Financial Derivatives and Risk Management

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Lecture 59

Value at Risk: Definition and Computation

We have been talking about risk throughout this course. Whenever we talk about risk, we talk about fluctuations and when we talk about fluctuation we invoke the concept of standard deviation as a measure of such fluctuations. Indeed, risk must necessarily be tagged to fluctuations. If an asset's prices show no fluctuations, there is no possibility of its deviating from a targeted outcome because the target would be set on the basis of price trend and there being no fluctuations in the asset prices, the target so fixed would definitely be achieved. It is akin to a deterministic model where the projected path is represented by a deterministic curve. We can a priori determine whether a point will lie on the curve or it will not. If it does lie on the projected curve, then the particle will with 100% certainty pass through it in its real trajectory. In such scenario, because the outcome is deterministic and there is no chance of the actual trajectory not coinciding with the projected one, there is no risk. Thus, risk must necessarily arise only when the evolution of a system is stochastic i.e. there is a random component in the trajectory of the system. In such a situation, it becomes impossible to project with certainty whether the trajectory of the system will pass through a pre-determined point (target). Stated otherwise, risk emanates on account of the possibility of random fluctuations in the future evolution of the system. This leads us to standard deviation being accepted as a measure of risk. Nevertheless, the acceptance of standard deviation as a measure of risk in supersession of other measures is also because of its simplicity, mathematical compatibility and cosmetic appeal. It is not that standard deviation is unequivocally, unambiguously the best, the most optimal, most appropriate measure of risk. It has its limitations, that is why we look at other measures of risk. One of the most prominent among those other measures of risk, which is now used being used internationally is the concept of value at risk.

Value at Risk (VaR) is an attempt to provide a single number summarizing the total risk in a portfolio of financial assets. It has become widely used by corporate treasurers and fund managers as well as by financial institutions. Bank regulators also use VaR in determining the capital a bank is required to keep for the risks it is bearing. Value at risk is also being employed in exchanges for working out margins. In India, we have the span (Standard Portfolio Analysis of Risk) software, software which computes the margins on the basis of a certain level of value at risk.

What is value at risk

Value at risk is a measure of potential loss on a portfolio due to an unlikely adverse event in otherwise normal market conditions. VaR indicates the potential maximum loss that could occur given a certain probability in a certain time span. It is designated in units of money and it is a statistical measure of risk.

(i) VaR is a measure of potential loss from an unlikely adverse event in an otherwise normal market.

(ii) VaR indicates the potential maximum loss that could occur given a certain probability in a certain time span.
(iii) VaR is denominated in units of currency. e.g. INR.
(iii) VaR is a statistical measure of risk exposure.

Inputs to the value at risk

(i) A given portfolio W whose VaR (risk) is to be computed/ reported.
(ii) A probability 0<p<1 called the confidence level.
(iii) A time horizon N.

Definition of Value-at-Risk

VaR attempts to answer the question:

I am p percent certain that my portfolio (W) loss will not exceed D over N days.

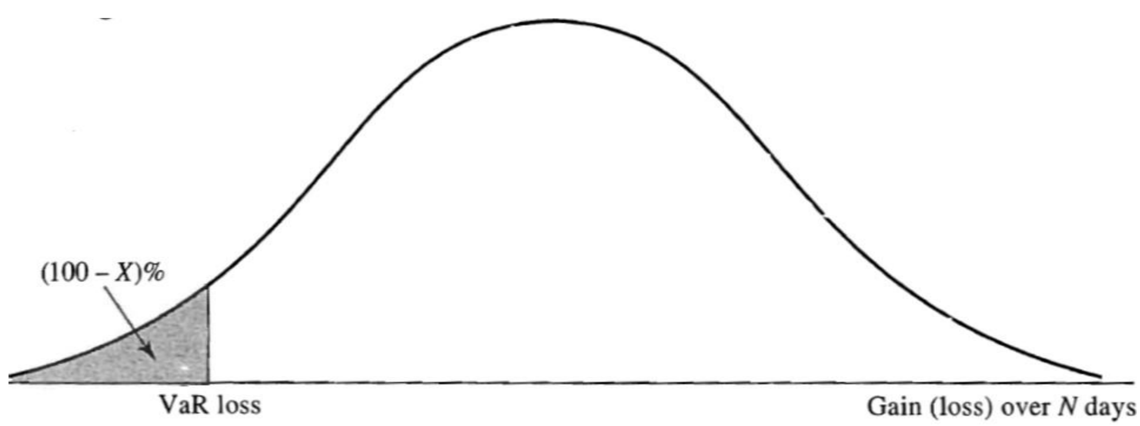
D is the VaR of the portfolio;
p is the confidence level; and
N is the time horizon.

