Financial Derivatives & Risk Management
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Lecture 54
Option Greeks: Role in Trading Strategies Contd.; Swaps

Vega based strategies

(i) Short premium positions like Iron Condors or Butterflies will be negatively impacted by an increase in implied volatility, which generally occurs with downside market moves. This is because such strategies have negative vegas.

(ii) Therefore, when entering negative vega (Iron Condors or Butterflies) strategies, it makes sense to start with a slightly short (negative) delta bias. If the market stays flat or goes up, the volatility will decrease, and due to negative vega, the portfolio value will increase. But the negative delta will pull it down. The short premium will come in and our position benefits. However, if the market goes down accompanies by a rise in volatility, the negative vega will eat into the portfolio value. Nevertheless, the short delta will compensate this downfall as it will precipitate a value rise.

(iii) Following the same logic, it makes sense to start vega positive trades like calendars slightly delta positive. Such trades would benefit from an increase in implied volatility, which generally occurs with downside market moves. But if the underlying goes up with a fall in volatility, the positive vega effect will precipitate a decline in portfolio value but the positive delta will hedge his potential fall in portfolio value.

(iv) It also makes sense to use vega positive strategies like calendars when implied volatility is rising and vega negative strategies like Iron Condors when implied volatility is falling.

Positive vega strategies

(i) Long Call
(ii) Long Put
(iii) Long Straddle
(iv) Long Strangle
(v) Long Calendar Spread
(vi) Vertical Debit Spread (Bull Call Spread, Bear Put Spread)

Negative vega strategies

(i) Short Call, Short Put
(ii) Short Straddle, Short Strangle
(iii) Vertical Credit Spread
(iv) Covered Call Write, Covered Put Write
(v) Iron Condor
(vi) Butterfly

Option greeks & moneyness - summary

	Delta	Gamma	Theta	Vega
In the money	INCREASE -> 1.00	DECREASE ->0.00	DECREASE	DECREASE
At the money	± ->0.50	± ->MAX	-->MAX	+->MAX
Out of money	DECREASE ->0.00	DECREASE ->0.00	DECREASE	DECREASE

Option greek & expiration: summary

	Delta	Gamma	Theta	Vega
In the money	INCREASE	DECREASE	DECREASE	DECREASE
At the money	±	INCREASE	INCREASE	DECREASE
Out of money	DECREASE	DECREASE	DECREASE	DECREASE

Rho

Rho is the another option greek, perhaps not so important. It is not so important for simple reason that the risk-free rate or the risk-free rate of interest is not as volatile as any of the other variables. As a result of this, ρ does not assume relevance when talking about trading strategies, at least for short-maturity options because over that period, the variation is interest rate is unlikely and insignificant, by and large.

Rho is the first partial derivative of the OPM with respect to the riskfree interest rate

Black Scholes ρ

$$\rho = \frac{\partial c}{\partial r} = S\mathbf{N}'(d_1)\frac{\partial d_1}{\partial r} - Ke^{-r(T-t)}\mathbf{N}'(d_2)\frac{\partial d_2}{\partial r} + (T-t)Ke^{-r(T-t)}\mathbf{N}(d_2)$$

$$= (T-t)Ke^{-r(T-t)}\mathbf{N}(d_2) + S\mathbf{N}'(d_1)\left(\frac{\partial d_1}{\partial r} - \frac{\partial d_2}{\partial r}\right) \sin ce\, S\mathbf{N}'(d_1) = Ke^{-r(T-t)}\mathbf{N}'(d_2)$$

$$= (T-t)Ke^{-r(T-t)}\mathbf{N}(d_2) + S\mathbf{N}'(d_1)\frac{\partial}{\partial r}\left(\sigma\sqrt{T-t}\right) = (T-t)Ke^{-r(T-t)}\mathbf{N}(d_2)$$

Trading strategies based on option greeks: near-maturity trades

(i) If you expect a big move, go long with near-expiration trades. This will create large positive Γ and positive υ. Thus, if the large move materializes, the positive Γ will bring in large profits. If, at the same time, the volatility also increases (as is usual in bearish markets), the vega will serve as an add-on bringing in additional profits. The flip side, however, is that if the large expected large movement does not materialize, gamma profits will vanish and if the volatility is also unchanged, vega profits will also not arise. But, these near-maturity long trades create large negative thetas. These thetas relentlessly erode the value of the portfolio as maturity approaches (irrespective and independent of gamma and vega effects). Hence, while the gamma and vega profits will not arise, theta will continue eating into the value of your portfolio with each passing day cumulating to large losses. Thus, in such a case, if the large move doesn't materialize, you will start losing money much faster, it becomes a "theta against gamma" fight.

