Financial Derivatives and Risk Management
Professor J. P. Singh
Department of Management Studies
Indian Institute of Technology Roorkee
Lecture 09: Basics of Futures Hedging

Determinism & randomness

It was the belief of Albert Einstein that the universe is intrinsically deterministic. He had once said that "God does not play dice", so let us first understand what is determinism. Determinism means that, given the state of a physical system at any point in time, say t=0, we can precisely, exactly predict the state of the same system at any future point in time.

The process of doing that is a standard problem in physics. We identify a set of variables that provide a complete description of the system and then we use physical laws to evolve these variables forward in time. These physical laws usually manifest themselves as differential equations, so that the solutions to these differential equations with the appropriate boundary conditions provide the complete future state of the system.

The important thing is that the values of these variables and therefore the state of the system can be completely determined at any future point in time. Einstein, Laplace and most other physicists of the era other than the advocates of quantum mechanics, firmly believed that the universe evolved in a deterministic manner.

But then as quantum mechanics came along, the concept of determinism took a huge beating. Quantum mechanics has a certain inbuilt indeterminacy in distinction to randomness. I think we should use the word indeterminacy as pointed out by Heisenberg.

To understand the meaning of classical randomness, let us first talk about quantum randomness. The first thing that we must know is that quantum effects are perceptible at subatomic scales, at scales which are very-very small compared to everyday life. Physics at the scales of everyday life are well explained by Newtonian physics or by the classical physics. We do not really perceive the quantum effects in everyday life.

However, at the subatomic scales, for example at the Large Hadron Collider, quantum effects become very pronounced. The issue in quantum mechanics is that of what happens when we make a measurement at that scale. For example, if we want to measure the position of an electron, we do it by bombarding it with a beam of very high-frequency radiation e.g. gamma rays and analyzing the scattered beam which hits the electron and comes back to us and on that basis we are able to determine the position of the electron. Now, what happens is that as soon as the photon comprising the high frequency incident beam hits the electron, there is a transfer of energy and momentum between the two and as a result of it the state of the electron changes and therefore, if we determine the position and momentum of the electron at the same instant of time, it is not possible to make a precise simultaneous determination of both. The interaction of the measuring apparatus with the measured object during the measurement process makes it impossible to measure completely the state of a quantum system with perfect precision. This is the kind of indeterminacy which is built into quantum mechanics.

However, at classical levels these effects are not pronounced. At the classical level randomness was perceived by Einstein as an absence of information for the complete description a given physical system. He used the word ‘ignorance’. The ignorance of the complete physical attributes of a system manifests itself as randomness.

Let us take an example, the toss of a coin is widely accepted as a random process in the sense that the outcome is unknown at tossing. It is unpredictable. However, let us assume that by some mechanism we are able to ascertain precisely all the influences and all other relevant information that affect the dynamics of the coin on tossing including how all these factors, their influences, interactions evolve in time e.g. the angle of flip, the torque imparted, the viscosity of air, the temperature gradient, wind speed etc, then, at least, in principle it should be possible to predict with certainty the outcome of the coin toss.

Proponents of determinism argue that given the initial condition we can determine precisely the final outcome (at least, in principle) provided all the requisite information is available. They feel that it is our lack of knowledge, lack of information about either the innumerability of the factors, their mutual interaction or interaction with the system under observation or the physical laws governing the time evolution or the mathematical representation of these laws etc that causes the imprecision in the ascertainment of the future physical state of a system. It is our inadequacy of knowledge, incompleteness of knowledge that results in the unpredictability of the outcome and what we term as a randomness.

When the evolution of a system includes an element of unpredictability and, therefore, it is impossible to ascertain the future state of a system with absolute precision. Classical randomness due is to inadequacy of information (Einstein). Quantum randomness is due to the measurement process (Heisenberg, Hawking).

The other school that believes that the universe evolves in a random manner. Votaries of this school are equally convinced that there is certain intrinsic underlying randomness in this universe, which we have accepted, at least at the quantum level. Neither perfectly precise predictions nor perfectly precise measurements are possible and therefore, there would be some measurement error which also contributes to the randomness.

