Production and Operation Management
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Lecture 10
Time Series Forecasting - Exponential Smoothing-II (Classification)

Welcome friends. So, now we are coming to end of week 2 of this course, we are discussing about forecasting since last 2, 3 sessions. In the last session, we started the discussions on exponential smoothing models. Exponential smoothing models are very, very efficient models of smoothing. And these models help us effortless updation of our forecast.

And with the help of these exponential smoothing models, we can do forecasting of large amount of items which are available in our stores. We discussed basic exponential smoothing model in our last session. The concept of basic exponential smoothing model will now be extended to those models were trend, seasonality, other kind of characteristics are also available in our historical data.

And with the help of that, now, we will be able to use this exponential smoothing models for more generalized kind of forecasting activities.

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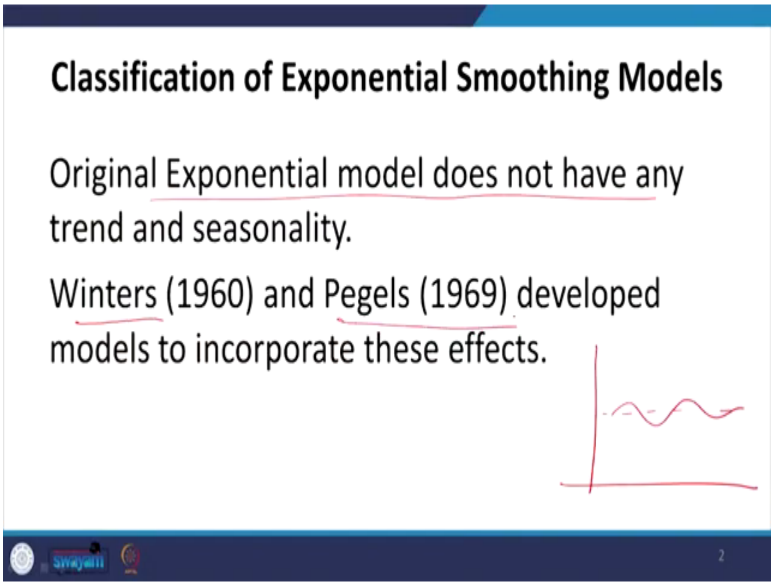

So, the original model or the basic exponential smoothing model, which we discussed in our previous class, it was having only one component that was the base component and we said that the fluctuations are around this base value and we wanted to smoothen those fluctuation. So, that my actual demand data actual demand curve is coinciding with that dotted line. But we all know that our historical data may have different types of pattern.

So, how to use exponential smoothing models for those types of data that was done by Winters and Pegels, over the period of those 10 years, from 1960s to 1970, they developed different types of forecasting models, which were based on exponential smoothing and these models were incorporated the fact of trend as well as the fact of seasonality. Now, in this particular class, first we will see that what are the particular combinations with those trends and seasonality.

So for that purpose first, we will see what are the different types of trend? And what are the different types of seasonality? Which are there? And then we will see the combinations of trend and seasonality also.

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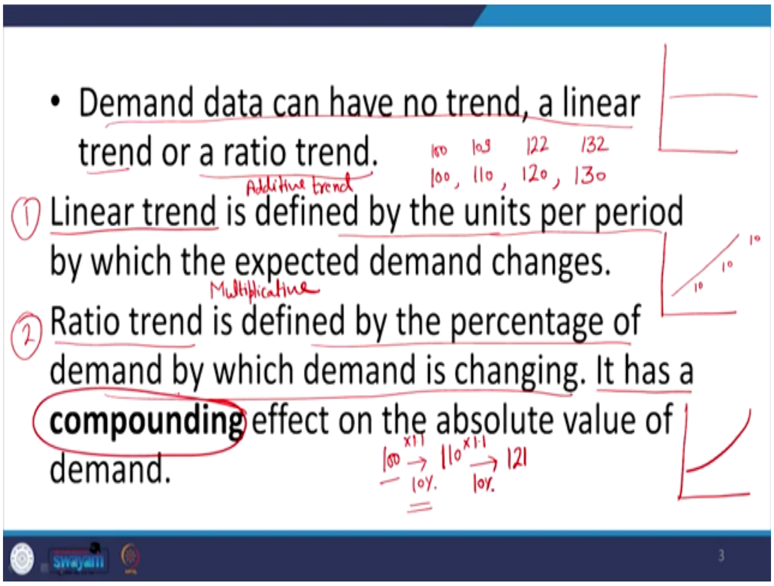

Now, demand data, historical demand data can have no trend or it can have a linear trend or it can have a ratio trend. So, there is a horizontal data like this, where no trend is there or it can have a linear trend or it can have a ratio trend. Now, linear trend like demand data is increasing over a period of time.

