Financial Derivatives and Risk Management
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Lecture 53
Option Greeks: Further Properties, Role in Trading Strategies Contd.

Recap

(i) Delta measures the rate of change of the call price with respect to the price of the underlying stock.
(ii) Positions with positive delta increase in value if the underlying goes up. Positions with negative delta increase in value if the underlying goes down.
(iii) $dc=\Delta.dS$ to first order. If Δ is positive, then positive dS will cause increase in call value and vice versa.
(iv) Call delta increases with increase in stock price from 0 for deep OTM calls to 1 for deep ITM calls.
(v) As expiration approaches, ITM call delta approaches 1 with increasing rapidity, OTM call delta approaches 0 with increasing rapidity. This is because time value erodes quickly as you approach maturity.
(vi) Delta hedging provides immunity against price changes, but only in an infinitesimal region. For perfect hedging continuous rebalancing of portfolio is required.
(vii) This is because of CURVATURE of the call price curve so that delta value changes with every move of the stock price.
(viii) This curvature is captured by GAMMA. If gamma is small, then the delta hedged portfolio is robust and rebalancing may be done at infrequent intervals

Gamma

(i) Gamma measures the rate of change for delta with respect to the underlying asset's price.
(ii) All long options have positive gamma and all short options have negative gamma.
(iii) In BSOPM, put and calls have the same gamma.
(iv) The gamma is maximum for ATM calls and approaches 0 as the stock price decreases or increases. The gamma of both OTM and ITM calls approaches zero.
(v) As expiration approaches, ATM call gamma increases very rapidly while gamma of OTM and ITM both approaches zero.
(vi) When volatility is low, the gamma of At-The-Money options is high while the gamma for deeply into or out-of-the-money options approaches 0.
(vii) When volatility is high, gamma tends to be stable across all strike prices.
(viii) If gamma of a position is small, delta neutrality is robust and frequent rebalancing is not required.
(ix) **Positive gamma can be used to book profits.**

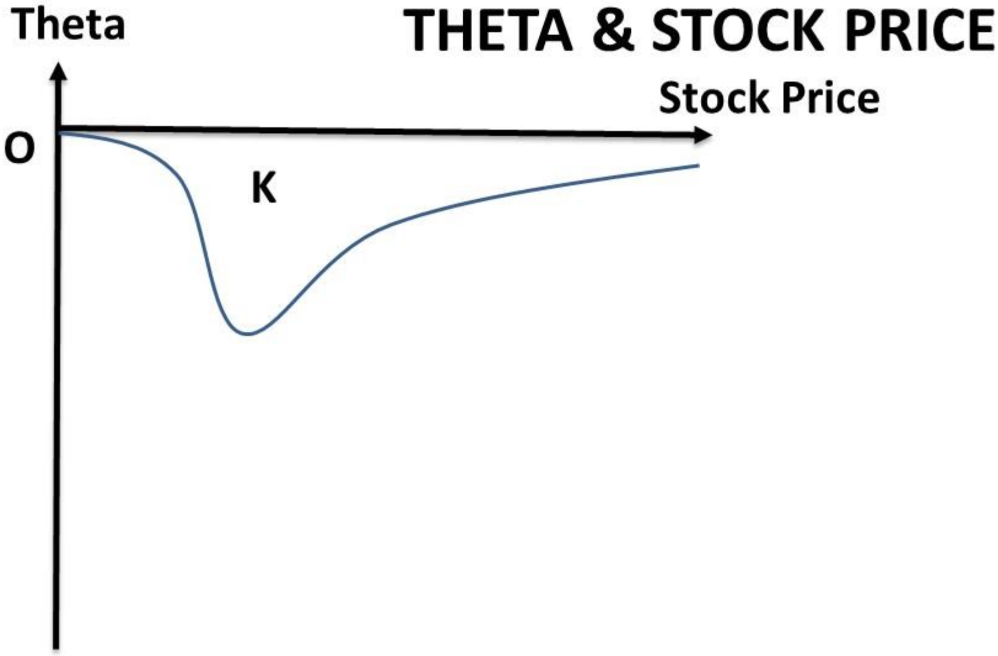
Theta

(i) The theta of an option is the rate of change of its value with respect to the passage of time with all else remaining the same. Theta is sometimes referred to as the time decay of the portfolio.

