Financial Derivatives and Risk Management
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Lecture 08: Exposure & Risk

Exposure

Consider an Indian firm that owns assets in the United States valued in USD at $V_{\$,t}$ at arbitrary time t=t. Let its corresponding value in INR at that time be $V_{Rs,t}= V_{\$,t}*S_t$

Obviously, $V_{Rs,t}$ will depend on the INR/USD exchange rates at that time S_t.

At this instant let there be an unanticipated change ΔS in S with a change ΔV in the value of assets. This is the change in the value of foreign asset in terms of INR.

There may, of course, also be other factors contributing to change in the value of the assets ΔV. At this point we need to understand certain things, the immediate and logical reaction would be that ΔV should proportional to ΔS. However, if one does a plot of ΔV with ΔS for different values of ΔS that occurred historically at different points in time, one does not necessarily get a straight line. Therefore, the inference us that ΔV is influenced not merely by ΔS alone by there are other factors that contribute to the variation of ΔV. Let $\Delta\hat{V}$ be the portion of ΔV that varies directly as ΔS and let *a* be the portion not influenced by ΔS so that $\Delta V = \Delta\hat{V} + a$. Since we model these other factors, about the identity and influence of which we have little knowledge, as a random term, *a* is a random variable.

Assets held by an Indian firm in the US :

$$(in\ \$) = V_{t,\$} \xrightarrow{S_t} V_{t,Rs}\ (in\ Rs)$$

Let $\Delta V_{Rs} \equiv \Delta V$ *be the change in Rs value of the assets at time* $t=t$ *when an unanticipated change* ΔS *in ex rate occurs*

$$\Delta V_i = \beta\Delta S_i + a_i;\ a_i = \alpha + e_i$$

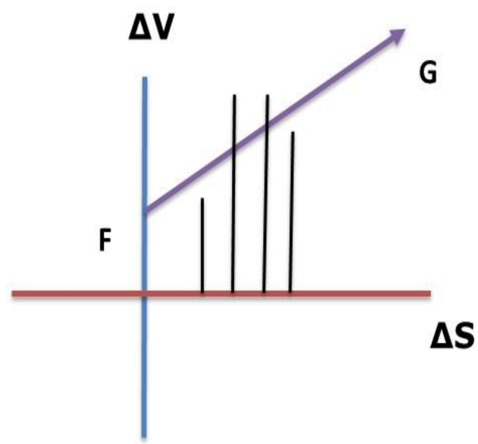
$$E(a) = \alpha;\ \text{so that } E(e) = 0$$

$$Cov(\Delta S, e) = 0$$

$$\Delta V_i = \beta\Delta S_i + \alpha + e_i$$

$$\Delta V = \beta\Delta S + \alpha$$

$$y = \beta x + \alpha\ (General\ \mathbf{Re}gression\ Equation)$$



Thus, when you plot ΔV against ΔS, you do not get a straight line, you get points which are clustered around in the straight line.

Since we have no information about the identity of these "other" factors and/or the nature of their impact on ΔV, we assume that these "other" factors collectively make a random contribution to ΔV.

Hence, we can write: $\Delta V_i=\beta^*\Delta S_i+a_i$ where index i indicates observation sequence.

We further write $a=\alpha+e$ with $E(a)=\alpha$ so that $E(e)=0$ so that $\Delta V_i=\beta^*\Delta S_i+\alpha+e_i$.

Now $\beta^*\Delta S_i$ is the change in the value of asset due to the unanticipated change in exchange rate ΔS_i while a_i is the random cumulative change in ΔV_i due to "other" factors.

It is logical to assume $Cov(e,\Delta S)=E[(e-0),(\Delta S-E(\Delta S))]=0$ i.e. that the partition of ΔV, as above, is orthogonal. Indeed, if we use OLS regression for determining the straight line of best line, both these assumptions viz $Cov(\Delta S,e)=E(e)=0$ are automatically ensured.

Thus, ΔV that results when there is an unanticipated change in the exchange rate has a component which is directly related to ΔS and another component which is random in nature, random in the sense that it encodes information about other factors which are either not identified or identifiable or their impact on ΔV not quantifiable. Sometimes it does happen that the cost of obtaining additional information outweighs the benefit that is derived from it.

Once we find that there are some other factors which are also influencing ΔV in some manner but we do not have access to complete information on those factors, we do the next best thing, we model them as a random variable. This process is called regression.

We can come up with some more factors which in addition to ΔS probably do a better job in explaining the variance of ΔV through Principal Component Analysis but most of the finance theory is built around a single factor model which we are using here.

Now, a systematic relationship exists when two variables move in a predicted manner "on the average". A systematic relationship between two variables exists when there is some predictable manner in which these two variables move ***on the average*** or over a period of time, over a set of observations. Given the relationship equation, $\Delta V=\beta^*\Delta S+\alpha+e$, the first term captures the systematic relationship between the independent variable ΔS and the dependent variable ΔV.