Linear trend is also known as additive trend. It is known as additive trend. The meaning is that demand is changing over a period of time by some number of units. For an example, I can say that demand is changing over a period by 10 units that from one period to another period, there is an increment of 10 units in the demand.

In January demand is 100 units in February it becomes 110, in March it becomes 120 then it becomes 130. So, if this type of increases there, if this type of trend is there, it is additive trend that demand is increasing by a fixed number or more or less a fixed number that is not a fixed number exactly, actually the demand will very like 100 to 109 to 122 then 132 something like that.

So, this is actually the variation of demand. On the other side, there is a different type of trend also, which is known as a ratio trend. The ratio trend is defined by the percentage of demand by which demand is changing. Now, here the data is like this, demand is changing from one period to other period in terms of some percentage, demand is 100 here, then demand is increased by 10 percent, it becomes 110 it is increasing by 10 percent it becomes 121.

So, like that over a period of time demand is changing by some percentage. So, this is known as ratio trend or it is also known as multiplicative trend where you have a multiplying factor, you have a multiplying factor. So, here the multiplying factor is 0.1, every period demand is increasing by 10 percent. So, the increment is of 0.1 factor and you multiply the demand of hundred with 1.1 you get 110, you multiply it again by 1.1 you get 121. So, that is a multiplying factor. So, you can also say that the ratio trend shows you a compounding effect on the absolute value of the demand.

Demand has a compounding effect because of these multiplying activities. So, that is the different type of trend which may be there in your historical data.

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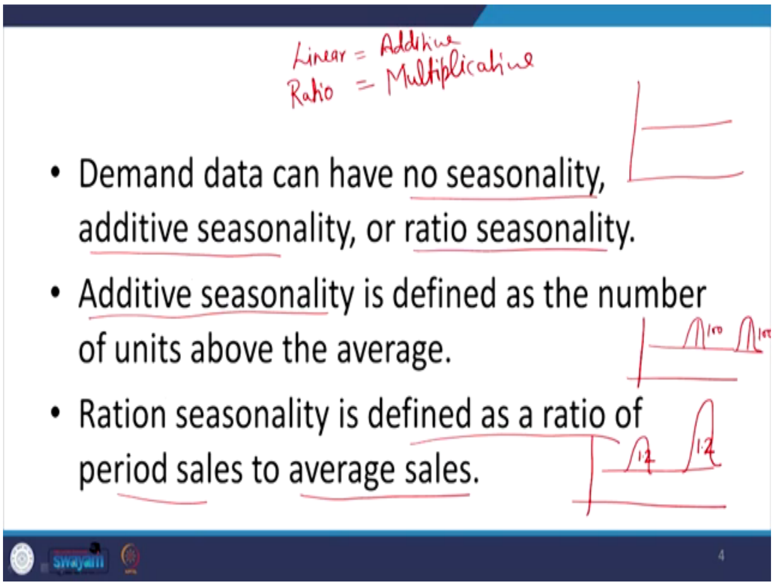

Similarly, seasonality is also there. Either your data has no seasonality. It can have additive seasonality, or it can have ratio seasonality as we discussed about trend, three characteristics of seasonality are also possible. Now, when, there is no seasonality. So, data is like horizontal there is no seasonality. So, it is like a horizontal demand data.

But when there is additive seasonality, when there is additive seasonality, it means that demand increases to a particular number of units above the average number. So, like this is the average values. Now, the period of seasonality comes and during that period of seasonality demand increases to a high number, then again period of seasonality comes, demand increases to a high number.

So, that is the meaning of seasonality that whenever seasonality comes demand increases by 100 units. So, you add that number to the basic value and that is the meaning of additive seasonality, the ratio seasonality, that number is in terms, the increases in terms of some multiplying factor.