Thus, VaR is the loss level over T days that has a probability of only (100 - p)% of being exceeded.

Equivalently, we may say that over a series of 100 days, in the case of p days, the loss on our portfolio (if a loss occurs) shall not exceed the VaR amount.

Thus, VaR is that amount (in money terms) of loss over N days that will not be exceeded with a pre-specified probability p. The investor is p% certain that the loss of his portfolio W, will not exceed D (the VaR) over N days.

So value at risk is that negative change in value (D) of the portfolio over N days whose magnitude will not be exceeded with probability p. p is called the confidence level, N is called the time Horizon and D is called the VaR. In other words, we can also say VaR is the loss level the probability of exceeding which is (100-p)%.



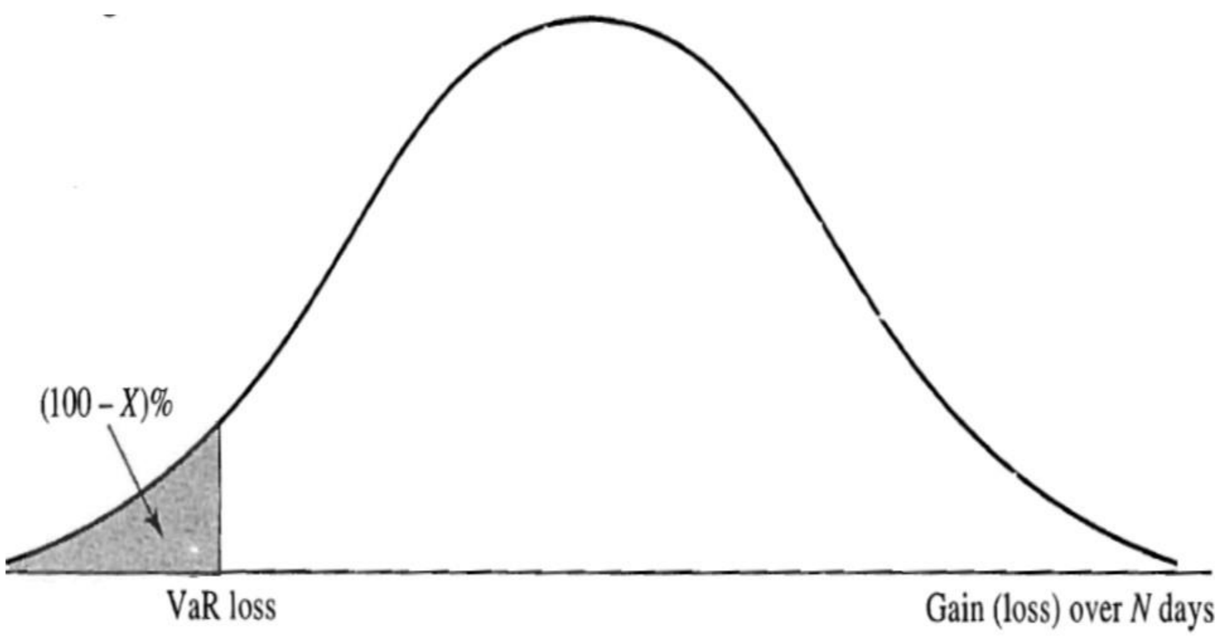
From the definition of VaR, we can also interpret, given a set of say 100 observations on changes in prices of the given portfolio over the desired time horizon and a confidence level p, there would be only be 1-p observations wherein the loss (fall in price) on the portfolio would exceed the Value at Risk.

Let us look at this distribution. This is a gain distribution. It represents the distribution of the change in value $Z=dS=S_N-S_0$ of a portfolio. Clearly, if $S_N>S_0$, the portfolio earns a profit over this period and vice versa. Since S_N increases as we move to the right, the profit also increases as we move to the right and the loss increases as we move to the left. The probability of the gain i.e. $Z=dS$ "exceeding" a certain value, say z^*, is equal to the area to the right of the ordinate at $Z=z^*$ i.e. $P(Z>z^*)$. This probability and hence, this area is called the "confidence level". Equivalently, we can also say that the probability of the magnitude of the loss i.e. -dS "not exceeding" a certain value, say $z_c=-z^*$, is equal to the area to the right of the ordinate at $Z=z^*=-z_c$ i.e. $P(Z>-z_c)$.