(ii) The Black Scholes theta is given

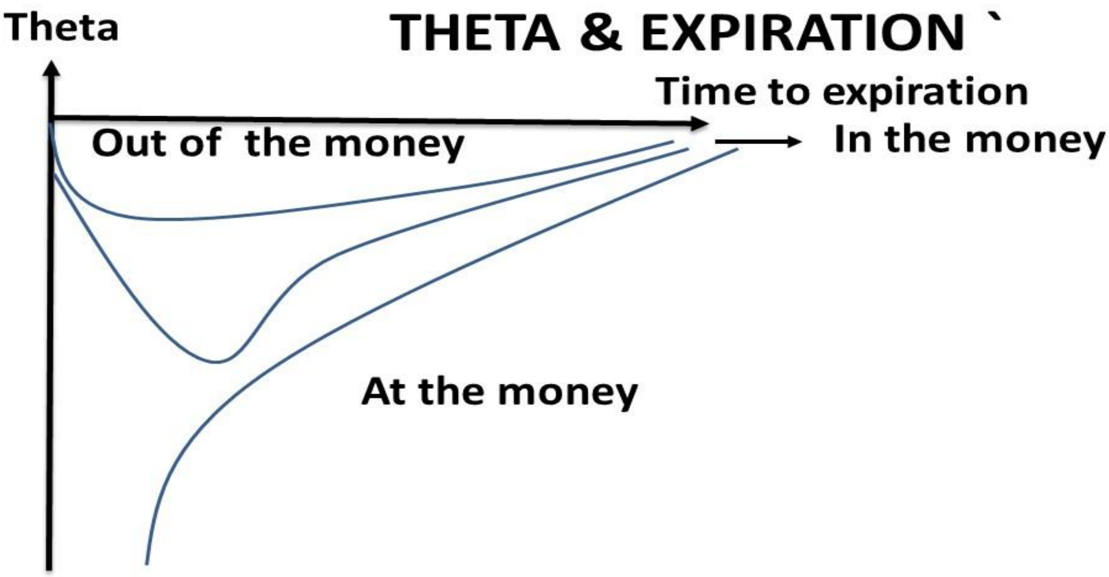
$$\Theta_c = \frac{\partial c}{\partial t};\ \ \Theta_p = \frac{\partial p}{\partial t};\ \Theta_c = -\frac{S\mathbf{N}'(d_1)\sigma}{2\sqrt{(T-t)}} - rKe^{-r(T-t)}\mathbf{N}(d_2)$$

(iii) When the stock price is very low, theta is close to zero. For an at-the-money call option, theta is large and negative. As the stock price becomes larger, theta tends to -rKexp(-rT) for BS options.



(iv) Theta values are negative in long calls. Initially, OTM options have a faster rate of theta decay than ATM, but as expiration nears, the rate of theta option time decay for OTM options slows and the ATM options begin to experience theta decay at a faster rate.

(v) As expiration nears, the rate of time decay for OTM options slows and the ATM options begin to experience theta decay at a faster rate. This is because time value is a much smaller component of an OTM option's price, the closer the option is to expiring.



Gamma Theta trade-off

If you buy at the money options close to expiration, they will create a significant positive gamma portfolio. Thus, if a significant and quick price move of underlying occurs, such options with closer expiration will gain significantly.

However, long positions in such short-dated options create high negative theta. Thus, they will undergo rapid time decay as maturity approaches.

The trade-off is that if the underlying doesn't move, the negative theta will start to kick off much faster but if the stock price fluctuates significantly, then the positive gamma will capture large profits.

Vega

Vega is the first partial derivative of the option with respect to the volatility of the underlying asset. Vega measures of an option's sensitivity to changes in the volatility of the underlying asset. Volatility measures the amplitude and frequency at which option price moves up and down. The higher the volatility, the higher the value of the option. Thus, if there is an increase in the volatility of the stock price, it will reflect as an increase in the call price as well whence vega of calls is positive. All long options have positive vegas.

