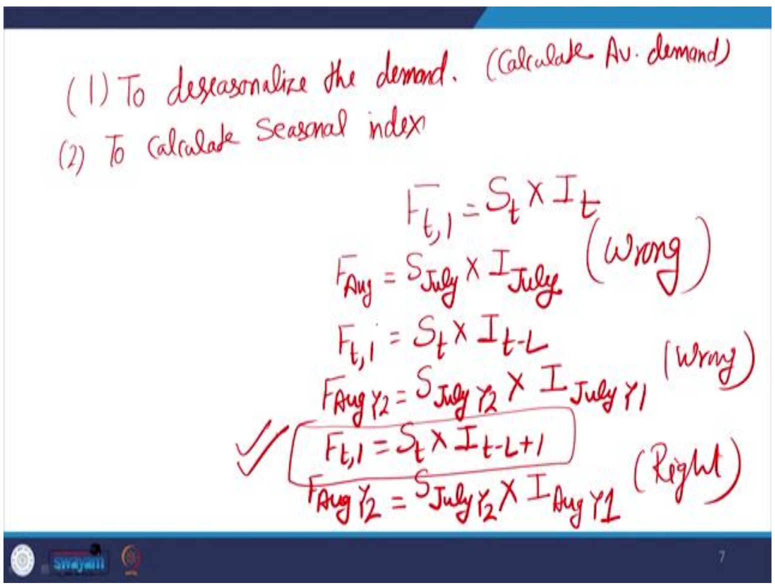
Example (Take $\alpha = 0.1, \gamma = 0.3$)

Nov	25	Base Value	I 1.17
Dec Y1	27	30 (Assumed)	1.27
Jan Y2 $F_{Jan\,Y2} = 26.82$	32	$F_{Jan\,Y2} = S_{Dec.\,Y1} \times I_{Jan\,Y1} = 30 \times .894 = 26.82$	
Feb Y2	31		
March Y2	25		
April Y2	27		
May Y2	26		
June Y2	23		
July Y2	29	$F_{Aug\,Y2} = S_{July\,Y2} \times I_{Aug\,Y1}$	



Example (Take $\alpha = 0.1, \gamma = 0.3$)

Nov	25	Base Value	I
Dec Y1	27	30 (Assumed)	1.27
Jan Y2	32		
Feb Y2	31		
March Y2	25		
April Y2	27		
May Y2	26		
June Y2	23		
July Y2	29		



We are starting data from January of year one and this data is flowing into two slides and it is going up to June of second year. And then we will forecast with the help of this for the period of August of year two. Now let us see how to do this. Now for this purpose we have two smoothing constant, one is alpha, which we know for base value or the average value. In some books you get this word average. Second is Gamma, gamma, do smooth the fluctuations of seasonality.

So, if you remember in our previous class, we used two smoothing constants alpha and beta. Beta was to smoothen the fluctuations of the trend. Now we are introducing the third smoothing constant that is gamma, which will use for smoothing the fluctuations of seasonality component and as already told you, these values of alpha, beta, gamma are always between 0 to 1. So, in this particular example, we are going to take value of Alpha as 0.1 and value of gamma as 0.3.

So, this is just our understanding that these values we are going to take, we can start with some other values, but we will do calculations right now with these values of Alpha and Beta, Alpha and Gamma. So, first we will use this equation for updated base, this equation for updated seasonal component and then we will use the value of updated base and the appropriate, please see, not updated seasonal component.

We will use forecast for next purpose for next period. By using St plus it is a case of ratio seasonality. So, Ft1 will be St into appropriate I. Now I will tell you, what is this appropriate I. Because this appropriate I is not It. Let me also write this that this appropriate I is not It. This is something else and we will see what is that something else. So, it is important that how do we develop this formula.

This is important thing in the part of modeling activity that how do we customize the formula building as per the requirement of that data, as per the philosophy of that particular type of concept. So, now going back to the data available to us. Now in this particular case, we are starting we are starting with the data of January for year one and these are the demands, actual demands from January 1 to October.