So, when we are talking of ratio seasonality, it is in terms of some number, which is giving you a multiplying factor. So, therefore, as you move from one season to another season, because this is a multiplying factor, let us say the seasonality is 20 percent. So, because you have some kind of increase in the demand also.

So, the increase in the seasonal factor becomes much, much more visible. So, it is a ratio of period sales to the average sales. So, it is also known as a multiplicative seasonality. It is also known as additive or linear seasonality. So, you have linear which is similar to additive. So, whether you use these words in case of trend or seasonality.

Similarly, you have a ratio or multiplicative. These words you can use either for trend or for seasonality. So, it is same for both these things. Now, you see, we have three conditions for trend, no trend, linear trend, ratio trend, we have three conditions for seasonality, no seasonality, linear seasonality, or ratio seasonality. So, 3 into 3 we may have 9 possibilities. So, let us see those different possibilities that how you can have the combination of these different type of phenomena.

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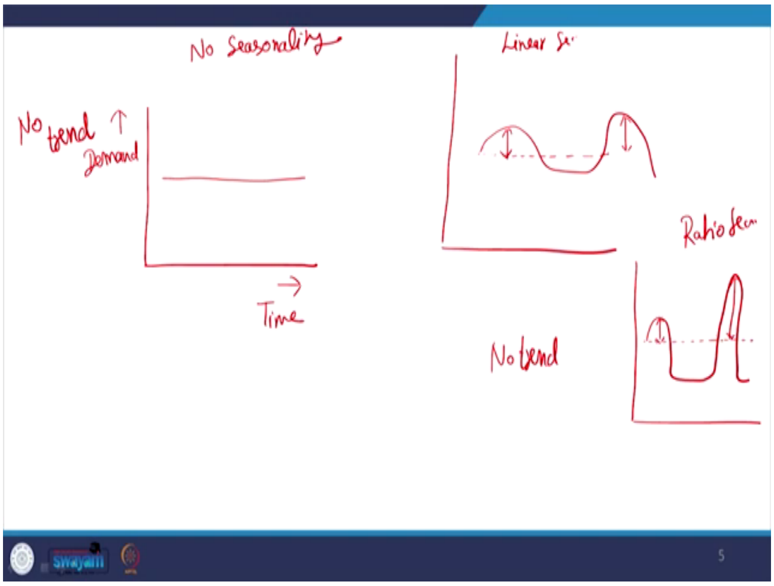

Now, first we are going to have a curve where, there is no seasonality and no trend, no trend, no seasonality, and this is a very simple kind of curve, which is a horizontal demand curve. On y axis, you have demand data and on x axis, it is time period. So, no seasonality, no trend, but you may have another combination where you have linear seasonality, but no trend.

When you have linear seasonality and no trend, in that case, what is going to happen that your demand data is going to behave like that where you will have this kind of average or low demand during the period of non-seasonality, whenever the period of seasonality comes, the demand increases to a high value.

So, this is the period of seasonality where only because of seasonality demand increases to a high value otherwise, it remains at a low level and then you have a third type of characteristic also no trend and ratio seasonality. Where there is no trend and ratio seasonality, so there is no trend so, this is the base data.

And here my seasonality factor is like this from period to period, my demand will increase in the terms of ratio, what I am going to have from base value to this peak value. So, these are the three types of curves, which are possible when there is no trend, but it will become more interesting when you have linear trend.

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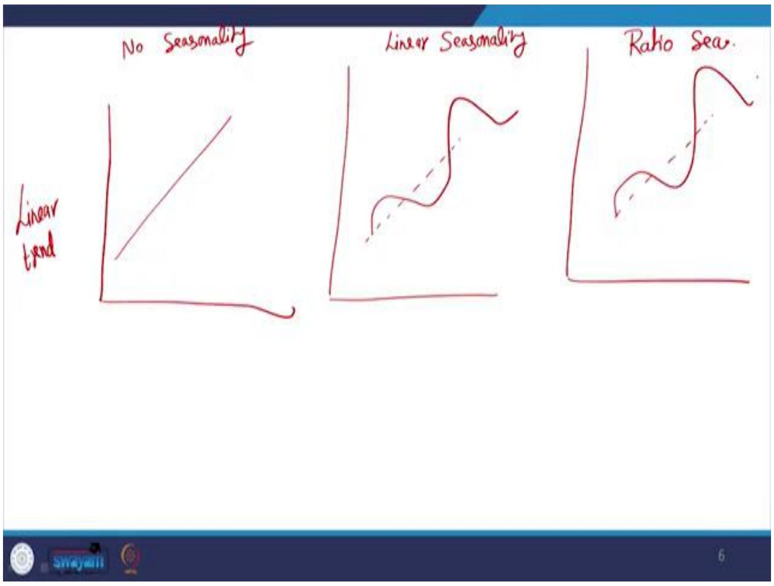

