**Financial Derivatives and Risk Management**
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**Lecture 55**
**Forward Rate Agreements; Swaps**

## Swaps

***A swap is an over-the-counter agreement between two companies to exchange streams of cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated.***

Usually one of the stream is independent of a specified market variable while the other bears a functional relationship with the same variable. The variable may be an interest rate, an exchange rate, or an asset price or other market variable.

Thus, swaps are contracts to exchange of streams of cash flows. They are over the counter agreements. They are tailor-made on the basis of negotiation between the parties and are not traded on recognized exchanges. They are usually facilitated by intermediaries/brokers that are usually large commercial banks or financial institutions.

In most of the swaps one of the stream of cash flows is fixed i.e. independent of a specified market variable while the other is determined with reference to the value partaken by that market variable i.e. it bears a functional relationship with the same variable.

For example, consider an interest rate swap (IRS). In this swap, one-party agrees to pay interest on a certain notional principal at a fixed rate while the other party agrees to pay interest on the same principal at a rate (floating rate) that is determined at regular intervals by reference to the value of a benchmark rate e.g. the LIBOR. The principal is called notional because it may not actually be exchanged between the parties. It may simply be the reference amount for calculating the amount of the interest streams, just like the face value of bonds.

The day count conventions for calculating the fixed rate and the floating rate streams may be the same or different, but are incorporated in the swap contract. Indeed, the swap contract contains all such provisions as are necessary for determining the two streams unambiguously e.g. the dates than which the cash flows are to be exchanged, the manner in which they are to be computed etc.

## Swap as a forward contract

Swaps can be viewed as forward contracts. Forward contracts entered into a given point in time (t=0) entail the exchange of an asset S at a future point in time (t=T) for a price (K) that is agreed upon at t=0, but the exchange of the price and the asset both take place at t=T. Now, at t=T, the asset will command a certain market value $S_T$. As such, this forward contract can be viewed as an exchange of cash flow K (that is determined at t=0) with a cash flow $S_T$ (which is floating at t=0 and determined at t=T). Thus, this forward transaction may also be viewed as a fixed vs floating swap wherein a fixed cash flow (K) is exchanged for a market based cash flow ($S_T$).

The forward contract is, therefore, equivalent to a swap where one party agrees that on maturity of forward, it will pay K and receive $S_T$, where $S_T$ is the market price of the asset on that date. So here again we have an exchange of cash flows where one cash flow is independent of the market and the other cash flow is dependent on the market price, typical constituents of a swap.

Whereas a forward contract is equivalent to the exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges on several future dates.

**Forward rate agreements (FRAs) and swaps**

A forward rate agreement (FRA) is an over-the-counter agreement designed to ensure that certain interest rates will apply to either borrowing or lending a certain principal during a specified future period of time.

In effect, the agreement implies that one party to the agreement shall pay a predetermined interest rate fixed at the inception of the FRA on a certain notional principal from a future date for a predetermined period irrespective of the market rate prevailing at the time the loan is actually disbursed while the other party will pay a market determined rate that is usually fixed on the basis of the rates prevailing in the market at the point in time close to the actual initiation of the lending period.

The long party to the FRA will pay fixed rate i.e. $R_K$ that is fixed at negotiation of the agreement ($T_0$) on a notional loan of amount L that starts at $T_1$ and is repaid at $T_2$.

The short party will pay floating rate i.e. $R_M$ that is determined and fixed at the date of disbursement $T_1$ on the basis of market rates prevailing at that time on the same loan L that starts at $T_1$ and is repaid at $T_2$.

The underlying for the FRA is, therefore, the interest rate applicable to a notional principal for a lending-borrowing transaction negotiated at $t=T_0$ scheduled for a future interval $(T_1,T_2)$. The long party pays the fixed rate $R_K$ and receives the floating rate $R_M$. $R_K$ is determined and fixed at the negotiation of the agreement at $t=T_0$, $R_M$ is determined and fixed at commencement of loan i.e. at $t=T_1$. The interest is payable on the notional principal L for the period from $T_1$ to $T_2$ at the respective rates. The agreement concludes at $T_2$.