And then in the next slide you have for November, December, and then you will go for calculation of forecast for the period of August. Now for the forecast of period of August, how we are going to do the first important step in this particular case is to deseasonlize demand. First, we need to do deseasonlisation of the demand. Now how to do the deaseasonalisation of the demand?

For that purpose let us go back to this particular table. Now here we have the demand available from the January of the year 1 to December of the year 1. Now we will take the average of year 1 demand. So, that will be sigma of demand January to December divided by 12. So, let us do this calculation. This is 19 plus 25 plus 19 plus 21 plus 20 plus 25 plus 23 plus 26 plus 25 plus 25 plus 27. So, this is the 255. Sigma becomes 255 divided by to 12, so the average demand becomes 21.25.

This is the average demand for year 1. Now based on this average demand, we will calculate the seasonal component, seasonal index of each period. So, now the second step is to calculate seasonal index. We will calculate the seasonal index and how, so to deseasonalize the demand we are going to first calculate average demand. Now the average demand is 21.25 and for calculating the seasonal index we will divide the actual demand by the average demand.

Actual demand divided by average demand. And that is going to be the value of 19 divided by 21.25, so this is coming 0.894. Second is 25 divided by 21.25, it is coming 1.1764, 19 divided by 21.25 it is coming 0.894, 21 divided by 21.25 it is coming 0.988. 20 divided by 21.25 is coming 0.9411 and so on you will calculate that we just completed for your reference quickly 1.176 then 23 divided by 21.25 it is 1.08, 26 divided by 21.25, it is 1.22, 25 divided by 21.25, 1.17. Then 25 divided by 21.25 the It values coming to be, It values is 1.17 and then 27 divided by 21.25 1.27.

So, now we have the demands and we have calculated the seasonal index from January of year 1 to December of year 1. So, we have these values ranging from 0.894 to 1.22 in the month of August and you can see that in the month of August. There is the peak seasonality because the seasonal index is highest in this case. When this seasonal index is less than 1, it means these are the period when demand is very low, less than average value.

And when the seasonal index is around 1, that is the average level of demand but when the demand is the seasonal index is more than 1, it means the period of seasonality is starting. Like if you see from June to October, the values of seasonal index are 1.17, 1.08, 1.2, 1.17 that means the seasonality is started from the month of June. It was at its peak in the month of August and then it started decreasing in the month of October and so on.

Now with the help of this information that you have the value of seasonal index for the period of January to December, now what I am going to tell you that is going to be very, very important thing if I am going to forecast for the month of August Y2, if I am going to forecast for the month of August Y2, what I will be doing? I will be using the most recent updated base value and the most recent updated base value will be for the month of July S July, Y2.

And since I am talking of ratio seasonality, since I am talking of ratio seasonality, I will multiply this with this value of base of July year 2 with the seasonal index of August of year 1 that is the most recent updated value of seasonal index available to me. Please be careful that do not use the value of seasonal index of July Y2 for calculation of forecasts of August Y2. You need to see that you are forecasting for the month of August.

So, you need to take the updated value of base for the month of August and the updated value of August for August seasonal index is available for year 1. When you will be forecasting for year 3, then the updated value of this, seasonal indexed will be from August year 2. So, if I say in a generalized way the appropriate I, which I said. I will tell you later, the appropriate I is in the general form we will be like this, St into It minus L plus 1.

So, L is here, period of seasonality. L is the period of seasonality, so here we have a monthly data availability, so the period of seasonality is 12 months and after 12 months, phenomena is going to repeat. If quarterly data is available to you, if quarterly data is available to you, so the period of seasonality will be 4. If half yearly data is available to you then period of seasonality will be 2. If weekly data is available to you, period of seasonality will be 52.

So depending upon what type of data is available, you will have accordingly the period of seasonality and why I am doing this plus one in this because T minus L when if you see this equation and when I am doing this so it means F August is S July in to I July which is wrong. Then I do Ft,1 St in to It minus L. So, what I am going to do F August for year 2, S July year 2 in to I July year 1 that is also wrong.