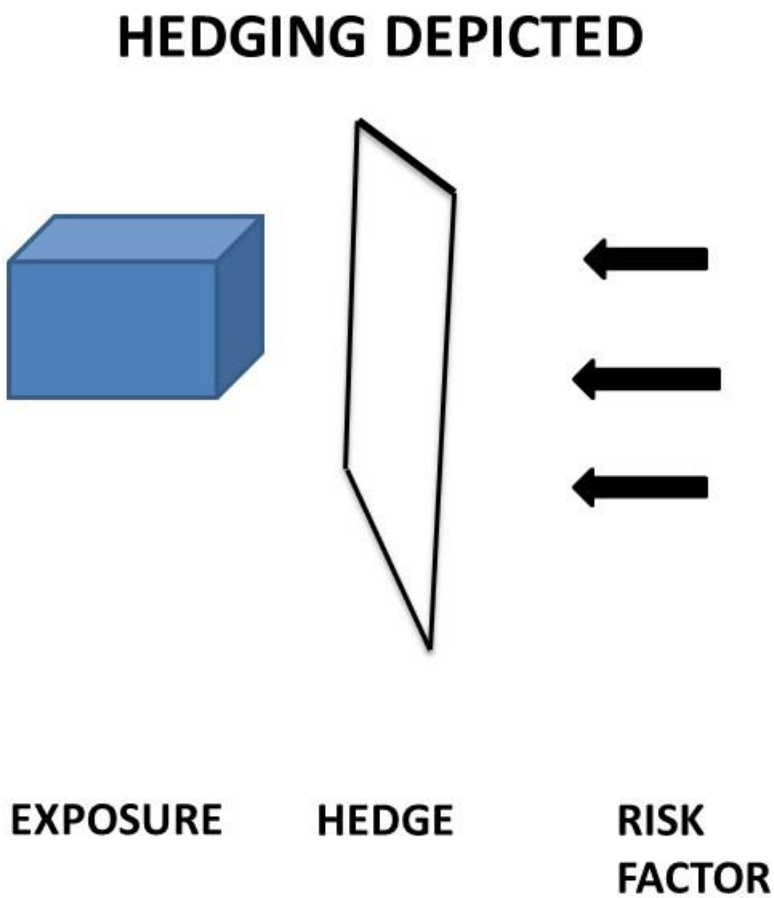
The Ehrenfest theorem in quantum mechanics validates that the cumulative effect of various quantum fluctuations results in a trajectory which in the mean value coincides with the classical path but it is essentially an aggregation of quantum phenomena. Quantum phenomena being random, the argument is that it is the cumulative effect of randomness that manifests itself as a perceptible determinism at the level of large scales but intrinsically the world is random.

This issue is still yet to be settled. However, for our purposes what we accept for the moment is that randomness does exist and it is probably due to our incompleteness of knowledge about the system. As a result of this we are not able to model each and every influencing factor in its entirety and therefore, predict the future evolution of the system with precision.

There is a quote by Stephen Hawking on this. It was in 1999.

"The classical view put forward by Laplace was that the future motion of particles was completely determined if one knew their positions and speeds at one time, this view had to be modified when Heisenberg put forward the uncertainty principle which said that 'one could not know both the position and the speed accurately'. But even this limited predictability disappeared when the effects of black holes were taken into account. The loss of particles and information down black holes meant that the particles that came out were random. One could calculate probabilities but one could not make any definite predictions. Thus, the future of the universe is not completely determined by the laws of science and its present state as Laplace thought 'God still has a few tricks up his sleeve'."

<u>Hedging</u>



Let us, now, take up hedging of a financial exposure. We have a risk factor S and we have an account V (an asset, liability or operating income). The value of this account V varies systematically with variation in S. We want to eliminate or, at least, minimize the impact of changes in this risk factor ΔS on this account V. This is the objective of hedging. If the risk factor value varies (ΔS) the value of the account also varies (ΔV). While the variation in the risk factor ΔS is believed to be exogenous and hence, uncontrollable, hedging attempts to minimize the impact of such exogenous changes on the value of the hedged asset V. Because entities prefer certainty to uncertainty, we want to evolve optimal mechanisms for the management of ΔV insofar as such changes in V relate to its systematic relationship with S. That is what we call hedging.

So, how do we do the hedging? We take a position in certain other financial instruments (called the hedging instrument H) such that when a change in the risk factor (ΔS) acts on the account V and as a result of it the value of this account changes (ΔV), it simultaneously acts on the hedging instrument H also (the same change in the risk factor (ΔS) acts on the hedging instrument H also) but it acts in such a way that the change in the value of the hedging instrument ΔH, is in opposite direction, and therefore, the changes in values of the account ΔV and the hedging instrument ΔH due to the same stimulus ΔS tend to annul each other and the value of the combination of the two (V+H) is minimally influenced by the variation in the risk factor ΔS.

Hedging is the practice of taking a position in one market to offset and balance against the risk adopted by assuming a position in a contrary or opposing market or investment.

Futures hedging, basis & basis risk

Now, when we talk about futures, a very important concept is the concept of basis.

Basis at any instant of time t is given by $b_t=S_t-F_t$. Since both S_t and F_t are stochastic processes and evolve in time with a random element, the basis is also a stochastic process. The basis is, therefore not precisely predictable at a future instant of time or that it will remain constant. This unpredictability of the basis causes imperfection in futures hedging termed as basis risk.