And first case is no seasonality. You have linear trend and no seasonality. So, in this case you have this type of demand data, some angle is there, because data is demand is increasing, but there is no seasonal component. Now, you have two more interesting figures, one is when you have linear seasonality and then you have ratio seasonality. So, here we take this as our baselines. Now, linear trend and linear seasonality this combination you can have like this.

And similarly, you can have a combination of ratio trend, ratio seasonality and linear trend. So, these are the combinations, where demand data is increasing naturally and on the top of that, we have a seasonal factor also. So, these are more complicated historical data cases.

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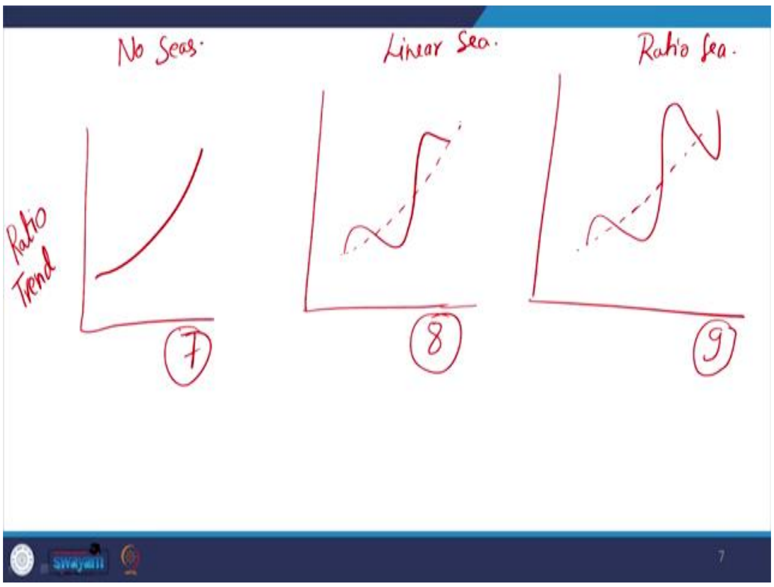

And now, you can have third possibility, where you have ratio trend and in that ratio trend also you have three cases no seasonality, linear seasonality and ratio seasonality. So, these are the three more possibilities. Now, when you have ratio trend, and no seasonality you have this type of curve which is giving you some kind of ratio trend multiplicative factor without seasonality.

When you want to have linear seasonality on the top of ratio trend, we draw these type of dotted lines, because we are doing all these things manually. So, maybe our curves are not very accurate, but it will give you a feel that how demand data is going to change in these cases.

So, you have linear trend and top of that, you have this kind of linear seasonality, you have linear trend, ratio trend and on the top of that, you have this type of ratio seasonality. So, if you give some kind of numbering, let us give this figure where ratio trend and linear seasonality, the figure number 9, this is figure number sorry 8 this is 9, this is 7.

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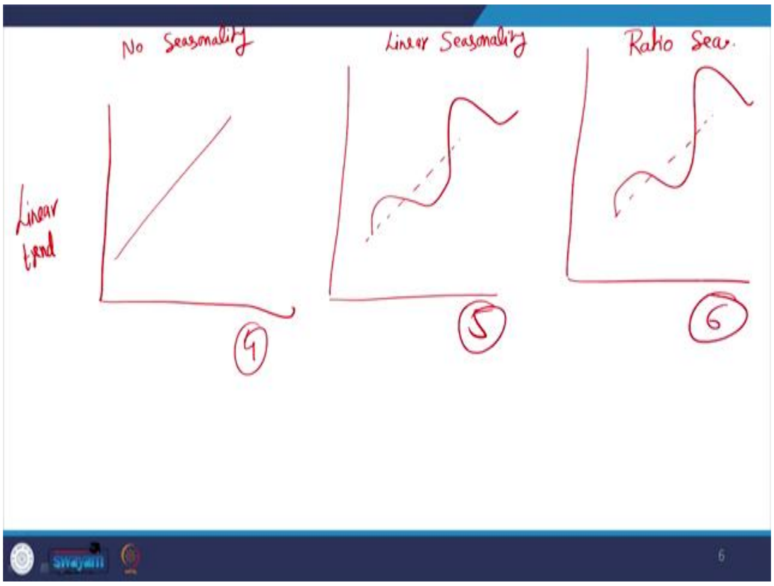