Now, value at risk is that maximum value of loss z_c such that this confidence level $P(Z>-z_c)$ is given, is pre-assigned. This confidence level (probability) is given and the maximum level of loss that can occur subject to this probability represents the value at risk.

The confidence level (probability) is the unshaded area and this value is given. The negative of the value of Z that bifurcates the entire area into the unshaded area (confidence level probability) and the shaded area (1-p) is the required VaR.

VaR & confidence level



From the above figure we see that:

Higher the confidence level $P(Z> z^{*}=-z_c)$, lesser would be the area to the left of $z^{*}=-z_c$, lesser would be the value of z^{*} and higher the value of z_c i.e. the maximum loss and hence, higher the VaR and vice versa i.e. the confidence level and VaR move in same directions.

Remember, confidence level is the probability of not occurring of the loss exceeding the VaR. Hence, higher the confidence level, higher the probability of not occurring of a loss exceeding VaR i.e. lower the probability of a loss exceeding VaR. Thus, VaR will increase with confidence level.

Let us say, we are working at 95% confidence level and we find that at $z=z_{0.95}$, the area under the pdf curve gets split up as 5% to the left of the ordinate at $z=z_{0.95}$ and 95% to the right. Now, suppose we move to 99% confidence level. We, now, need to identify a point $z=z_{0.99}$ such that the area to the left of the ordinate at $z=z_{0.99}$ is only 1% and that to the right is 99%. It is quite obvious that $z_{0.99}$ must necessarily be to the left of $z_{0.95}$, whatever be the nature of the distribution. Since VaR is the magnitude of these z values, VaR at 99% will unequivocally be higher than the VaR at 95%.

When the confidence level changes from 95% to 99%, the area to the right of the ordinate from the VaR value must increase from 95% to 99%. This will push the VaR point more to the left because the right hand side area should now be 99% instead of 95% of the entire area. The value at risk will increase in absolute terms. Note that value of risk (being a loss) is obviously a negative quantity but is usually presented as its absolute value.

So, higher the confidence level higher the probability of not occurring of a loss exceeding value at risk and lower the probability of loss exceeding value at risk.

Calculating the N day VaR from one-day VaR

N-Day VaR = One Day VaR x $\sqrt{N}$

This formula is exactly true when the changes in the value of the portfolio on successive days have independent identical normal distributions with mean zero. In other cases, it is an approximation.

It is conventional when we calculate value at risk, to calculate value at risk for 1 day and then extrapolate it over N days. For this purpose, we use the square root rule. The rule says that the standard deviation of the portfolio scales as the square root of time i.e. N-day VaR= 1-day VaR * $\sqrt{N}$.

Calculation of Value-at-Risk: Single asset case

Calculation of VaR can be done in two fundamental ways, either we can use an empirical approach (based on historical data) or we can work out the VaR assuming a certain model for the distribution of prices of the portfolio.

Empirical approach for computation of VaR

Let us assume that we want to determine the 5-day 99% VaR of a portfolio W.

Assumptions

The portfolio under consideration doesn't change over the forecast horizon.
The historical data used contains information useful in forecasting the loss distribution.
The historical data follows a specific distribution.

Step 1

We start by calculating VaR for a one-day time horizon. We shall then extrapolate it over 5-days by using the sq root rule.

The first step is to identify the market variables affecting the portfolio W. These will typically be interest rates, equity prices, equity market indices, commodity prices/indices and so on. All prices are measured in the domestic currency.

We assume that the value of our portfolio W is affected by a single variable e.g. the S & P BSE Sensex M. In other words, the portfolio W has a functional relationship with the S&P BSE Sensex M so that given the Sensex value M_t we can work out the corresponding portfolio value $W_t=f(M_t)$.

Step 2

Having identified the variables being tracked by our portfolio W, we collect a suitable number of historical data of each such variable.