Vega of BS calls

$$\upsilon = \frac{\partial c}{\partial \sigma} = S\mathbf{N}'(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-r(T-t)}\mathbf{N}'(d_1)\frac{\partial d_2}{\partial \sigma} = S\mathbf{N}'(d_1)\left(\frac{\partial d_1}{\partial \sigma} - \frac{\partial d_2}{\partial \sigma}\right) \textit{ as } S\mathbf{N}'(d_1) = Ke^{-r(T-t)}\mathbf{N}'(d_2)$$

$$\textit{Hence, } \upsilon = \frac{\partial c}{\partial \sigma} = S\mathbf{N}'(d_1)\frac{\partial}{\partial \sigma}\left(\sigma\sqrt{T-t}\right) = S\sqrt{T-t}\mathbf{N}'(d_1) = Ke^{-r(T-t)}\sqrt{T-t}\mathbf{N}'(d_2)$$

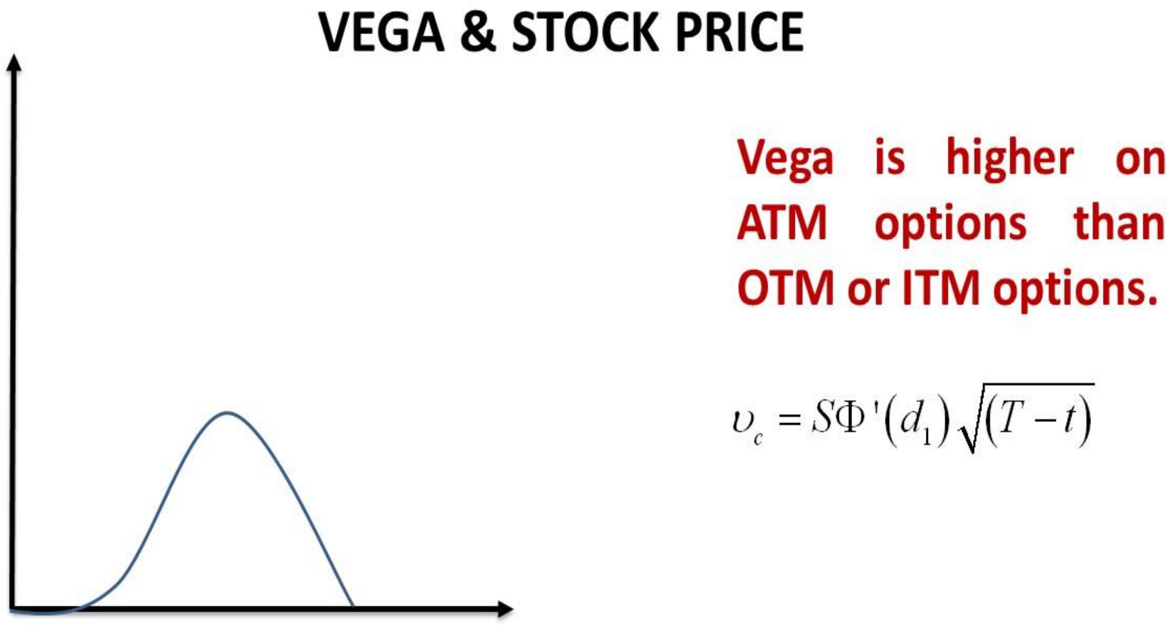
$$\upsilon_c = \frac{\partial c}{\partial \sigma};\ \upsilon_p = \frac{\partial p}{\partial \sigma};\ \upsilon_c = S\mathbf{N}'(d_1)\sqrt{(T-t)}$$

Each of the quantities on the RHS viz, S, **N**'(d_1) and √(T-t) are positive so that $\upsilon>0$. Indeed, higher the volatility, higher the time value of the option, because larger is the uncertainty in relation to the payoff from the option.

Remember, usually bearish markets are associated with higher volatility. Vega for all options is always a positive number because options increase in value when volatility increases and decrease in value when volatility declines.