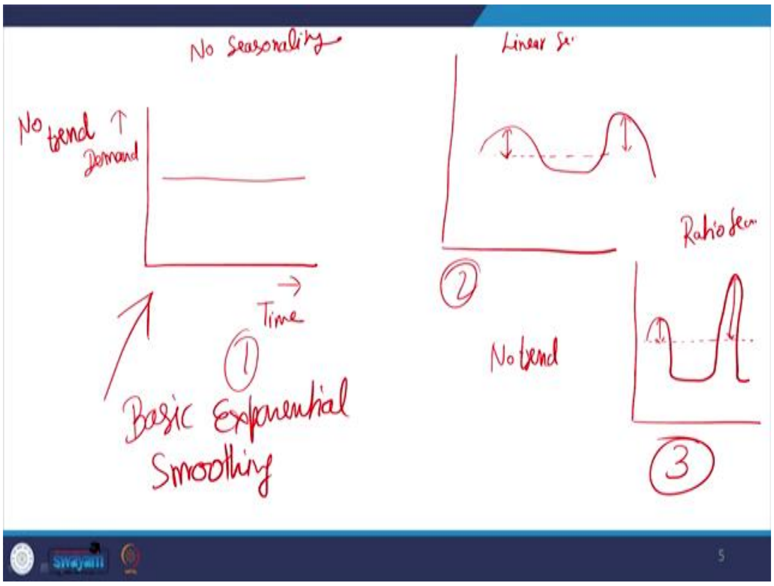

Then in the previous, these are 4, 5, 6 and in the previous it is 1, 2 and 3. So, we have in all these 9 different types of models and we can develop our extrapolative or exponential smoothing models for all these 9 types of models. It is quite possible that all these 9 models we can very easily develop, we have already discussed in our previous session, the basic exponential smoothing model which was applicable to this particular case.

When no trend and no seasonality was there. So, that basic exponential smoothing model where we used alpha as smoothing constant that was discussed in this particular case.

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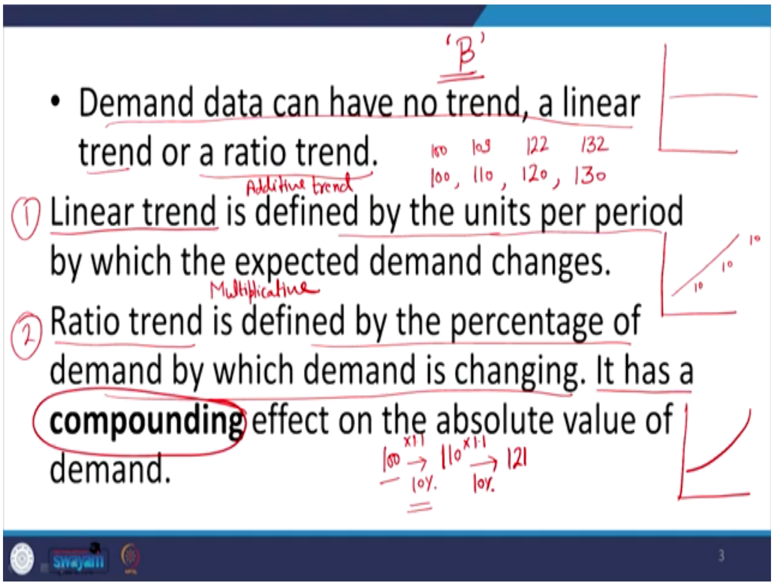

Now, when we are going to have these additional things also, when you want to have trend also in your data, you will use one more smoothing constant that is beta, beta will be used for smoothing the fluctuations of your trend data.