Systematic relationship implies that a change in the independent variable results in a predictable change in the dependent variable "on the average". If observations are taken a sufficiently large number of times, then on the average, we will get the values of the dependent variable predicted by the regression equation. The systematic relationship carries a predictive power only on the average over a sufficiently large number of observations. A particular observation may deviate from the predicted value by the regression equation. Systematic relationship holds over a sustained number of observations, but may be violated by individual observations.

So here ΔV is having a systematic relationship with ΔS and we have model the rest as a group and assume that group to create random variations in ΔV.

If we use the OLS regression for the determination of the parameters of the line of best fit, it automatically ensues that $Cov(\Delta S, a)= Cov(\Delta S, e)=0$ i.e. that the systematic variable ΔS and the random term *a* are uncorrelated. The regression also ensures that $E(e)=0$.

Just to recapitulate, we are analyzing the change in the value of a foreign asset designated in home currency due to an unanticipated change in the exchange rate. When we run a regression and fit the regression line by using OLS, the regression line that we obtain is such that $Cov(\Delta S,e)=0$ computed over all observations and $E(e)=0$.

β and exposure

Exposure is sensitivity of the value of an account to "unanticipated" changes in the value of a risk factor. Thus, given the regression equation, exposure is the slope of the regression equation which relates changes in the account value to unanticipated changes in risk factor.

We have a risk factor and we have an account which may be an asset, a liability or an operating income which is influenced by that risk factor. The sensitivity of that account, value of that account to the risk factor determines the exposure of the firm of the entity.

Clearly, we have $\Delta V=\beta*\Delta S+\alpha+e$ so that $E(\Delta V)=\beta*E(\Delta S)+\alpha+E(e)= \beta*E(\Delta S)+\alpha$.

This shows a systematic relationship between ΔS and ΔV. In other words, given some change in ΔS we can predict ΔV on the average because this is a statistical relationship. This is not a deterministic equation, *e* is a random variable, so $\Delta V=\beta*E(\Delta S)+\alpha$ will not hold in each and every case, *e* may take non-zero values. If it were so the ΔS-ΔV plot would have been a straight line fit for all observations. But on the average this relationship $\Delta V=\beta*E(\Delta S)+\alpha$ will hold.

Beta which represent the sensitivity of ΔV to ΔS (which is our risk stimulus) is called the exposure.

Exposure is a measure of how our asset/liability is reacting to the risk stimulus. Exposure is, therefore, a sensitivity, a slope.

Why unanticipated?

Now a very important point, why unanticipated changes in exchange rate?

Because all anticipated changes in the underlying risk factor (being existing information) are captured and incorporated in the price of the asset at t=0. Hence, all anticipated changes should not affect the price as and when they occur.

The point is, if any change in exchange rate is anticipated, it follows that the market is cognizant of the change. If the market anticipates a prospective change, it incorporates it in the price at a given point in time of the asset (the price of an asset reflects all known information about the future earning prospects of the asset at that point in time). Therefore, when the change actually occurs, its impact having already built into the price, any further change in price will not occur.

<u>Risk</u>

In general, risk is the variability in the values of the asset/ liability/operating incomes consequent to the unanticipated changes in the values of the stimulus. Now there can be two approaches to measurement of risk. As we know risk is measured by variability, if there is no variability in an asset's price, there is no risk. Therefore, one can measure risk:

(i) Either in terms of the variability of ΔV; or
(ii) In terms of the variability of ΔS itself.

If we use the first approach, the exposure automatically comes into play. In fact, we cannot have risk unless we have exposure. The variability of ΔV which is equal to $\beta\Delta S$, automatically brings the exposure β into the picture. If β is zero, then ΔV is not influenced systematically be ΔS so that if we measure risk by the variation of ΔV then there is no risk. Thus, if there is no exposure there is no risk.

So either one measures risk by the variability of ΔV or of ΔS. In the first case, if you have exposure you have risk and if you do not have exposure you have do not risk.

Conventionally it is the practice to use the variability of ΔV, the changes in the value of the dependent variable which is influenced by the risk factor as a measure of risk. We measure risk as the variability of the exposure in consequence to the variation in the risk factor. If we consider the equation $\Delta V=\beta\Delta S+a$, we find that ΔV is split up into two parts, the systematic component that varies with our chosen principal factor ΔS (whose impact on ΔV is being studied) and the random component a that models the influence on ΔV of the other factors that is not specifically incorporated. This may be due to these factors not being known or whose impact on ΔV is not understood well enough or even that we do not want to examine their specific influences on ΔV by design. We may believe that the predominance of the variation in ΔV is captured by the factor ΔS and therefore by design choose to model the rest of the variation in terms of a random variable a.

The need for the existence of a systematic relationship between the risk factor and the value of the exposed asset is imperative because if there is no relationship between them, then the risk factor is not deemed to influence the latter "on the average" and as such would cease to constitute a risk factor. It is only when the fluctuations in a variable influence the value of an account that it constitutes a risk factor. So, when we talk about risk, a systematic relationship between the risk factor and the value of the asset is implicitly understood.