When position vegas are generated, however, positive and negative signs appear. When you establish a position selling or buying an option, this will result in either a negative sign (for selling) or positive sign (for buying), and the position vega will depend on net vegas. It may be noted that the Vega of a portfolio consisting of number of derivatives will be equal to the weighted average Vega of the constituent derivatives.

Vega & stock price



We have, $\upsilon_c = \dfrac{\partial c}{\partial \sigma};\ \upsilon_p = \dfrac{\partial p}{\partial \sigma};\ \upsilon_c = S\mathbf{N}'(d_1)\sqrt{(T-t)}$ whence $\upsilon_c \to 0$ as $S \to 0$ since $d_1 \to -\infty$ and $\mathbf{N}'(d_1) \to 0$ so that call vegas approach 0 for OTM options. But as $S \to \infty$, $d_1 \to \infty$ and $\mathbf{N}'(d_1) \to 0$ so we need to look at the leading order behaviour. If we look at the normal pdf, it is seen to approach 0 very rapidly as S becomes larger & larger. On the other hand $S \to \infty$ linearly. So clearly, as S becomes large, the $\mathbf{N}'(d_1)$ term will dominate over the S term and the product will remain finite and $\to 0$ slowly. The implication is that ITM call vegas also approach 0 as the stock price rises e.g.

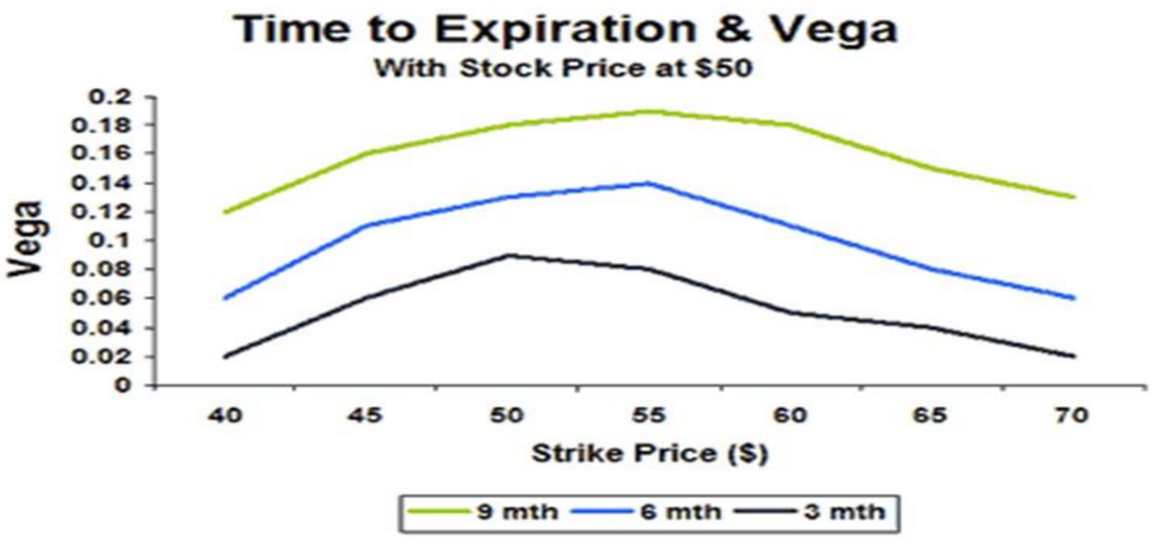
$$\mathbf{N}'(d_1) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}d_1^2\right)\ hence\ \nu = \frac{1}{\sqrt{2\pi}} S\sqrt{T_{\exp}} \exp\left(-\frac{1}{2}d_1^2\right)$$
$$= \frac{1}{\sqrt{2\pi}} \frac{S}{\exp\left(\frac{1}{2}d_1^2\right)} \sqrt{T_{\exp}} = \frac{1}{\sqrt{2\pi}} \frac{1}{\frac{1}{S}\left(1+\frac{1}{2}d_1^2+\ldots\right)} \sqrt{T_{\exp}} \xrightarrow{S,d_1\to\infty} 0$$

For ATM calls, both the factors S, $\mathbf{N}'(d_1)>0$ and are significant so that such calls have the maximum Vega. As S decreases and the calls go OTM or S increases and the calls go deep ITM, Vega approaches 0.