Now if I use Ft 1 St in to It minus L plus 1, it means F of August Y2 equal to S July Y2 in to I. Now T minus L was July 1 and plus 1, July plus 1 means August, so August Y1, that is right. So, we need to see that how are we going to take this calculation forward? And with this we will be able to update. We also recommend you to continuously update the values of base and update the values of trend, updated of values of base always will be used currently, but updated values of I updated values of these seasonality will be used for subsequent years.

These will not be used immediately, but these will be used when we are going to forecast for year 3. So, at the moment if I am just not interested to do calculation, I can skip this part of calculation, of updated values of seasonality index, but we recommend because we will be continuously doing this forecasting. So, we should do this calculation for seasonality index also, which will be useful for determining forecast of year 3.

Now, let us see how to do this calculation for our current requirement. So, we can start this calculation from the initial period. But I have skipped this whole data is not required and I am directly taking some intermediate value as our base value. So, I am assuming the base value for the month of December year 1. I am assuming just to simplify the calculation. There is no other purpose just to simplify the calculation I am taking the base value for December Y1 so that we can shorten our calculation process.

Now when we are assuming the base value for December and the base value for December is here. Now I am just demonstrating you how do we calculate the forecast for January. So, F January Y2 will be S December Y1 into I of January Y1. Now as December Y1 is given to you as 30 and I January Y1 we have just started this calculation and we got the initial value of I for January Y1 as 0.894.

So, this is 0.894. So, this is the forecast for the month of January Y2, that is 30 in to 0.894 and this becomes forecast for January Y2 that is coming to be 26.82. So, this multiplication is coming 26.82. It is also recommended now that we get the updated value of January's base and we get the updated value of seasonal index of January Y2 and with the help of this updated base value of January, we can calculate the forecast for February.

However, the updated value of seasonal index will not be used immediately. It will be used for January; it will be used for February year 3. So, but it is important that we do this in the iterative mode. Now let us go for calculation of the base value for this January Y2. Now, this is the formula for calculation of these base values and the updated value of the seasonal indexes. Now St that is the S January equals to alpha Dt.

Now this means Dt upon It minus L that means we have deseasonalize the current demand plus 1 minus alpha in to St minus 1. So, Dt is right now demand of January Y2 and it will be divided by seasonal index of January Y1 plus 1 minus alpha into St minus 1. So, let us do this calculation.