The floating rate is, thus, floating for the interval between $T_0$ and $T_1$, it will remain undetermined between $T_0$ and $T_1$, it will be crystalized at $T_1$ in accordance with then prevailing market rate, whatever the benchmark is in the agreement. The seller of the agreement will pay interest on the same notional principal for the period from $T_1$ to $T_2$ at the interest rate that is finalized at $T_1$.

So in that sense it is floating, it is floating in the sense that it is fixed at the beginning of the period at which the loan is deemed to be disbursed. It is concluded when the agreement is concluded at $T_2$.

A forward rate agreement can be viewed as a simple example of an interest rate swap. However, a FRA involves exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges on several future dates. So swaps may be viewed as a cascade of FRAs bundled together.

The buyer of the agreement pays a fixed rate, the rate is fixed at the time of negotiation of the agreement.

The seller of the agreement pays a floating rate, floating in the sense that it is not fixed at the date of negotiation of the agreement, but it is fixed at a date of disbursement of the loan or deemed disbursement of the loan at a future date and it concludes of course both the interest will be for the period T 1 to T 2 same period.

Ancillary issues like day count would be contained in the forward rate agreement itself.

**Payoff of FRAs**

Consider an FRA negotiated at $t=T_0$ that entails payment by the long party of interest at a fixed rate (determined at $t=T_0$) of $R_K$ on a notional loan principal of L for the period from $T_1$ to $T_2$ in exchange of interest at a market rate $R_M$ (determined at the commencement of the notional loan transaction i.e. $t=T_1$ with reference to a certain pre-specified benchmark) on the same principal for the same period. For simplicity we ignore issues of day count conventions.

Payment by long party @ fixed rate $R_K$: $-LR_K(T_2-T_1)$
Receipt by long party=Payment by short party @ market rate $R_M$: $+LR_M(T_2-T_1)$
Net receipt by long party: $L(R_M-R_K)(T_2-T_1)$

As per standard practice, interest on borrowings is settled at the conclusion of the loan. Hence, the above exchange should take place at the end of the notional loan transaction i.e. at $t=T_2$.

However, it is usually the practice in the case of FRAs that the above cashflow is settled by a payment at $t=T_1$ rather than $t=T_2$, by transacting the present value of the above amount so that if the FRA is settled at $T_1$ instead of $T_2$, then amount received by the long party (maybe negative): $L(R_M-R_K)(T_2-T_1)/[1+R_M(T_2-T_1)]$.

The present value is worked out at the current rate $R_M$. So this is the net amount that will be exchanged between the long party (received) and the short party (paid) and this exchange will take place at $T_1$. This is a usual practice that instead of the exchange been transacted at $T_2$, the exchange is made at $T_1$ with the present value of the payoff (arising at $T_2$) worked out at $T_1$.

**Example**

Consider an FRA where we will receive a rate of 6%, measured with annual compounding, and pay LIBOR on a principal of USD 100 million between the end of year 2 and the end of year 3. Calculate the payoff of the FRA at the end of year 2. The current 2 year and 3 year spot rates are respectively 3% and 4% with continuous compounding.

**Solution**

This FRA operates between end of year 2 and end of year 3. The spot rates for 2 & 3 years are respectively $S_{02}$ =3.00% & $S_{03}$ =4.00% with continuous compounding. By no arbitrage:

$$e^{2\times S_{02}}e^{f_{23}}=e^{3\times S_{03}} \text{ so that } 2\times S_{02}+f_{23}=3\times S_{03} \text{ or } f_{23}=3\times S_{03}-2\times S_{02}=6.00\%$$

Thus, the forward rate $f_{23}$ works out to 6.00% continuous compounding. The equivalent annual compounding rate is $e^{0.06}$-1=6.1836%.

Hence, effectively the FRA involves a payment @ 6.00% on USD 100 million for one year by the long party against a receipt of 6.1836% on the same amount for the same period.

Thus, the net amount receivable by the long party is USD 100*(0.061836-0..06) = USD 0.1836 million.

However, this transfer would take place at the end of the third year. If it is desired to settle the agreement at the end of the second year, the payoff will be the present value of the above amount i.e. USD [0.1836/(1+0.061836)] = USD 0.1729 million

**Valuation of FRA after inception**

Consider an FRA A that is executed at t=0 that pays strike (fixed) rate of $R_A$ for a borrowing of notional principal L for the interval ($T_1$,$T_2$) against receipt of market determined $R_M$.