For convenience we choose to write $a=\alpha+e$ where α=E(a) so that E(e)=0. β represents the sensitivity of ΔV to ΔS and hence, the exposure. α is the mean of the random errors while e is the deviation of the random term about its mean.

Standard deviation as a measure of risk

The first thing is that risk relates to fluctuations, variability. If there is no fluctuation in price due to a factor, no risk will arise. For example, Treasury bills are considered less risky as their prices do not fluctuate much. The change in prices of Treasury bills are usually only due to the passage of time on account of time value of money but the yield itself does not change frequently. Stock prices on the other hand change with very high frequency. Stocks are considered as epitomizing financial risk.

Risk is the possibility or the chance of not meeting a targeted outcome. It is very clear that greater is the price fluctuation, greater is the chance that the return may undershoot or overshoot the target. If there is no fluctuation then one is literally assured of meeting the target, because the target would have been set on that basis.

The “uncertainty” in achieving targets is directly related to the level of fluctuations or “dispersion” about the mean value i.e. higher the amplitude of swing about the mean value, higher is the uncertainty of achieving the targeted return.

Now that being risk, it is an immediate corollary that risk relates to variability and one of the most common measures of variability is standard deviation. ***Accordingly, we measure risk usually by the standard deviation of the value of ΔV in response to the fluctuations in the value of the risk factor ΔS.***

However, this raises the issue that the investor is not bothered with the upside fluctuations, but is more concerned about the downside fluctuations whereas standard deviation gives equal weightage to upside and downside fluctuations. An appropriate response is that empirical studies demonstrate the prices of financial assets are symmetric in their movements about the mean. While risk is generally associated with downside outcomes, the return structure of securities is usually assumed symmetric so that the level of downside fluctuations equals the level of upside fluctuations.

So if you measure the fluctuations on the up or on the downside of the mean position it really makes a little difference because of the symmetry embedded in them. Now this is not exact but pretty close.

Standard deviation coupled with expected return gives a comprehensive theory of portfolio optimization of Harold Markowitz, Markowitz mean-variance optimization. SD provides coherence with other concepts of e.g. of microeconomics.

There is another positive of SD. Stock prices are usually modeled as lognormal variables. That being the case, mean and standard deviation are adequate to describe the entire distribution.

Systematic & unsystematic risk

Let us look at one more implication when we use standard deviation as a measure of risk. We have:

$$\sigma_{\Delta V}^2 = E\left[\Delta V - E(\Delta V)\right]^2 = E\left[\alpha + \beta\Delta S + e - E(\alpha + \beta\Delta S + e)\right]^2$$

$$= E\left[\alpha + \beta\Delta S + e - \alpha - E(\beta\Delta S)\right]^2 = E\left[\beta\Delta S + e - E(\beta\Delta S)\right]^2 = E\left[\beta\Delta S + e - \beta E(\Delta S)\right]^2$$

$$= E\left\{\beta\left[\Delta S - E(\Delta S)\right] + e\right\}^2 = E\left\{\beta\left[\Delta S - E(\Delta S)\right]\right\}^2 + E(e)^2 + 2E\left\{\beta\left[\Delta S - E(\Delta S)\right], e\right\}$$

$$= \beta^2 E\left[\Delta S - E(\Delta S)\right]^2 + E(e)^2 = \underset{SYS\ \ RISK}{\beta^2\sigma_{\Delta S}^2} + \underset{UNSYS\ \ RISK}{\sigma_e^2}$$

$$\Sigma_{\Delta V} = \begin{pmatrix} \beta\sigma_{\Delta S} \\ \sigma_{e_i} \end{pmatrix};\ \sigma_{\Delta V}^2 = \Sigma_{\Delta V}^T . \Sigma_{\Delta V}$$

What we are getting here is an orthogonal bifurcation of the total risk of a position. The first component is the systematic risk and the second one the unsystematic risk. This is indeed a manifestation of the fact that when we use OLS regression, the procedure of minimization of the sum of the square terms results in the vanishing of covariance between ΔS and *e*. Thus, the splitting of ΔV into the systematic component βΔS and the unsystematic component ***a*** is orthogonal.

Therefore, we are able to split the variance into two orthogonal parts viz the systematic part $\beta^2\sigma_{\Delta S}^2$ and the unsystematic or the random part σ_e^2. σ_e^2 is the risk due to factors which we modeled as a random variable and $\beta^2\sigma_{\Delta S}^2$ is the risk which arises because of the systematic relationship between ΔS and ΔV.

α is the intercept and β is the slope of the regression line.

We also note that E(ΔV)=βE(ΔS)+E(*a*)= βE(ΔS)+α. Since the random term is absent in this equation, it indicates a systematic or a predictable relationship ***on the average***, not for individual observations but on the average.