(ii) When you expect an increase in volatility (e.g. before earnings), positive vega will generate profits. It's a "theta against vega" fight, and the large gamma is the added bonus.

(iii) When you are net "short" options, the opposite is true. For example, Iron Condor is a vega negative and theta positive trade. That means that it benefits from the decline in implied volatility (IV) and the time decay. If you initiate the short trade when volatility is high and is declining during the life of the trade, the trade wins twice: from the declining implied volatility (negative vega) and the time passage (positive theta). However, it is also gamma negative and the gamma accelerates as we get closer to expiration. Any big move of the underlying will cause big losses due to a large negative gamma.

(iv) If you expect a big move but you do not know the direction of the move, the first thing that you would do is that you will create a positive gamma. The higher the gamma, the higher would be a profit if that perception of the big move turns out to be correct. In other words, you will net long gamma on your portfolio.

Now, if you perceive that the move is likely to be downside with increasing volatility i.e. the market is likely to turn bearish, you can add value to your portfolio by making it long vega, large positive vega. Then, volatility increase will add to your positive gamma profits.

Now positive gamma, positive vega can be created by long calls. But if you do that, where do you lose out? You lose out on theta, long calls close to expiration have very high negative thetas. In other words, as time passes, the amortisation of their value is very rapid. Thus, if this anticipated big move does not materialise, the negative theta is going to rapidly eat into the value of your portfolio and you could be burdened with losses due to the decay of time value of the calls.

Conversely, if you carry the perception that the volatility is going to fall with stable stock prices, a negative vega, gamma neutral strategy could be considered. A negative vega, positive gamma strategy could be optimal in a rising (bullish) market with falling volatility and so on.

Vega positive theta neutral trades

These trades provide another variant where the portfolio has a positive vega but is made theta neutral. Because of theta neutrality, the portfolio is relatively insulated against time decay i.e. loss in value due to mere passage of time. Normally, if the stock price does not move, the portfolio tends to lose value due to theta decay or time decay. By having a theta neutral portfolio, this time decay is substantively arrested. Thus, while the portfolio will benefit due to an increase in volatility (positive vega), in the event that the increase does not materialize, we do not lose value rapidly due to passage of time (neutral theta).

How this is done? To illustrate, we have theta positive calendar spreads and theta negative straddles. Both have positive vegas. A combination of these can be structured to have the desired properties.

So that is the interplay between the various option greeks, gamma versus theta, vega vs theta etc.

<u>Option greeks & trading</u>

Line	Max Profit?*	VEGA POSITION	DELTA POSITION	Max Profit?*	PORTFOLIO BIAS**
1		Long Vega	Short Delta		BEARISH
2		Short Vega	Short Delta		NEUTRAL
3		Long Vega	Long Delta		NEUTRAL
4		Short Vega	Long Delta		BULLISH

Line	Max Profit?*	VEGA POSITION	DELTA POSITION	Max Profit?*	PORTFOLIO BIAS**
1	↓	←Long Vega	Short Delta →	↓	BEARISH
2	↑	←Short Vega	Short Delta →	↑	NEUTRAL
3	↓	←Long Vega	Long Delta →	↓	NEUTRAL
4	↑	←Short Vega	Long Delta →	↑	BULLISH

<u>Example</u>

An investor X has taken short positions in 2000 calls and 2000 puts on the stock of XYZ Ltd. Over the price range of interest to X, the price of calls increases (decreases) @ 70% of the increase (decrease) in price of the underlying stock whereas the price of puts decreases (increases) @ 30% of the increase (decrease) in the price of underlying stock. What is the number of shares of the underlying stock that he must purchase/short-sell in order to create a delta-neutral position?

<u>Solution</u>

Delta of portfolio = Weighted average delta of its constituents
=(-2,000)*(0.70)+(-2,000)*(-0.30)=-800
Since one unit of stock creates delta of +1, no of units of stock required =800 (long).

Example

Suppose that a portfolio held by a US bank can be made delta neutral with a short position of 6.20 million pounds. The position is, however, proposed to be made delta neutral by taking a short position in a 9-month forward contract rather than the spot short position in pounds. Risk-free rates are 66% in the US and 6% in the UK. What is the number of pounds to be shorted under the forward contract (in millions of pounds)?

Solution

Since the portfolio can be made delta neutral by a short position in GBP 6,200,000 (GBP is the underlying asset) and one unit i.e. GBP 1.00 creates a delta of +1, it follows that the delta of the portfolio is +6,2000,000.