Consider another FRA B that is executed at t=t' that pays strike (fixed) rate of $R_B$ for a borrowing of notional principal L for the same interval ($T_1$,$T_2$) against $R_M$.

We need to value the FRA A at t=t'. Since FRA B is initiated at t=t', its value at the point of initiation t=t' is 0.

Construct a portfolio Π consisting of a long position in FRA A and a short position in FRA B.

Then long FRA A pays fixed $R_A$ and receives $R_M$. Short FRA B receives fixed $R_B$ and pays $R_M$. Thus, the portfolio Π receives $R_B$ and pays $R_A$.

Thus, net receipt on the portfolio Π at t=$T_2$ is $L(R_B-R_A)(T_2-T_1)$.
The present value of this receipt at t=t' is $L(R_B-R_A)(T_2-T_1)\exp[-R(T_2-t')]$ where R is the appropriate riskfree rate for the period (t',$T_2$).

But the value of the FRA B at t=t' is zero because it is negotiated at t=t'. As we know the value of a forward contract at its inception is zero. FRA is also a forward contract, so the same rationale holds for FRAs. However, as time passes, because of fluctuation in market variables that influence the payoff of the FRA, the FRA acquires a positive or a negative value. In fact, it a zero sum game, so the positive value for one party is equivalent to a negative value for the other party. Hence, the value of the portfolio Π at t=t' is equal to the value of FRA A at t=t' =$V_A$(t').

Hence, we must have $V_A(t') = L(R_B-R_A)(T_2-T_1)\exp[-R(T2-t')]$

Thus, for valuing an FRA we:

(i) Calculate the payoff on the assumption that forward rates are realized (that is, on the assumption that $R_M = R_{Fwd}$).
(ii) Discount this payoff at the risk-free rate for the appropriate maturity.

**Interest rate swap: Plain vanilla swap**

As I mentioned a FRA is an exchange of a fixed rate payment and a floating rate payment. If we have a series of such exchanges i.e. a series of FRAs at equal periodic intervals, then it constitutes an interest rate swap.

In this swap the one party (A) agrees to pay cash flows calculated on a notional principal at interest at a predetermined fixed rate for a predetermined number of years. In return, the other party (B) pays interest at a floating rate on the same notional principal for the same period of time.

**Example of interest rate swap**

An agreement by XYZ Ltd to receive 6-month LIBOR & pay a fixed rate of 5% per annum compounded semi-annually, every 6 months for 3 years on a notional principal of 100 million. The agreement is effective from March 5, 2019.

**Determination of floating rate**

The life of the IRS is divided into 6-monthly periods. For each period, the rate of interest is set as per the benchmark for setting the floating rate i.e. the 6-month LIBOR rate at the beginning of the period. Interest is, then, paid at the end of the period. Let us continue with our illustration.

**ONE POSSIBLE OUTCOME FOR CASH FLOWS TO XYZ Ltd.**

| Date | LIBOR | Floating CF (Receive) | Fixed CF (Pay) | Net CF |
|---|---|---|---|---|
| Mar 5, 2019 | 4.20% | | | |
| Sep 5, 2019 | 4.80% | +2.10 | −2.50 | −0.40 |
| Mar 5, 2020 | 5.30% | +2.40 | −2.50 | −0.10 |
| Sep 5, 2020 | 5.50% | +2.65 | −2.50 | + 0.15 |
| Mar 5, 2021 | 5.60% | +2.75 | −2.50 | +0.25 |
| Sep 5, 2021 | 5.90% | +2.80 | −2.50 | +0.30 |
| Mar 5, 2022 | | +2.95 | −2.50 | +0.45 |

In this example, XYZ receives 6-month LIBOR and pays fixed 5% per annum every 6-months for 3 years on a notional principal of 100 million.

Now, there is no issue as far as the fixed rate is concerned. It is fixed upfront @ 5%p.a. compounded semi-annually i.e. 2.50% for every 6-nmonth period. So XYZ pays 2.50% every 6-months.

