

Production and Operation Management
Professor. Rajat Agarwal
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 11

Time Series Forecasting: Working Example of Exponential Smoothing 1

Welcome friends. So this is the eleventh session of this course, and we are entering into the third week of the course. In last few sessions, we started discussions on Forecasting and we started discussions particularly on Exponential Smoothing Methods in our last two sessions. We discussed the basic concept of exponential smoothing method that how it is an improvement of our weighted average moving method.

And at the same time, we also discussed different types of classification of these models, how we can incorporate trend, how can we incorporate seasonality and different types of trends and different types of seasonality, whether it is linear or ratio, so all that we have already discussed in our last two sessions.

Now in this particular situation, we will see that how we can use these models in our calculations, how we can use these models to forecast and for that purpose we have some data with us and with the help of data, we will use our different types of forecasting models, particularly exponential smoothing model.

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Example

Month	Actual Demand	Base Value
Initial	---	20 = S_0
Jan	19	$F_{Jan} = 20 = S_0$
Feb	25	F_{Feb}
March	19	
April	21	
May	20	
June	25	S_{June} (updated base for June)
July	---	F_{July} (Forecast for July)

Handwritten notes on the slide:

- $S_0 = F_1$
- $S_{June} = F_{July}$
- $D_{Jan} = 19$
- $D_{Feb} = 25$
- $F_{Jan} = 20 = S_0$
- S_1 (pointing to Feb)
- S_{June} (pointing to June)
- updated base for June
- Forecast for July

Now in this table we have the data of past six months from January of a month to June. Now we want forecast for July. So, you have actual demand data for these six months from January to June and with the help of this actual demand data we can forecast for July. We have already seen how can, we use simple moving average method and weighted moving average method where we will take, average of some most recent periods according to our choice of moving average according to our choice of moving average.

So, if my moving average period is let us say 3, so I will take the average of April, May, June's demand and that average will be the forecast for July. If I am going with weighted moving average method I will assign, let us say weight of 0.5 to the demand of June, 0.3 to the demand of May and 0.2 to the demand of April and then the product of weight and actual demand and the submission of that products will give me the demand of, will give me the forecast for the month of July.

So these two methods we have already discussed in our previous classes. Now in today's session we will discuss that how we can apply the exponential smoothing model for determining the forecast for the July. Now we know that in theory we have already discussed that the forecast is going to move and going to randomize around a base value. So, we assume a base value for initiating our process.

And now my job is to get the updated base value for the month of June and the updated base of June you can write it as, S_{June} . This S_{June} is going to be the forecast for July. This is forecast for the month of July. So, now in my simplest method of exponential smoothing, what I am going to do that I will do iterations to update the value of this initial base of 20 and you see, this initial base of 20 is actually the forecast for, January forecast is 20.

That is S_{naught} , this initial base is nothing but S_{naught} . Now the S_1 , which we will calculate that is the forecast for the month of February and this data D_1 these are the demand of January. This is the demand of February. So, now the purpose of telling you is to familiarize with the conventions that what type of conventions we are going to follow, so that you understand what is the meaning of S , what is the meaning of D , what is the meaning of F .

Now it is clear that S_{naught} is going to be the forecast for the period month, so similarly following this convention as I just wrote S_{June} will be the forecast for July. So, my job is to

determine the value of S June. Now, for that purpose, if you remember in our last session we discussed that the base value have fluctuations because of variations in actual demand.

So, in this exponential smoothing method we are going to smoothen the fluctuation of this actual demand so that it comes closer to our base value. So, for that purpose, we need to use a smoothing constant.

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Example (Take $\alpha = 0.2$) Possible range 0 to 1
1 to 3
popular Value

Month	Actual Demand	Forecast	Base Value
Initial	---		20 = S_0
Jan	19 = D_{Jan}	20	19.8
Feb	25 = D_{Feb}	19.8	$S_{Feb} = 2 \times 25 + 8 \times 19.8$ $= 5 + 396/25$
March	19 = D_{Mar}	$\frac{521}{25}$	$S_{Mar} = 2 \times 19 + 8 \times \frac{521}{25}$
April	21		
May	20		$S_{May} = 2 \times D_{May} + 8 \times F_{May}$
June	25		$S_{June} = 2 \times D_{June} + 8 \times F_{June}$
July	----	F_{July}	

$$S_t = \alpha D_t + (1-\alpha) S_{t-1}$$

$$S_{Jan} = \alpha D_{Jan} + (1-\alpha) S_0 \quad S_0 = F_{Jan}$$

$$\frac{2(19) + 8(20)}{3 \cdot 8 + 16} = 19.8$$

$S_{Jan} = \frac{20\%}{80\%}$ to the current demand to the forecast (previous base)

$= \alpha = 0$
 $\alpha = 1$

So, here we have only considered the fluctuations in the base value. So, we require only one type of smoothing constant. You can recall that we discuss that in a very, generalized condition there

may be a requirement of three smoothing constants. In our coming sessions we will do one case where all three smoothing constants are required.