We believe that 501 units of data is adequate for the purpose of our exercise and accordingly collect historical Sensex values (e.g. day closing values) over the most recent 501 days e.g. t=-500 to t=0. Let these values be denoted by M_t e.g. M_{-500} is the Sensex value on day -500 etc.

We collect 501 day closing Sensex values going back in time, say from today (t=0) to (t=-500). We are collecting historical data, so data for today, yesterday, day before yesterday and so on, we continue up to 500 days going backwards. So the earliest data will be for t=-500 and the latest data for t=0 i.e. today.

Step 3

Using this pool of 501 historical Sensex values, we project the Sensex value (M_1) at t=1 day (tomorrow) under 500 different scenarios S_i, i=1,2,..,500.

The projections under the different scenarios are arrived at as follows:

The projected value of M_1 under scenario 1 i.e. M_1^1 is arrived at on the premise that the percentage change in the Sensex value between tomorrow and today under this scenario (scenario 1) is equal to the percentage change in the value of Sensex between day -499 and day -500 so that:

$$\frac{M_1^1 - M_0}{M_0} = \frac{M_{-499} - M_{-500}}{M_{-500}} \text{ or } M_1^1 = M_0 \frac{M_{-499}}{M_{-500}}.$$

Similarly, the projected Sensex for tomorrow under Scenario 2 i.e. M_1^2 is arrived at on the premise that the percentage change in the Sensex value between tomorrow and today under this scenario (scenario 2) is equal to the percentage change in the value of Sensex between day -499 and day -499 so that: $M_1^2 = M_0 \frac{M_{-498}}{M_{-499}}$. Proceeding in this way, we generate the projected Sensex for tomorrow under 500 different scenarios.

Step 4

Since we know the functional relationship between the Sensex value and the value of portfolio W, we can compute the value of W tomorrow i.e. W_1 corresponding to each of these 500 projected values of Sensex corresponding to the 500 scenarios e.g. $W_1^i = f\left(M_1^i\right),\ i = 1, 2, ..., 500$.

Step 6

Knowing the projected values of our portfolio W tomorrow (t=1) corresponding to each scenario $W_1^i,\ i = 1, 2, ..., 500$, we work out the projected change in the value of the portfolio over this 1-day period (t=0 to t=1 day) corresponding to each scenario i.e. $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$ where W_0.is today's value of the portfolio i.e. portfolio value corresponding to Sensex value M_0.

Step 7

Using these value of the changes in value $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$ of the portfolio between t-0 and t=1-day for different scenarios, we construct a probability distribution for the change in price over the 1-day period t=0 to t=+1day. Alternatively, we arrange these value $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$ in descending order. The 1-day VaR is then, given by the 5th lowest value of $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$. If we arrange the changes is value of our portfolio in the different scenarios $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$ in descending order, the fifth worst possible loss or the 99th percentile will give us the 99% VaR for one day.

Step 8

We obtain the 5-day VaR by using the square root rule i.e. VaR(5-day)=VaR (1-day)*√5.

Example

Suppose today (t=0) is Sept 25, 2018. Suppose that an investor, today, on September 25, 2018, desires to compute the 5-day 99% VaR of a portfolio worth INR 10 million that mimics one stock index, the S & P BSE Sensex.

Data on the closing values of Sensex for the latest 501 days i.e. from August 7, 2016 onwards is collected and tabulated.

DATA ON SENSEX FOR HISTORICAL SIMULATION

Day	Date	SENSEX
0	Aug. 7, 2016	11,219.38
1	Aug. 8, 2016	11,173.59
2	Aug. 9, 2016	11,076.18
3	Aug. 10, 2016	11,124.37
499	Sept. 24, 2018	10,825.17
500	Sept. 25, 2018	11,022.06

GENERATION OF SCENARIOS FOR SEPT 26, 2018: SCENARIO 1

Value of Sensex on Sept 25, 2018: 11,022.06
On August 8, 2016, it was 11,173.59, down from 11,219.38 on August 7, 2016.
Since, in scenario 1, we require that the index change from Sept 25, 2018 to Sept 26, 2018 by the same proportion as from August 7, 2016 to August 8, 2016, the scenario 1 value of Sensex for Sept 26, 2018 is: 11,022.06*11,173.59/11,219.38 = 10,977.08

Similarly, the Sensex values corresponding to the other scenarios are worked out.