The floating rate is 6-month LIBOR. This will be fixed every 6-moths at the beginning of the period to which it relates on the basis on the LIBOR prevailing at that point in time. Thus, for the first 6-month period starting from March 5, 2019, the rate that XYZ will receive i.e. the floating rate will be based on the LIBOR that prevails on this date i.e. March 5, 2019. Let us assume that this LIBOR (March, 5, 2019) = 4.20% p.a. or 2.10% for the 6 months.

Then, for the 6-month period beginning from March 5, 2019 i.e. March 5, 2019 to September 4, 2019, XYZ will pay 2.5% and will receive 2.10% i.e. XYZ will pay a net amount of (2.50-2.10)% = 0.40% on a principal of 100 million i.e. 0.40 million at the end of the 6-month period beginning March 5, 2019 i.e. on September 4, 2019.

For the next 6-months i.e. the 6-month period commencing September 5, 2019 the fixed rate is unaltered i.e. it remains at 2.50% (XYZ pays).

Recall that the floating rate is 6-month LIBOR. For the 6-month period starting from September 5, 2019, this rate will be fixed at the beginning of this 6-month period i.e. on September 5, 2019 on the then prevailing LIBOR. Let us assume LIBOR (September 5, 2019) =4.80% p.a. i.e. 2.40% for the 6-month period. Thus, XYZ will receive 2.40% for the 6 months starting September 5, 2019 and will pay 2.5% for the same period. Thus, the net payment will be of 0.10% on 100 million =0.10 million. But this payment will be at the end of the relevant 6-month period i.e. March 4, 2020.

This is how the swap will progress over its life-span of three years. The cardinal feature is that while the rate payable by XYZ remains unchanged over the life of the swap, the floating rate that XYZ receive will be fixed at the beginning of every six-month period to which it relates. However, the actual transfer of the net amount shall occur at the end of the said 6-month period.

Then, for the 6-month period beginning from March 5, 2019 i.e. March 5, 2019 to September 4, 2019, XYZ will pay 2.5% and will receive 2.10% i.e. XYZ will pay a net amount of (2.50-2.10)% = 0.40% on a principal of 100 million i.e. 0.40 million at the end of the 6-month period beginning March 5, 2019 i.e. on September 4, 2019.

So, party A pays the fixed rate, fixed rate in the sense that the rate is fixed at the time of negotiation of the swap and receives floating rate i.e. the rate that is fixed at the beginning of every period to which it relates. These rates are then used for calculating interest during that interval. However, the net cash stream is exchanged at the end of that period.

## <u>Swap as exchange of fixed and floating rate bonds</u>

**IRS AS EXCHANGE OF FIXED RATE TO FLOATING RATE BONDS**

| Date | LIBOR | Floating Rate Bond (Long) | Fixed Rate Bond (Short) | Net CF |
|---|---|---|---|---|
| Mar 5, 2019 | 4.20% | | | |
| Sep 5, 2019 | 4.80% | +2.10 | −2.50 | −0.40 |
| Mar 5, 2020 | 5.30% | +2.40 | −2.50 | −0.10 |
| Sep 5, 2020 | 5.50% | +2.65 | −2.50 | + 0.15 |
| Mar 5, 2021 | 5.60% | +2.75 | −2.50 | +0.25 |
| Sep 5, 2021 | 5.90% | +2.80 | −2.50 | +0.30 |
| Mar 5, 2022 | | +102.95 | −102.50 | +0.45 |

The cash flows in the third column of this table are the cash flows from a long position in a floating-rate bond. The cash flows in the fourth column of the table are the cash flows from a short position in a fixed-rate bond. The table shows that the swap can be regarded as the exchange of a fixed-rate bond for a floating-rate bond. XYZ whose position is described by the Table is long a floating-rate bond and short a fixed-rate bond.

From the above table, we see that a swap can also be interpreted as an exchange of a fixed rate bond and a floating rate bond because the notional principal is the same for both legs so that we can simply add that notional principal to the final cash flow whence, the two cash flow streams partake the character of interest payments on fixed and floating bonds of the face value equal to the notional principal.

Thus, the cash flows in the third column are essentially the cash flows that would be received by an investor invested in a floating rate bond while those in column 4 are representative of an entity that has issued fixed rate bond. The issuer of a fixed rate bond would pay periodic interest at every 6-months interval at a fixed rate and redeem the principal on maturity of the bond.