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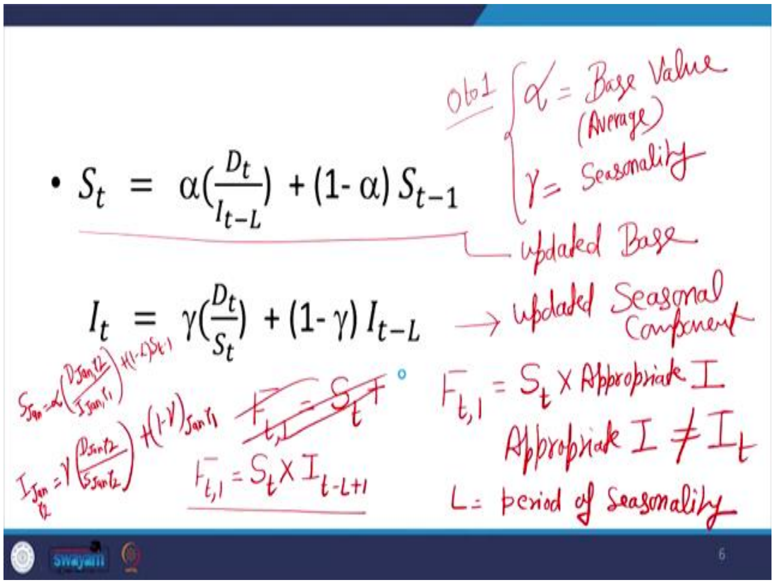
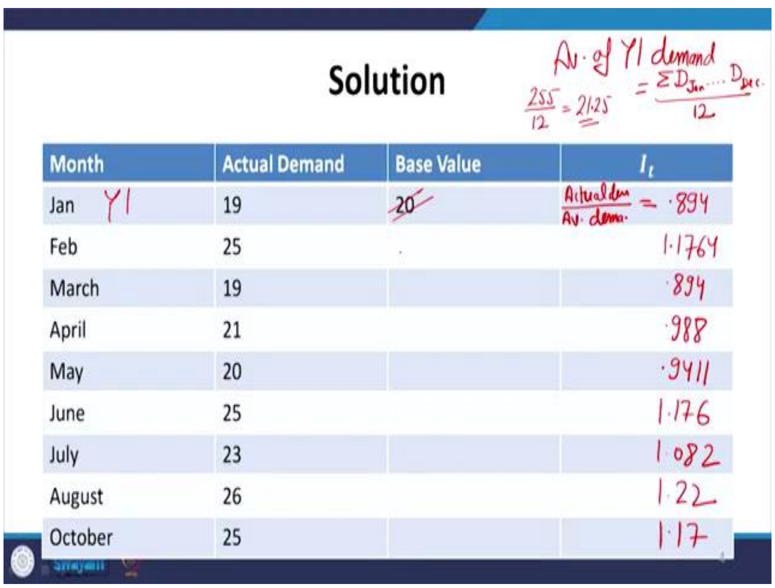
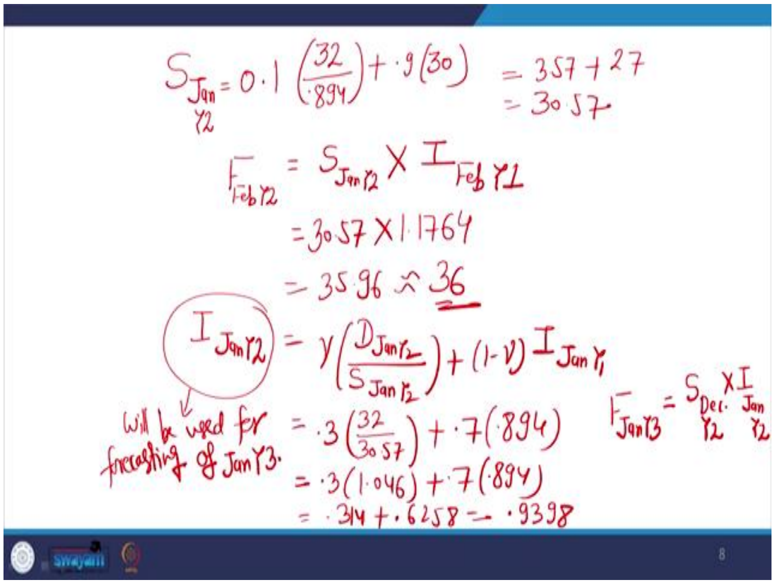
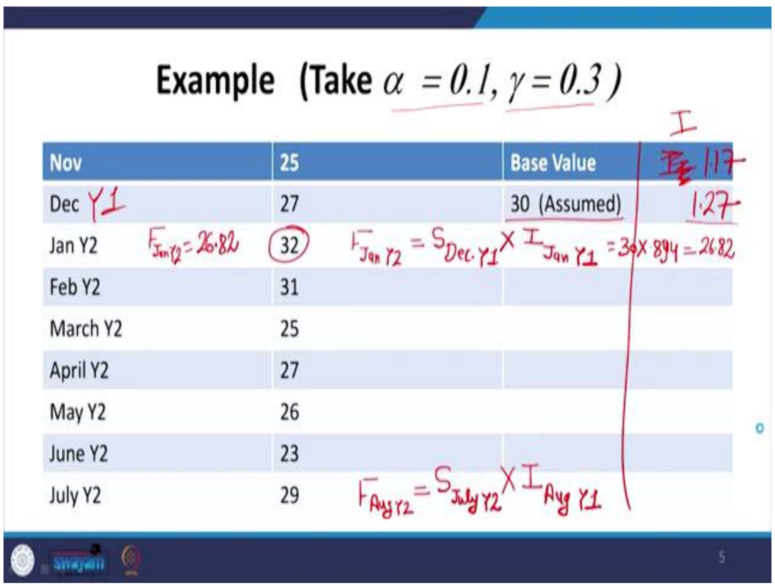