In case of calls which are deeply OTM or deeply ITM, if the volatility changes by a small amount their moneyness is not going to change significantly. Therefore, their price is also not going to change significantly due to small change in volatility. Hence, their Vega is small.

However, in the case of ATM calls, small changes in volatility can result in a significant change in moneyness and as a result of which Vega for such calls is larger.

Vega & expiration



Higher is the time to expiration, higher would be the effect of a change in volatility on the option price because the option's time value is going to be influenced by this change in volatility over a longer period. Let us explore the case of ATM options. We have,

$$\nu = S\mathbf{N}'(d_1)\sqrt{T_{\exp}} \; \textit{where } d_{1,ATM(S=K)} = \lambda\sqrt{T_{\exp}} \; \textit{so that}$$

$$\mathbf{N}'(d_1) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\lambda^2 T_{\exp}\right) \; \textit{hence } \nu = \frac{1}{\sqrt{2\pi}} S\sqrt{T_{\exp}}\exp\left(-\frac{1}{2}\lambda^2 T_{\exp}\right)$$

$$= \frac{1}{\sqrt{2\pi}} S \frac{\sqrt{T_{\exp}}}{\exp\left(\frac{1}{2}\lambda^2 T_{\exp}\right)} = \frac{1}{\sqrt{2\pi}} S \frac{1}{\frac{1}{\sqrt{T_{\exp}}}\left(1+\frac{1}{2}\lambda^2 T_{\exp}+\ldots\right)} \xrightarrow{T_{\exp}\to\infty} 0$$

$$0 = \frac{\partial \nu}{\partial T_{\exp}} = \frac{1}{\sqrt{2\pi}} S \frac{\exp\left(\frac{1}{2}\lambda^2 T_{\exp}\right)\frac{1}{2\sqrt{T_{\exp}}} - \sqrt{T_{\exp}}\frac{1}{2}\lambda^2\exp\left(\frac{1}{2}\lambda^2 T_{\exp}\right)}{\exp\left(\lambda^2 T_{\exp}\right)} = 1 - \frac{1}{2}\lambda^2 T_{\exp}$$

$$\textit{whence } T_{\exp,\max} = \frac{2}{\lambda^2} \; \textit{where } \lambda = \frac{r+\frac{1}{2}\sigma^2}{\sigma}$$

The above quantitatively establishes that, at least for ATM calls, Vegas of long maturity calls approach 0. Further, at maturity Vega of all types of calls must necessarily vanish since any change in volatility has no time to act on the call and, indeed, no uncertainty remains with respect to the payoff. However, there is a point $T_{\exp,\max} = \frac{2}{\lambda^2}$ *where* $\lambda = \frac{r+\frac{1}{2}\sigma^2}{\sigma}$ at which vega takes a maximum value for ATM calls.

For practical maturities it is seen that vegas tend to increase with time as a change in volatility impacts the call over a longer time. In the diagram, we observe that

(i) the 9-month call has the largest vega among the 3,6 and 9 month calls plotted.

(ii) Further, in each of the calls, the maximum vega occurs near about the ATM moneyness.

Thus, to summarize,

(i) Vega for long calls is positive, so when an investor buys calls, vega is his friend as his portfolio gains if the volatility rises.
(ii) Vega is higher on ATM options than OTM or ITM options.
(iii) Vega decreases as the option approaches expiration.

Vega decreases as calls approach maturity. This is contrary to the behaviour of Gamma and Theta. Gamma and Theta both increase in magnitude towards maturity although gamma is positive theta is negative. Theta becomes more negative in that sense the absolute value increases. Vega, however, decreases as we approach expiration.

Vega changes when there are large price movements in the underlying asset. Vega is a function of S so Vega will change when S changes. Vega will, therefore, change from point-to-point along the call price curve.

<u>Construction of a vega-neutral portfolio</u>

A position in the stock has zero vega. However, the vega of a portfolio can be changed, similarly to the way gamma can be changed, by adding a position in a traded option.