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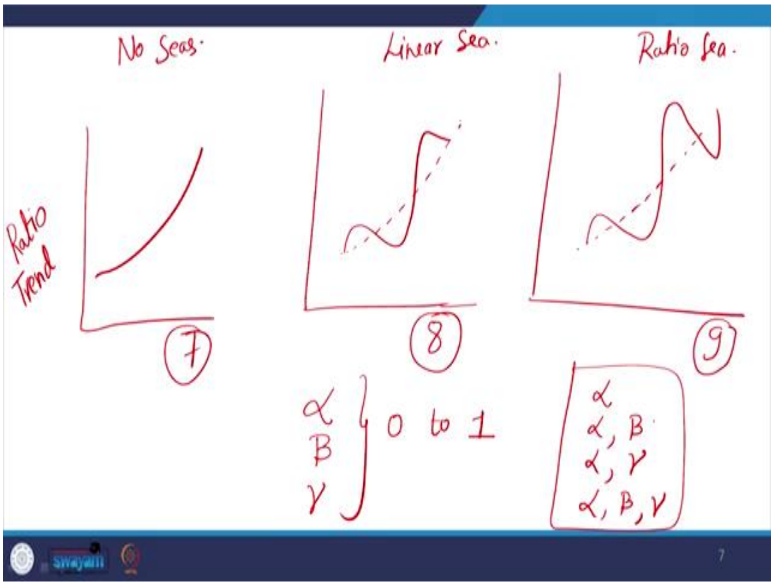

When you are going to use seasonality also in your demand data, you will use one more smoothing constant that is gamma. So, we have in a very general situation, three types of smoothing constants, alpha, beta and gamma values of alpha beta gamma. Values of alpha, beta and gamma varies from 0 to 1 because all three are smoothing constants.

In a basic exponential smoothing model we use only alpha when trend is also present, you have to use alpha and beta when only seasonality is present, you have to use alpha and gamma. And when trend and seasonality both are present, you have to use all three smoothing constants alpha, beta, gamma.

So, you have possibility of using either alpha alone, you can use alpha and beta, you can use alpha and gamma and you can use alpha, beta, gamma altogether. So, depending upon what type of demand data you have, these are the possibilities of using my smoothing constant when I am using smoothing constants, it is possible, means there is no hard and fast rule, it is possible that the values of all smoothing constants are same or it may be different also.

So, depending upon situation to situation you can have alpha, beta both 0.2, 0.2 or for alpha it can be 0.2, for beta it can be 0.1, in alpha beta gamma, one value of alpha is 0.3, beta’s value can be 0.2 and gamma’s value can be 0.1 and it is also possible that we have 0.1, 0.1, 0.1 as the values of alpha, beta, gamma.

So, you have no restriction on the combination of alpha beta gamma’s value, the only limitation is the boundary condition the values will vary between 0 to 1. Now, if you remember the basic exponential smoothing model we discussed already in our previous session.

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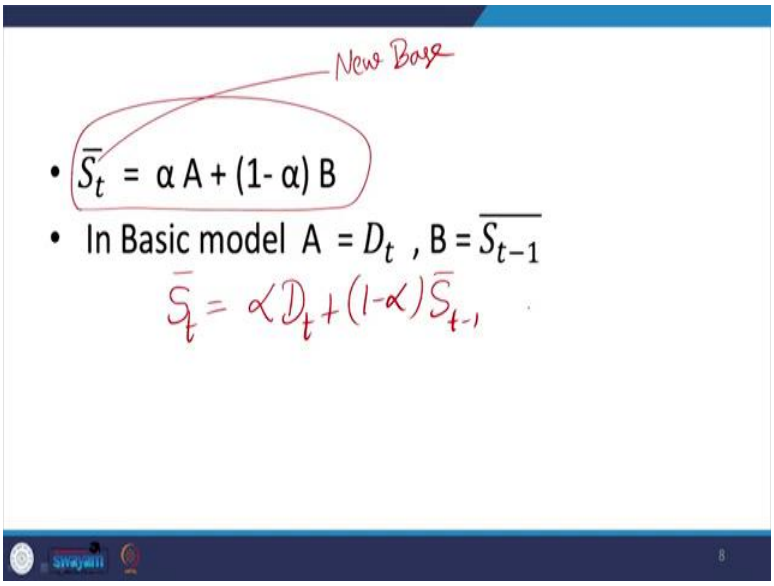

So, the equation which we developed is of this type St that is the updated base, St is the updated base or the new base. It was alpha A plus 1 minus alpha B, A and B. We are using these two things in a generalized fashion right now, in my exponential smoothing model A was Dt and B was S t minus 1.