SCENARIOS GENERATED FOR SEPT 26, 2018

Scenarios number	Sensex
1	10,977.08
2	10,925.97
3	11,070.01
499	10,831.43
500	11.222.53

VALUE OF PORTFOLIO FOR SEPTEMBER 26, 2018, UNDER SCENARIO 1

Value of the portfolio under scenario 1 = Investment in the portfolio * Sensex under Scenario 1/ Sensex as on Sept 25, 2018 =10*10,977.08/11,022.06=9.959 million.

The portfolio, therefore has a loss of 0.041 million under Scenario 1.

GAIN/LOSS OF THE PORTFOLIO UNDER VARIOUS SCENARIOS

The gain/loss of the portfolio under all the 500 scenarios is calculated and the gains are ranked in descending order.
The one-day 99% value at risk can be estimated as the fifth-worst loss.
We can, then, calculate the 5 day VaR using the square root rule.

VaR for a portfolio tracking many indices

The VaR for a portfolio of assets is not equal to the weighted average of the VaRs of the constituent assets. We can, however modify the above algorithm to determine the VaR of a portfolio of assets that tracks a number of indices, prices or rates rather than a single index, price or rate.

We may have a portfolio having some content of foreign investment, foreign currency, fixed income bonds etc. The respective constituents of such a portfolio would track the relevant indices/prices/interest rates. Therefore, the value of the portfolio as a whole will be functionally dependent on several of these parameters. The basic thing is that we must be able to determine the value of the portfolio given these various parameters.

Let us assume that we are required the determine 5-day 99% VaR of a portfolio W that tracks n indices M_j, j=1,2,…,n, so that: $W = \sum_{j=1}^{n} w_j f\left(M_j\right)$. The value of the portfolio today (t=0) is: $W_0 = \sum_{j=1}^{n} w_j f\left(M_{j,0}\right)$ where $M_{j,0}$ j=1,2,…,n is the value of the j^{th} index today (t=0).
We collect data on a sufficiently large recent historical period e.g. latest preceding 501 days i.e. t=-500 to t=0 for each of these indices M_j, i=1,2,…,n.

On the basis, of this data, we project various scenarios for each index j tomorrow, given its value today. For example, we work out the first scenario projection of the value of the index j on the premise that the percentage change in the value of index j between tomorrow and today equals the percentage change in the value of the same index between day -499 and day -500 so that: $\frac{M_{j,1}^{1} - M_{j,0}}{M_{j,0}} = \frac{M_{j,-499} - M_{j,-500}}{M_{j,-500}}$ so that $M_{j,1}^{1} = M_{j,0} \frac{M_{j,-499}}{M_{j,-500}}$ and similarly for the second scenario $M_{j,1}^{2} = M_{j,0} \frac{M_{j,-498}}{M_{j,-499}}$ and so on. We work out the projections for each of the j indices under each of the 500 scenarios.

Corresponding to each of these scenarios, we calculate the value of our portfolio W tomorrow $W_1^1 = \sum_{j=1}^{n} w_j f\left(M_{j,0}^{1}\right)$. Similarly, we work out portfolio values for each scenario $W_1^i = \sum_{j=1}^{n} w_j f\left(M_{j,0}^{i}\right),\ i = 1, 2, ..., 500$. Thus, we get 500 scenarios for the value of our portfolio tomorrow.

Knowing the projected values of our portfolio W tomorrow (t=1) corresponding to each scenario $W_1^i,\ i = 1, 2, ..., 500$, we work out the projected change in the value of the portfolio over this 1-day period (t=0 to t=1 day) corresponding to each scenario i.e. $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$ where W_0 is today's value of the portfolio.

Using these changes in value $\Delta W_1^i = W_1^i - W_0,\ i = 1, 2, ..., 500$ of the portfolio between t=0 and t= 1-day for different scenarios, we construct a probability distribution for the change in price over the 1-day period t=0 to t=+1day. Alternatively, we arrange these value $\Delta W_1^i = W_1^i - W_0,\ i = 1,2,...,500$ in descending order. The 1-day VaR is then, given by the 5^{th} lowest value of $\Delta W_1^i = W_1^i - W_0,\ i = 1,2,...,500$. If we arrange the changes is value of our portfolio in the different scenarios $\Delta W_1^i = W_1^i - W_0,\ i = 1,2,...,500$ in descending order, the fifth worst possible loss or the 99^{th} percentile will give us the 99% VaR for one day.