But here this is a simple case. So, we are using only one smoothing constant that is alpha. You can also recall that the value of smoothing constants vary from 0 to 1. The values vary from 0 to 1 and we also discussed that the popular values of alpha vary from point 1 to point 3. These are popular values and this is the possible range. So, here randomly, we have taken a value alpha equals to 0.2. We have taken the value alpha equals to 0.2, and by taking this alpha equals to 0.2, we will update the value of base table

And therefore we will use this equation, if you recall that our S_t equals to αD_t plus 1 minus αS_{t-1} . This equation you may recall, which we have discussed in our previous sessions. So, now S_t is S January equals to α into D January plus 1 minus αS_{naught} .

So, if you go with our available data, demand of January is 19, S_{naught} is 20. So D_{Jan} is 19 and S_{naught} is 20. Value of alpha which we have taken is 0.2 and this becomes 0.8. So, here this becomes 3.8 and this becomes 16, so it becomes 19.8. So, this 19.8 will be the value of base which you can put here, or this is the base of January or it is a forecast for the month of February. Now using this 19.8, how did we do this calculation?

You can use this complete formula. Now we will do this calculation in this table itself so that you can understand that how this data is flowing and we can also add one more column of forecast. So, the forecast for the month of January was 20, now the forecast for the month of February is 19.8. So, you can understand that these values of previous periods are the forecast for the next period.

Now this D February is known to us. So, now my job is to determine the value of S February. Again, use the formula αD_t , so 0.2 in to D_t that is 25 plus 1 minus α that is 0.8 into S_{t-1} that is 19.8. If I do this calculation, this becomes 5 plus 19.8 into this 0.8, I solved it. And we get 396 by 25. That becomes plus 5, so this becomes 521 by 25. So, that is the value of S February and this is going to be the forecast for the month of March.

Now again, in this iterative manner we can go forward. This is D of March, now D of March we will use to get the updated value of S of March which is 0.2 in to D_t that is 19 plus 0.8 in to this

S_{t-1} , that is 521 by 25. And in this fashion, you keep on doing the iteration and finally, you will find the value of S_{June} and that will be the forecast of July. So, it is a very sequential way of calculation of the forecast.

Now the simpling of this whole project is that, if I have the value of let us say, this previous period that is S_{May} which is 0.2 in to demand of May plus 0.8 into forecast of May. Now with this, if I am going ahead and I have the demand data of June. So, only by doing a slight change you can calculate the value of F_{July} . So, the iterative nature of this calculation is so simple that whenever data gets developed, whenever new data is available, you will be able to update your forecast without much effort.

You need not to go back. You need to consider only the current equation and by putting your new values into that current equation, you can update your forecast. So, this is the forecast for July. So, in this case, we took the monthly data so the forecast are updated on the monthly basis. Similarly, you can do it on the weekly basis. You can do it on the quarterly, basis so you can choose a period of forecasting.

And depending upon the requirement of your organization, you can develop this kind of table. And nowadays it is very simple that even the simplest software like Excel can help you in doing this iteration automatically. You can put this formula like $\alpha D_t + 1 - \alpha S_{t-1}$, this formula you can put in the Excel and Excel this formula. This is the important formula. And by putting this formula, you can automatically get iteration step by step and your forecasting will be done.

So this is the simplest method of exponential smoothing. Now you can see that the weightage of our in this particular case, if you remember, here we took S_{January} where we are giving 20 percent weightage, where we are giving 20 percent weightage to the demand of current period and 80 percent weightage we are giving to the forecast of January. This S_{naught} is basically F_{January} . Now you can recall it.

So here in this formula, you see 20 percent weight to the current demand and 80 percent to the forecast or previous base. Now you can recall we have already discussed again, I would like to reiterate that point. In case, this weightage is like that, α is equals 0. Now in this case, α

is equals to 0, you are giving 0 percent weightage to the current demand and 100 percent weightage to the previous case.

So, it means there are some fluctuations in the current period which are of extreme temporary nature. These are not going to be recurring, these are not going to affect your future demand. So, you want to discard those temporary changes you do not want to include that phenomena in your future calculations. So, in that case, you take alpha equals to 0 that is one extreme case. Alpha equals to 1, when you are considering that 100 percent changes need to be incorporated.

Here, your base has shifted to a new value and you do not want to continue with the previous base. In that case, you take alpha equals to 1. So these are the extreme conditions. It is important to remember these things again and again, and therefore you will also be able to appreciate the meaning of popular values of alpha that why the smaller values of alpha have more smoothing effect.