So, the model was like alpha Dt plus 1 minus alpha St minus 1. This was the model which we have already discussed in our previous session. Now, this particular equation will be the base for developing the models for our advanced algorithms.

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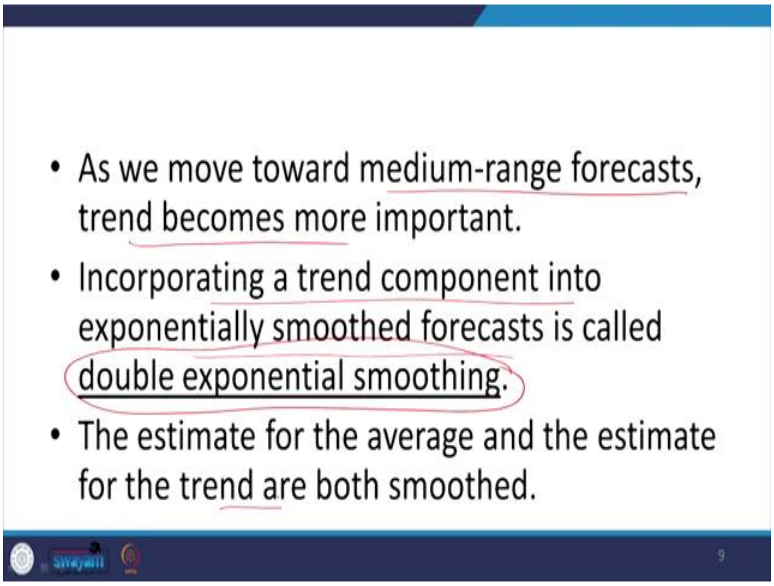

Now, in this particular case, we will see that now we are moving towards medium range forecast. We can do those forecasts, where trend data is having some kind of importance. Seasonality data has some kind of importance, if you do day to day forecast, in that case, trend and seasonality data may not have that importance, but whenever you are forecasting for a longer period for some two months, three months of period, then the trend data, seasonality data may have some kind of importance.

So, one important thing about the usefulness of this model is that though we discussed in our previous session, that exponential methods are more suitable for short term period, immediate period, but that is the basic exponential smoothing model. When we are going for trend or seasonality inclusive models.

These are suitable for medium range forecast. Then incorporating a trend component into exponential smoothing forecast is also known as double exponential smoothing, why double exponential smoothing? As I already told you that we require two smoothing constants, alpha and beta one for basic smoothing and the second is for smoothing of the trend.

So, therefore, these are also known as double smoothing models and the estimate for the average and the estimate for the trend are smoothed separately. So, we required to have separate smoothing for these two types of data.

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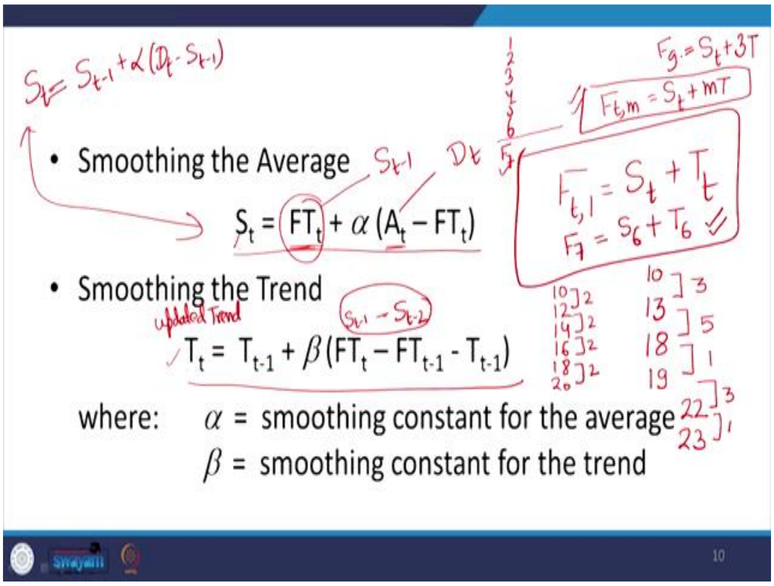

Now, how do we do that? For that purpose, you have these two formula. One is for smoothing the fluctuations of your base value, and second the smoothing the fluctuation of your trend value. And then with the help of the smooth values of base and smooth value of trend, we will get the forecast for the next period.