Now, the relation between spot price and forward price is $F=Se^{(r-q)T}$. It follows that the delta of a forward is $\frac{\partial F}{\partial S}=e^{(r-q)T}$. In the given problem, r= home country riskfree rate =0.66; q= foreign country riskfree rate =0.06; T=0.75 years. Hence, $\Delta_{forward}=e^{(0.66-0.06)*0.75}=1.5683$

Since, we need to create a delta of -6,200,000, the number of units required to be shorted under the forward contract =-6,200,000/1.5683=-3,953,295, (-) indicates shorting.

Example

Calculate the delta of an at-the–money six-month European call option on a non-dividend-paying stock when the risk-free interest rate is 10% per annum and the stock price volatility is 25% per annum.

Solution

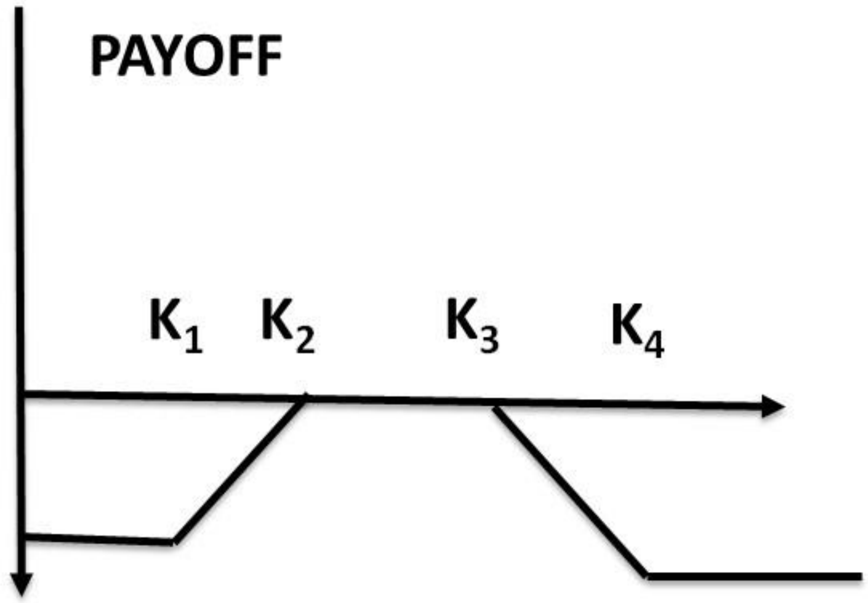
Since the option is at the money S_0=K. Now, $d_1=\frac{\ln\left(S_0/K\right)+\left(0.1+0.25^2/2\right)0.5}{0.25\sqrt{0.5}}=0.3712$

so that $\Delta=\mathbf{N}(d_1)=\mathbf{N}(0.3712)=0.64$

Appendix

Iron condor & regular condor

Iron condor



An iron condor consists of the following:

(i) Long OTM put (X) at K_1
(ii) Short OTM put (Y) at K_2
(iii) Short OTM call (A) K_3
(iv) Long OTM call (B) K_4

$K_1<K_2< K_3<K_4$

Initial investment

Let us first look at the initial investment.

(i) Let us, first look at the put spread. We have long OTM put X at K_1 and short OTM put Y at K_2 with $K_1<K_2$. Now, for put options, the premium varies directly with the exercise price i.e. higher the exercise price, higher the premium since put is a right to sell the asset at the exercise price. Since, the long put (purchased) is at a lower exercise price and the short put (sold) it at a higher exercise price, there will be a net cash inflow from this combination of put options when the spread is created.

(ii) Now, the call spread. We have short OTM call A at K_3 and long OTM call B at K_4 with $K_3<K_4$. For call options, the premium varies inversely with the exercise price i.e. higher the exercise price, lower the premium since call is a right to buy the asset at the exercise price. Since, the long call (purchased) is at a higher exercise price and the short call (sold) it at a lower exercise price, there will be a net cash inflow from this combination of call options at the creation of the strategy.

(iii) Thus, both the put spread and the call spread result in a cash inflow at inception so that the iron condor strategy will generate a cash inflow at inception.

<u>Payoff of iron condor</u>

Let us now look at the payoff at maturity of the condor. As usual, we divide the stock price spectrum as $0<S_T<K_1$; $K_1<S_T<K_2$; $K_2<S_T<K_3$; $K_3<S_T<K_4$; $K_4<S_T$. Then,

S_T	$(0,K_1)$	(K_1,K_2)	(K_2,K_3)	(K_3,K_4)	$(>K_4)$
$X(Long\ Put)$	K_1-S_T				
$Y(Short\ Put)$	S_T-K_2	S_T-K_2			
$A(Short\ Call)$				K_3-S_T	K_3-S_T
$B(Long\ Call)$					S_T-K_4
$TOTAL$	$K_1-K_2<0;$	$S_T-K_2<0;$	0	$K_3-S_T<0;$	$K_3-K_4<0$

(i) Now, long put option X has a strike of K_1, so it will be exercised if $0<S_T<K_1$ and will generate a payoff of K_1-S_T.