Larger values will give you a very impulsive kind of forecast, but smaller of alpha will give you a, more smooth values of forecast. So, we use more, smaller values so that your curve of forecast is more, smooth one? Now we go further in this discussion and here taking the same data you remember these old table.

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Example (Take $\alpha = 0.2, \beta = 0.1$)

$\alpha = 0.15, \beta = 0.05$
 $\alpha = 2, \beta = 0.15$

Month	Actual Demand	Base Value	Trend
Initial	---	Forecast 20 S_0	0 T_0
Jan	19 ✓	$F_1 = S_0 + T_0$ $S_1 = 19.8$	$T_1 = -0.02$
Feb	25 ✓	$F_2 = S_1 + T_1$ $= 19.78$	$T_2 = 0.1(S_2 - 19.8) + 0.9(-0.02)$
March	19 ✓	$F_3 = S_2 + T_2$	
April	21 ✓		
May	20 ✓		
June	25 ✓	S_{June}	T_{June}
July	-----	$F_{July} = S_{June} + T_{June}$	

$F_{t,1} = S_t + T_t$

$F_{July} = S_{June} + T_{June}$

$F_{June} = S_{May} + T_{May}$

- Smoothing the Average

$$S_t = \alpha D_t + (1 - \alpha)(S_{t-1} + T_{t-1}) \quad (A)$$
- Smoothing the Trend

$$T_t = \beta(S_t - S_{t-1}) + (1 - \beta)T_{t-1} \quad (B)$$

where: α = smoothing constant for the average
 β = smoothing constant for the trend

$$S_t = \alpha D_t + (1-\alpha)S_{t-1} \quad \checkmark$$

$$S_{Jan} = \alpha D_{Jan} + (1-\alpha)S_0 \quad S_0 = F_{Jan}$$

$$\frac{2(19) + .8(20)}{3.8 + 16 = 19.8}$$

$$S_{Jan} = \frac{20\%}{80\%} \text{ to the current demand to the forecast (previous base)}$$

$$= \alpha = 0$$

$$\alpha = 1.$$

The first was 19, 25, 19, 21, 20, 25. These were the demands of six period. So, the same data which we were discussing. Again we are going to use the same data. Now the base value again we have assumed 20. But now I feel that my data may have some more characteristics also and I am thinking that my data has linear trend. So, now I am going to use a smoothing model, which requires two smoothing constants, and therefore I have taken alpha and beta as my smoothing constants.

Alpha to smooth the fluctuations of the base value. Beta to smooth the fluctuations of the trend value. So, I am going to use two smoothing constants alpha for base, beta for trend. There may be another complicated situation, where I may use alpha, beta, gamma all three, but we will go for that situation in our coming classes. In this case, we are going to use two smoothing constants alpha and beta. Now in this particular case, we are assuming to start our discussions. We are assuming that the initial base value is 0. Base value is 20 and initial trend value is 0.

Now you see here what we need to do, we have to smooth the base value using this formula. You can say that this is formula A. Then we also need to smooth the trend, this is formula B. And then we also need to know what is the forecast for the next period. So the forecast for the next period is since it is a case of linear trend will be this.

Forecast for the next period will be the sum of the base value or the average value and the trend value, so updated trend plus updated base will be the forecast for the next period. Meaning is if I am forecasting for July, so the base of June and trend of June, their sum will give me the forecast

for July. If I am forecasting for June, so base of May and trend of May will give me the forecast for the month of June.

So this way, earlier in the previous discussion, previous example, the forecasting was only limited to this particular component. The second step trend was not there. But now I am saying that data may have trend and therefore I am taking the advance model where we have this base as well as trend component also present in to my data and here I am going to use two smoothing constant and that is how the forecast, if you see will be the sum of these two.

So this is S_{naught} , this is T_{naught} . So this is F_1 , so this is F_1 equals to S_{naught} plus T_{naught} . Similarly this will be S_1 , this is T_1 . So, F_2 will be S_1 plus T_1 and so on if I talking of F July. This is S_{June} , this is T_{June} . So, F_{July} will be S_{June} plus T_{June} . So, the same iterative process will be followed in this case but here, we need to continuously update two values one of the base and second of trend.

And how we are going to do that formula is available to us that first you will update the value of base using αD_t plus 1 minus α and this you see S_{t-1} plus T_{t-1} . This is actually the forecast. This is the actually the forecast this S_{t-1} plus T_{t-1} , this is actually the forecast for the current period.

So, you can now relate it with our earlier model that this T_t , this is T_t and T_t means the current trend. So here what we are doing, the component of previous trend and the difference of this since this is a linear trend. So, the updated base minus the previous base that will give you the value of current trend and the previous trend and on the basis of that, you will calculate the updated smooth trend and the sum of these two will give you the value of your forecast. Now, let us do calculation of this and for that purpose, let me have some values with us.