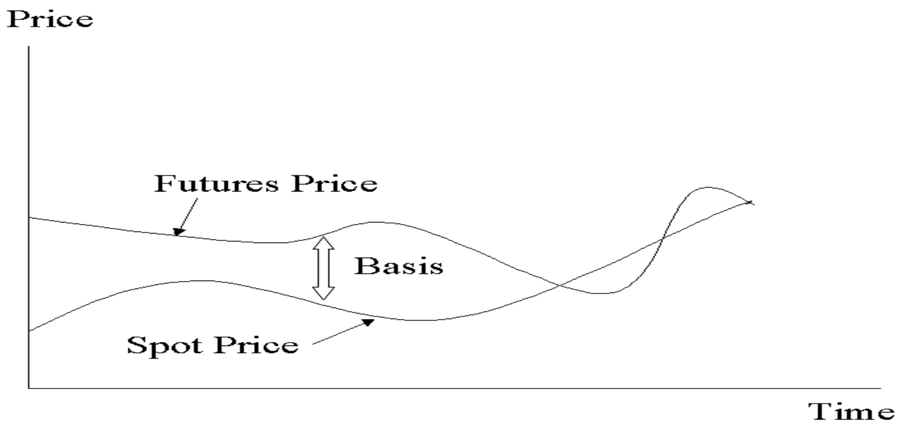
Because both spot prices and futures prices are stochastic processes that evolve in time as random variables, therefore the basis also is a random variable which evolves in time and it is also a stochastic process. Thus, the basis evolves in an unpredictable manner because the spot price and futures price both have embedded randomness.

Convergence of basis

.

Although basis does evolve as a stochastic process, being the difference of two stochastic processes, it has a certain special intrinsic property which is deterministic and which is enforced upon it be requirements of no-arbitrage in efficient markets.

It is that at the point of the maturity of the futures contract, the basis must necessarily converge to zero i.e. the spot price of the underlying and futures price, both on the date of the maturity of the futures contract, must converge.



Why? This is necessitated by considerations of arbitrage. Suppose the spot price exceeds the futures price on the futures maturity. The arbitrageur will simply long the futures, take delivery under the futures (which is to occur on the same date, since it is the maturity of the futures) at the futures price (lower) and sell (at the higher price) the asset so acquired in the spot market. Conversely if the futures price is higher, he will simply short the futures, buy the asset spot (at lower price) and deliver against the short futures position the same day and receive the futures price (higher). So, neither status can exist for a long time and, as a result, the two prices must converge.

It may be noted that the futures price here is the price at which the futures is traded on the maturity date i.e. on the date on which delivery and payment under the futures is envisaged i.e. F_T. F_T is the price of the futures contracts on the date of delivery itself. Thus, convergence requires that $F_T=S_T$. S_T is the spot price on the same day, spot price on the date of maturity of the futures contract.

How does a futures hedge operate

So, how does the futures hedge operate? There, usually, exists a positive correlation between spot and futures prices. Hence, if one has a portfolio consisting of opposite positions in the spot and futures markets, the price changes in one market will substantially offset the price changes in the other market. The degree of offsetting will depend on the level of correlation between the two markets.

Indeed, due to arbitrage considerations, the forward prices move in tandem with the spot prices while futures prices align with forward prices. Therefore, within marginal deviations, the futures prices should also move in line with spot prices so that there should be a strong positive correlation between spot & futures markets.

Thus, if one has a long portfolio of certain cash held securities, one can arrest the impact of a fall in spot prices of those securities by adding short futures positions on those or similar securities. If the price of these securities falls in the cash market, as a result of which the given portfolio loses value, the prices of the same securities in the futures market would also register a fall due to the positive correlation between spot and futures markets. However, because the investor has taken a short position in the futures market, this fall in prices in the futures market would generate a profit for the investor, thereby neutralizing part of the loss.

It is important to point out that the hedge lifting t=N need not necessarily coincide with the maturity of the futures t=T. Indeed, it is common practice that the hedge is liquidated before the futures actually mature for delivery. Usually, the hedger lifts the hedge by closing out the futures position when the hedged cash flow is to precipitate.

Hence, although the hedge may be lifted by an investor at any point in time t=N by ending his exposure in the futures market i.e. closing out his futures position, the futures contracts continue to be traded until their maturity.

Now, because we do have a mechanism by which we can transform a sum of money from one point in time to another point in time i.e. compounding & discounting, it really does not matter at what point in time the hedge is lifted. The profit/loss at hedge lifting can be translated to the point in time at which the underlying cash flow is executed using compounding/discounting for assessing hedge effectiveness.

But we normally accept the fact that the lifting of the hedge would coincide with the point at which the exposure materializes, although it is by no means obligatory. If the hedger perceives that he is getting an unusually better price in the futures market at an earlier date or that the futures position is worth carrying beyond the maturity of the exposure to make more profit, he could very well opt for the same without any impediment. The only issue here would be the induction of an element of speculation into the exercise, into the strategy.