We obtain the 5-day VaR by using the square root rule i.e. VaR(5-day)=VaR (1-day)*√5.

Model based VaR

The standard practice is to calculate the 1-day VaR at the relevant confidence level and thereafter to extrapolate it over the desired time horizon of N days by the square root rule. Since, we work in the 1-day time frame, the differential Black Scholes model is a sufficiently adequate model. The differential Black Scholes model, is thus, invoked for ascertaining the VaR of equity and derivative portfolios. The model gives the change in price as:

$dS=\mu Sdt+\sigma dW_t=\mu Sdt+\sigma Z\sqrt{dt}$. For a 1-day time horizon, we have dt =1 day so that

$dS_{day}=\mu_{day}S+\sigma_{day}SZ$ whence $E(dS_{day}) = \mu_{day}S$; $SD(dS_{day}) = \sigma_{day}S$.

It is instructive to look at the relative order of magnitudes of μ_{day} and σ_{day}. Suppose, that Microsoft has an expected return and volatility of 20% p. a. Over a 1-day period, the expected return is 0.20/252, or about 0.08%, whereas the standard deviation of the return per day is 0.20/√252 =1.26%. Apparently, the difference is significant. It is, therefore, the usual practice to ignore $\mu_{day}S$; compared to $\sigma_{day}S$ and approximate $E(dS_{day})=0$. The expected change in the price of an asset over a short time period is generally small when compared with the standard deviation of the change. Hence, we have that the change in the value of the portfolio over a 1-day period has a standard deviation of σ_{day} *S and a mean of zero. We assume that the change is normally distributed in the Black Scholes differential model. Thus, we can assume that $dS_{day} \xrightarrow{distribution} N\left(0,\left(\sigma_{day}S\right)^2\right)$. Therefore, given the volatility per day (in %), σ_{day}, we can work out the standard deviation of daily changes in the value of the position in absolute value (money terms) as $SD(dS_{day}) = \sigma_{day}$*S where S is the absolute value (money value) of the position. We also have, in this formulation: $SD(dS_{day}/S) = \sigma_{day}$ so that the daily volatility, σ_{day}, is equal to the standard deviation of the percentage change in the asset price in one day.

Now, given a confidence level p, we define 1-day VaR as the maximum loss over 1-day i.e. the least profit over 1-day, the least value of dS over 1-day i.e. least value of dS_{day} (say dS^*_{day}) such that $P(dS_{day}>dS^*_{day})=p$. But $dS_{day} \xrightarrow{distribution} N\left(0,\left(\sigma_{day}S\right)^2\right)$ so that $Z=\dfrac{dS_{day}}{\left(\sigma_{day}S\right)}$ or $dS_{day}=Z\left(\sigma_{day}S\right)$. Also, $dS^*_{day}=z^*\left(\sigma_{day}S\right)$. Thus, we have $p=P\left(dS_{day}>dS^*_{day}\right)$

$= P\left(Z\left(\sigma_{day}S\right) > z^*\left(\sigma_{day}S\right)\right) = P\left(Z > z^*\right)$. Then, $1-day\ VaR = \left|dS^*_{day}\right| = \left|z^*\right|\left(\sigma_{day}S\right)$ and $N\text{-}day\ VaR = \left|z^*\right|\sigma_{day}S\sqrt{N}$.

Thus, the procedure for VaR computation is simple. We have the following steps:

Fix the VaR parameters viz. the time horizon, N, the confidence level, p so that we are interested in the loss level over N days that we are p% confident will not be exceeded.

Convert the volatility per year to volatility per trading day: $Vol_{year} = \sigma_{day} * \sqrt{(252)}$

In the Black Scholes stock price model, the daily volatility, σ_{day} is equal to the standard deviation of the percentage change in the asset price in one day. Therefore, given the volatility per day (in %), σ_{day}, we can work out the standard deviation of daily changes in the value of the position in absolute value (money terms) as $SD(dS_{day}) = \sigma_{day} * S$ where S is the absolute value (money value) of the position.

Work out the value of z^* corresponding to the given confidence level from the normal tables. For example, given a confidence level of 99%, we have, from the tables, N(-2.33) = 0.01. This means that there is a 1% probability that a normally distributed variable will decrease in value by more than 2.33 standard deviations. Equivalently, it means that we are 99% certain that a normally distributed variable will not decrease in value by more than 2.33 standard deviations.