Similarly, an investor in a floating rate bond will receive interest as the floating rate and at the end of the maturity period of the bond will receive the principal. This is precisely what is happening. Thus, a long position in an IRS is equivalent to a long position in a floating rate bond and a short position in a fixed rate bond of the same nominal value.

**<u>Example: Designing a swap</u>**

Consider two entities BBB Ltd & AAA Bank. Each of them requires USD 100 million of funds. The nature of their requirement of funds and their sourcing costs are as follows:

| | | BBB Ltd | AAA Bank |
|---|---|---|---|
| Requirement | | 5 yr Fixed Rate $ | Floating Rate $ |
| Cost: | Fixed Rate | 8% | 6.5% |
| | Floating Rate | Prime+0.75% | Prime |

If the two entities borrow as per their requirements:

| | |
|---|---|
| BBB's cost of funds: | 8.00% |
| AAA's cost of funds: | P |
| Aggregate cost of funds (A): | P+8.00% |

If the two companies enter into a swap i.e. swap their sourcing:

| | |
|---|---|
| BBB borrows floating rate at: | P+0.75% |
| AAA borrows fixed rate at: | 6.5% |
| Aggregate cost of funds (B): | P+7.25% |

| | |
|---|---|
| Gross saving (C)=(A)-(B): | 0.75% |

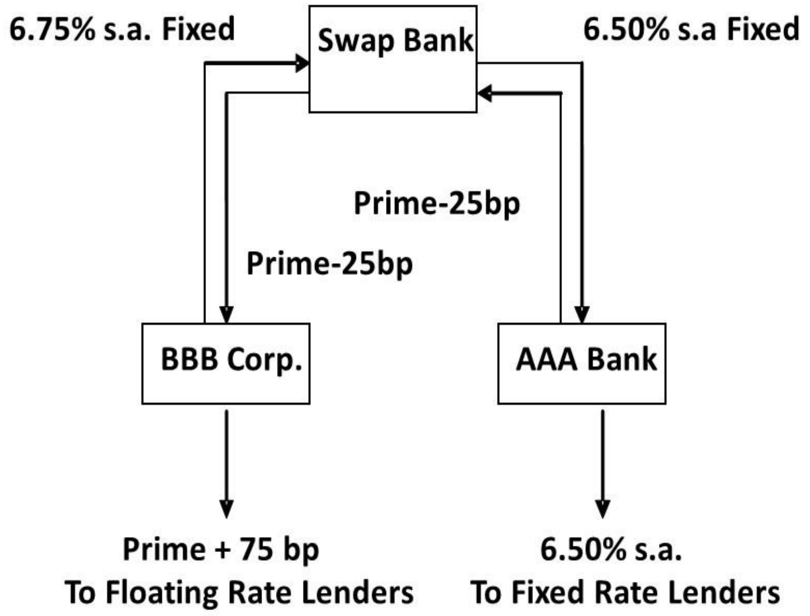
Let us assume that the swap was administered through a swap bank (broker) PQR who charged a commission of 0.25% for arranging & organizing the swap. Further, let us assume that BBB & AAA share the net savings equally. Then, we have:

| | |
|---|---|
| Gross saving (C): | 0.75% |
| Less broker commissions: | 0.25% |
| Net saving: | 0.50% |

Share of either party: 0.25%

Hence, net of cost of funds: BBB: 7.75% AAA: P-0.25%

A possible design of the swap would be as follows:




(i) AAA borrows USD 100 million at 6.5% fixed. It receives 6.50% from the swap bank. It pays P-0.25% to the swap bank.

(ii) BBB borrows USD 100 million floating at P+0.75%. It receives P-0.25% from the swap bank. It adds 1.00% and passes on P+0.75% to its floating rate lenders. It also pays 6.75% to the swap bank.

(iii) Swap bank receives 6.75% from BBB, takes its cut of 0.25% and passes on 6.50% to AAA. It receives P-0.25% from AAA and simply passes on this stream to BBB.

It may be noted that the savings on account of the swap can be shared in any negotiated proportion between the various parties involved in the swap including the broker.