(ii) Short put Y has strike K_2 and will be exercised if $S_T<K_2$. It will generate a payoff of S_T-K_2.

(iii) Short call A has strike K_3 and will be exercised if $S_T>K_3$ generating a payoff of K_3-S_T.

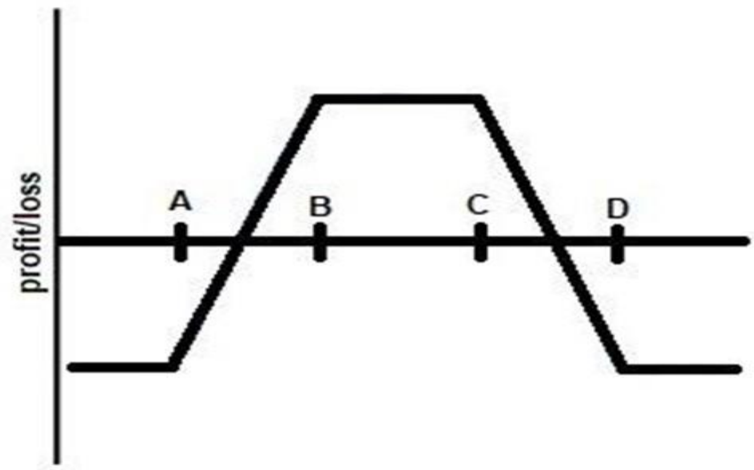
(iv) Long call B has strike K_4 and will be exercised if $S_T>K_4$ generating a payoff of S_T-K_4.

If we aggregate all these payoffs, we find that in every scenario except when $K_2<S_T<K_3$, the payoff is negative. And even when $K_2<S_T<K_3$, the payoff is zero. Thus, in the best case scenario the payoff is zero, otherwise it is always negative. However, we had a positive cash flow at the inception of this strategy.

Thus, if the stock price ends up as $K_2<S_T<K_3$ at options maturity, the investor makes a zero loss at maturity but retains the positive cashflow of $-P_X+P_Y+P_A-P_B$ i.e. the net premium from the strategy which is equal to the difference between the premia of the two calls and the premia of the two puts that constituted the strategy at the time of inception.

So, the iron condor at maturity does not give a positive payoff but it does give a positive inflow at the time of creation of the strategy.

<u>Regular condor</u>



A regular condor consists of the following:

(i) Long call (A) at K_1
(ii) Short call (B) at K_2
(iii) Short call (C) at K_3
(iv) Long call (D) at K_4

$K_1<K_2< K_3<K_4$

<u>Payoff of regular condor</u>

Let us now look at the payoff at maturity of the condor. As usual, we divide the stock price spectrum as $0<S_T< K_1$; $K_1<S_T<K_2$; $K_2<S_T<K_3$; $K_3<S_T<K_4$; $K_4<S_T$. Then,

S_T	$(0,K_1)$	(K_1,K_2)	(K_2,K_3)	(K_3,K_4)	$(>K_4)$
$A(Long\ Call)$		S_T-K_1	S_T-K_1	S_T-K_1	S_T-K_1
$B(Short\ Call)$			K_2-S_T	K_2-S_T	K_2-S_T
$C(Short\ Call)$				K_3-S_T	K_3-S_T
$D(Long\ Call)$					S_T-K_4
$TOTAL$	0;	S_T-K_1;	K_2-K_1;	$-(S_T+K_1)$ $+(K_2+K_3)$	K_2+K_3- (K_1+K_4)

(i) Now, long call option A has a strike of K_1, so it will be exercised if $K_1<S_T$ and in all the later cases with a payoff of S_T-K_1.
(ii) Short call B has strike K_2 and will be exercised if $K_2<S_T$. It will generate a payoff of $K_2- S_T$.
(iii) Short call C has strike K_3 and will be exercised if $K_3<S_T$ generating a payoff of K_3-S_T.
(iv) Long call D has strike K_4 and will be exercised if $S_T>K_4$ generating a payoff of S_T-K_4.

If we aggregate all these payoffs, we find that in every scenario the payoff is non-negative. Thus, in the worst case scenario the payoff is zero, otherwise it is always positive. Hence, by no-arbitrage requirements, it must carry a positive cost for setting up the strategy.