The 1-day 99% VaR for our portfolio consisting of S valued position is therefore 1-day VaR = $2.33 * \sigma_{day} * S$

The square root rule

The square root rule, actually follows from the properties of Brownian motion and the differential Black Scholes model that is based on BM. The variance of BM scales as its time of evolution so that $SD(dW_t) = \sqrt{(dt)}$ whence $dW_t = Z\sqrt{(dt)}$ so that dW_t is normally distributed with a mean of 0 and a variance of dt. It, then follows from the following stock price model

$dS_t = \mu_{day}Sdt + \sigma_{day}SdW_t = \mu_{day}Sdt + \sigma_{day}SZ\sqrt{dt}$ that $E\left(dS_t\right) = \mu_{day}Sdt$ and $SD\left(dS_t\right) = \sigma_{day}S\sqrt{dt}$ or $dS_t \xrightarrow{distribution} N\left(\mu_{day}Sdt, \left(\sigma_{day}Sdt\right)^2\right)$ thereby establishing that the standard deviation of the changes in stock price scales as the square root of the time of evolution.

Example

Consider a position consisting of a USD 100,000 investment in asset A. Assume that the annual volatility of the asset returns is 30% and there are 252 trading days, what is the 5-day 99% VaR for the portfolio?

Solution

We have, amount of investment (S)=100,000, σ=0.30, Trading days=252, $z_{0.99}$=-2.33, N=5 days
VaR=2.33*100,000*0.30*(√5)/(√252)=9846.05.

VaR of 2-asset portfolio

The variance (in money value) of a portfolio Π comprising of two assets X, Y with investment (in money terms) of S_X & S_Y, standard deviations σ_X and σ_Y and correlation between returns of ρ_{XY} is given by:

$$\sigma_{X+Y}^2(\$)=S_X^2(\$)\sigma_X^2(\%)+S_Y^2(\$)\sigma_Y^2(\%)+2\rho_{XY}S_X(\$)S_Y(\$)\sigma_X(\%)\sigma_Y(\%)$$

The procedure for working out the VaR is given by:

Calculate the volatility per day $\sigma_{X,day}$, $\sigma_{Y,day}$ (in %) of each asset's return from the annual volatility.

Calculate the SD (in money terms) of the value changes of each asset X, Y i.e. SD(dX)= $\sigma_{X,day}{}^{*}S_X$ and SD(dY)= $\sigma_{Y,day}{}^{*}S_Y$

Calculate the SD of the 1-day changes in value of portfolio Π from SD(X) and SD(Y) and the correlation coefficient ρ_{XY}.

$$\sigma_{X+Y}^2(\$)=S_X^2(\$)\sigma_X^2(\%)+S_Y^2(\$)\sigma_Y^2(\%)+2\rho_{XY}S_X(\$)S_Y(\$)\sigma_X(\%)\sigma_Y(\%)$$
$$\sigma_{X+Y}^2(\$)=\left[SD(dX)\right]^2+\left[SD(dY)\right]^2+2\rho_{XY}SD(dX)SD(dY)$$

The appropriate z-value is selected from the normal distribution table in accordance with the given confidence level. For a level of 99%, the z-value is -2.33

The 1-day 99% VaR is then given by: 2.33*σ_{X+Y}

Example

Consider a position consisting of a \$100,000 investment in asset A and a \$100,000 investment in asset B. Assume that the daily volatilities of both assets are 1% and that the coefficient of correlation between their returns is 0.3. What is the 5-day 99% VaR for the portfolio?

Solution

$$S_A=S_B=100000,\ \sigma_A=\sigma_B=1\%$$
$$\textit{Variance of portfolio's daily change in value}$$
$$\sigma_{A+B}^2=S_A^2\sigma_A^2+S_B^2\sigma_B^2+2\rho\sigma_A\sigma_B S_A S_B=2,600,000;\ \textit{Hence}, \sigma_{A+B}=1,612.45$$
$$\textit{Thus, SD of 5 day change is } 1,612.45\sqrt{5}=3,605.55.$$
$$\textit{The z value corresponding to } 99\% \textit{ confidence } z^*=-2.33$$
$$\textit{Hence}, 5-day\ 99\%\ VaR=2.33\times 3,605.55=8,401$$