The first step is to make the portfolio vega-neutral by taking appropriate positions in the traded options. If υ is the vega of the portfolio and υ_T is the vega of a traded option, a position of $(-\upsilon/\upsilon_T)$ in the traded option makes the portfolio instantaneously vega neutral.

Unfortunately, a portfolio that is gamma neutral will not in general be vega neutral, and vice versa. If a hedger requires a portfolio to be both gamma and vega neutral, at least two traded derivatives dependent on the underlying asset must usually be used.

We illustrate the procedure by a comprehensive example:

<u>Example</u>

Consider a portfolio that is delta neutral, with a gamma of -5,000 and a vega of -8,000. The options shown in the table below can be traded. Calculate how the position can be made gamma and vega neutral by including positions in Option 1, 2 and the underlying stock.

	Delta	Gamma	Vega
Portfolio	0	-5000	-8000
Option 1	0.6	0.5	2.0
Option 2	0.5	0.8	1.2

<u>Solution</u>

To make the portfolio gamma and vega neutral, both option 1 and Option 2 can be used. If w_1 and w_2 are the quantities of Option 1 and Option 2 that are added to the portfolio, we require that $-5,000+0.5w_1+0.8w_2=0$ and $-8,000+2.0w_1+1.2w_2=0$. The solution to these equations is w_1 = 400 w_2 =6,000. The portfolio can therefore be made gamma and vega neutral by including 400 units of Option 1 and 6,000 units of Option 2. The delta of the portfolio, after the addition of the positions in the two traded options, is 400x0.6 +6,000x 0.5 =3,240. Hence, 3,240 units of the asset woukd have to be sold to maintain delta neutrality.

Thus, the portfolio is made vega, gamma and delta neutral by a long position in 400 units of Option 1, a long position in 6000 units of Option 2 and a short position in 3240 units of the stock.

Thus, we, first, created vega and gamma neutrality by introducing two new options. By taking appropriate positions in these options, we made the portfolio vega and gamma neutral.

However, due to the introduction of these new options, the delta of the portfolio was disturbed. The delta neutrality was recreated by taking positions in the stock. Now, this addition of stock to the portfolio does not affect the vega or gamma of the portfolio because the stock has 0 values of vega and gamma. Thus, the vega and gamma neutrality created earlier is not disturbed.

Remember, one unit of the underlying stock contributes one unit in terms of Δ, 0 units of υ and 0 units of Γ. Thus, stock addition or shorting in the portfolio does not influence the υ and Γ.

So the procedure is simple, you construct a vega & gamma neutral portfolio first. Once you have the vega & gamma neutral portfolio, work out its delta and then neutralize delta by taking position in the stock.

Vega based strategies

Let us now look at some vega based strategies e.g. short premium positions like Iron condors, butterflies. Short premium positions mean positions where the investor gets net premium on setting up the strategy, the inflow on account of premium exceeds the out flow on account of the premium.

Short premium positions like iron condor or butterflies will be negatively impacted by an increase in implied volatility. Implied volatility increases when we have a bearish trend in the market. Thus, bearish trend in the market increases implied volatility that leads to a negative impact on short premium positions. This is because this short premium positions have negative vegas. Negative vegas implies an increase in implied volatility will result in a decline in value.

When entering Iron Condors or Butterflies (negative vega), it makes sense to start with a slightly short (negative) delta bias. If the market stays flat or goes up, the volatility will decrease, and due to negative vega, the portfolio value will increase. But the negative delta will pull it down. The short premium will come in and our position benefits.

However, if the market goes down, implied volatility will rise. The short (negative) vega position will cause a fall in portfolio value. But the portfolio has a short (negative) delta. Hence, the fall in the market will result in a rise in the portfolio value.

Following the same logic, it makes sense to start vega positive trades like calendars slightly delta positive. Such trades would benefit from an increase in implied volatility, which generally occurs with downside market moves. But if the underlying goes up, the positive delta will hedge potential implied volatility decrease.

It also makes sense to use vega positive strategies like calendars when implied volatility is low and vega negative strategies like Iron Condors when implied volatility is high.