Month	Actual Demand	Base Value	I_t
Jan Y1	19	~~20~~	$\frac{\text{Actual dem}}{\text{Av. dem}}$ = .894
Feb	25		1.1764
March	19		.894
April	21		.988
May	20		.9411
June	25		1.176
July	23		1.082
August	26		1.22
October	25		1.17





So, S January equals to alpha we have taken I remember as 0.1 in this case so alpha is equals to 0.1 T January Y2, it is important to write year otherwise we will be lost. So, this is the value of S January Y2, the value of demand is 32 and the seasonal index of January Y1 is 0.894 plus 1 minus alpha that is 0.9 into St minus 1, St minus 1 we have assumed as 30 here that was the December year 1, so this becomes the formula for calculation of S January Y2 and this becomes, the calculation becomes 32 divided by 0.894, that becomes 35 into 0.1. This is 3.57 plus 0.9 into 30 that becomes 27, so 27 plus 3.57 this comes to be 30.57.

So, this is the value of S January Y2 with the help of S January Y2, if I want to forecast for F February Y2 this will be S January Y2 multiplied by I of February Y1. So, S January Y2 that is

30.57 and I February Y1 let us check. I February Y1 is 1.1764. So, this becomes 1.1764. So, it becomes 35.96 or you can round off it to be 36. So, this is the forecast for February year 2. But one task is still incomplete.

We also need to calculate the updated value of all I January Y2. We also need to calculate the updated value of I January Y2 and I January Y2 will be the will be calculated by using this expression. Now what does it say? That is It that is January Y2 equals to gamma Dt that is the current demand D January Y2 divided by S January Y2 plus 1 minus gamma It minus L because we are updating the base of previous year for the same period that is January Y1.

So you see I January Y2 will be gamma of demand of January Y2 divided by S of January Y2 plus 1 minus gamma in to I of January Y1. Y's value we have taken as 0.3. So, this is 0.3 D January Y2 that is let us check what is the value D January Y2 that is 32. S January Y2 we have just calculated is 30.57 plus 1 minus gamma that is 0.7. I January Y1 if I recall, it was around 0.8 something 0.894.

So, this becomes 32 divided by 30.57, 0.3 in to 1.046 plus 0.7 into 0.894. So this is 0.3 so 0.314 plus 0.7 in to 0.894 it becomes 0.6258 plus 0.314 plus 2.9398. Now, interestingly, as we discussed again and again, this value of I January Y2 will be used for forecasting of January Y3. This value of I January Y2 will be used for forecasting of January Y3 later let me also tell you how.

F January Y3 the formula will be S December Y2 into I of January Y2. So, this value of I January Y2 will be used here for getting the forecast of January Y3. So, we are just keeping these values available with us so that after one year when the when we will be forecasting for that period, we can use those most updated values of our seasonal index.

So this is the case where we discussed in detail and now the request is that I have given you a simple calculation of updating the base value of getting the seasonal indexes and how the multiplication of appropriate value of seasonal indexes and the appropriate value of the base value will give you the forecast.

Now my request is that using this method, using this iterative process, you complete this remaining part of the table and calculate the value of F August Y2. So, we can discuss the answer of this F August Y2 in our forum.

So, you complete the table, you may share the table on the forum and discuss the example that what is the value of F August 2 coming. So, with this we are coming to end of this session and we will discuss one example where all three types of possibilities are there alpha beta gamma we will use all three smoothing constants in one particular case, so that we will discuss in our next class. Thank you very much.