So, we start with the first value of D January, so D January is given to us as 19. S_{naught} is given to us as 20, T_{naught} given to us as 0 and with the help of this. You can also say that F_1 or F January equals to 20, 20 plus 0, so F January becomes 20. Now my job is to calculate the value of S January and T January.

S January and T January will give me F February. So, now let us start our calculation for S January and T January. Now formula is again go back to this slide, S_t equals to αD_t plus 1

minus αF_t . So S_{January} equals to αD_t plus 1 minus αF_t . So this becomes α is what we have taken as α 's value, 0.2 . So, this we can write as 0.2 into D_t , D_t is the current demand that is of January 19 plus 1 minus α 0.8 and F_t , that is the forecast for January that is 20 .

So, this becomes 19 into 2 , 3.8 plus 16 that become 19.8 . This is the value of S_{January} that is 19.8 . Now I want to calculate the T_{January} . Now T_{January} is S_t minus S_{t-1} plus 1 minus βT_{January} minus 1 means T_{naught} . Now β 's value we have taken here as 0.1 , so $0.1 S_t$ we have just calculated is 19.8 . S_{naught} was 20 plus 0.9 , T_{naught} was given as 0 . So, here this is 0.1 into 19.8 minus 0.2 plus 0 . So, it becomes -0.02 that is the value of trend for the January.

So, you can now put these values that S_1 is coming 19.8 and T_1 is coming -0.02 . So, your F_2 will become 19.8 minus 0.02 . So, F_2 will become 19.78 . Now, this value will be used for calculation of S_2 and T_2 and it is possible now that we can do this calculation in this table itself so that we need not to go again to next slides and come back. So, now S_2 will be, you see this formula αD_t plus 1 minus αF_t . So, α is 0.2 into D_t that is 25 plus 0.8 into F_2 that is 19.78 .

T_2 will be β 0.1 , so this value of S_2 minus S_1 , that is 19.8 plus 0.9 into T_1 that is -0.02 , so you get the value of S_2 and T_2 now by doing this calculation, you can get the values of S_2 and T_2 and this S_2 and T_2 will give you the value of F_3 that is S_2 plus T_2 . So, now as we just discussed in the previous case, in this case also, you can develop an iterative method of calculation and that iterative method of calculation will help you in doing this calculation in a very simple, straightforward manner.

It is a more cumbersome method because you are simultaneously need to smoothen two values base and trend better if you just develop a standard process in Excel. It is very simple, just by updating the values, you keep on getting the new value and finally, you get the value of F_{July} . That is sum of S_{June} and T_{June} . Now the important point which we need to see that we have done this calculation for the same set of actual demand $19, 25, 19, 21, 20, 25$.

In this case, we consider that trend is present in the data and therefore we took these two smoothing values. In the previous case, we did not consider the trend. We only took as our base values and there was no presence of trend with both these methods we got the values of

forecasting. Now it is important to see which method is more useful in the given situation. So as a forecasting manager, as operation manager, it is nobody going to tell you that which method you apply.

It is your wisdom that which method is more suitable for my given data and for that purpose we are going to discuss in one of our coming sessions forecasting errors. Use of the concept of forecasting errors will help us in determining the suitability of a particular method. If our forecasting error measures are giving lower values that method is a suitable method.

If those values are higher, then we need to see some other method because if forecasting is done with a lot of mistakes, if it is not accurate, then whole production planning will suffer. You will not be able to take the advantage of the production planning. So, therefore it is important that we understand this particular thing properly and it is not only this suitability of method. In this particular case, we took the value of alpha equals to 0.2 and beta equals to 0.1.

Now since I have shown you manual calculation, you may be a bit afraid, but when we use calculators, when we use our software for this purpose, you can play with the values of smoothing constants also. You can take some other combination you take alpha equals to 0.15, beta equals to 0.05. You can take alpha equals to 0.2, beta equals to 0.15, you can take alpha equals to 0.25, beta equals to 0.15.

So, you can take any combination of alpha and beta's value and just by if you have everything done in your Excel spreadsheet, it becomes very, very convenient to do this kind of play with your data. Now which ever set of alpha beta values give you minimum forecasting errors that is a suitable values of alpha beta for your case because it is very, very difficult from naked eyes to tell that this particular value of alpha and beta is suitable for this particular data.

Only by iterations, only by putting different values of alpha and beta and checking those alpha beta values for forecasting errors will tell you that this set is a suitable value, so we need to see and we will see that when we will discuss the forecasting error in our coming session.

So, with this, we discussed in this particular session that how to apply basic exponential smoothing model and then we also discuss the same set of data for exponential smoothing with

linear trend. In our next session, we will discuss one more example where we see the seasonality component also included in our demand data. So, for this thank you very much.