So, Ft 1 will be the St plus Tt. This will be the forecast for my coming period. Now, if you see the development of this formula in line with my basic exponential smoothing method, so you can very well appreciate that St minus, St was equal to St minus 1 plus alpha into Dt minus St minus 1. This was the formula we used for development of our new base.

So, same is here. We are using different conventions in different slides. The very purpose is that if you are following different types of books, so, different books may give you different conventions. So, you are not going to stick with a particular convention, you should be able to apply the convention of book with actual understanding of the concept.

So, conceptual clarity is more important rather following a particular convention of writing those equation. So, here I am using this word Ft that Ft is the forecast, forecast of the period, current period and At is the actual demand of the current period. So, this is the formula of this it is actually FTt is nothing but St minus 1. So, just to familiarize you with the convention and this At is nothing but Dt.

So, you can now compare this formula and this formula these are same. So, we have done the smoothing of average or the base value. The second is now, we also need to smooth the fluctuations of trend, why I am saying the fluctuations of the trend, demand was turned in the first period, it increased to 13 in the second period, it increased to 18 in the third period, then it increase to 19, it increased to 22, 23 and like that.

So, here demand increased by 3 units. Here demand increased by 5 units, here demand increased by 1 unit, here demand increased by 3 units, here demand increased by 1 unit. So, this is the trend value, that first period to second period demand increased by 3 units, then by 5 minutes, then by 1 unit, then by 3 units, and then by 1 unit. But these values are also not constant. There is a fluctuation in this trend value also.

So, if there would have been a situation like 10, 12, 14, 16, 18, 20 in each period demand is increasing by exactly 2 units, then I say that there is no fluctuation in the trend data and in that case, there is no need of this second part of this process, because there is no fluctuation in the trend data.

So, what to smooth? But here we see the trend data has fluctuations, it is 3, 5, 1, 3, 1. So, I need to (fluct) I need to smooth the fluctuations of this trend data also, increment from one period to another period is not constant. So, that is the second part of this process Tt that is the current value or the updated trend. This is updated trend and this is the trend in the previous period.

Now, this beta is the smoothing constant for the trend, beta into FTt that is St minus 1 minus Ft minus 1, this is St minus 1 and this is St minus 2. So, you understand this is giving you this particular value or this particular data is giving you the value of trend. This is the value of trend available right now, and with the help of this value of trends you are going to update the current trend.

And when you have St and Tt updated St and updated Tt, you will sum them, you will sum them, and that will give you the forecast for the next period. Now, when we have data for, let say 6 period, and 1, 2, 3, 4, 5, 6, I want to do forecast for the seventh period. So, what I am going to do for seventh period forecast, I will do the calculation which involves getting the updated value of base for the sixth period and getting the updated value of trend for this sixth period.

So, T 6 plus S 6 will give me the F 7. But sometime it is also possible that with this data you want to forecast for ninth period, you want to forecast for the ninth period with the help of data available up to sixth period. So, in that case, we assume that trend is going to remain constant, we assume trend is going to remain constant.

And here because we are here in the sixth period, ninth period is 3 period ahead. So, you say, it will be a consideration of three period ahead and the current trend is considered to be constant. So, it will be written as 3T. So, if I want to write that I want to forecast for m period ahead, if I want to forecast for m period ahead, it will be St plus m into T that will be the equation which will be used, though, I also like to warn you, that the quality of forecast will not be very accurate with this type of long forecastings, you will have the best forecasting for the next period.

So, the best results will be available for this, but in case you require some forecasting in this particular type of situation, we can use this equation also Ftm where with the current base and current trend, you can forecast for m period ahead and F 9, F 10, F 11, etc. can also be determined in the similar fashion.

So, with this we come to end of this session. In our next session, we will handle some numerical examples to actually demonstrate that how do we do all these calculations and how do we play with the historical data. Because, this is one example where we are considering only this linear trend.

Similarly, you can also consider ratio trend. And then we also need to see how to handle the seasonality in our historical data. So, in next session, we will do many numerical examples, which will help you better understand the concepts which we have just discussed about using trend data in our exponential smoothing models. Thank you very much.