Financial Derivatives & Risk Management
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Lecture 50 - Option Greeks: Definition and Properties

Option greeks: Definition & Motivation

Let us start with the generalized Black Scholes option pricing formula for the value of a call option on a yield-bearing underlying asset:

$$c(S,t) = Se^{-qT}\mathbf{N}(d_1) - Ke^{-rT}\mathbf{N}(d_2) \text{ where}$$

$$d_1 = \frac{\ln\frac{S^{-qT}}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\frac{S}{K} + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \text{ and}$$

$$d_2 = \frac{\ln\frac{S^{-qT}}{K} + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\frac{S}{K} + \left(r - q - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

It follows from this formula, that the instantaneous value of a call option is dependent on the following variables:

(i) The instantaneous stock price S;
(ii) The term to maturity T;
(iii) The stock volatility σ;
(iv) The riskfree rate r;
(v) The yield on the underlying asset q.

The strike price is not a variable. It is the exercise price and it is fixed by contract and remains constant over the life of the option.

Thus, in order to study the impact on the call price of each of these variables, we examine the change in the call price due to infinitesimal changes in each of these variables. We do so by doing a Taylor expansion of the call price around a certain value predetermined value fixed by some initial values of these variable i.e. $c_0 \equiv c(S_0, T_0, \sigma_0, r_0, q_0)$. We have, on Taylor expanding:

$$c(S_0 + \Delta S, T_0 + \Delta T, \sigma_0 + \Delta\sigma, r_0 + \Delta r, q_0 + \Delta q) = c_0 + \left(\frac{\partial c}{\partial S} = \Delta\right)\Bigg|_{S=S_0} .\Delta S +$$

$$\frac{1}{2}\left(\frac{\partial^2 c}{\partial S^2} = \Gamma\right)\Bigg|_{S=S_0} .(\Delta S)^2 + \left(\frac{\partial c}{\partial T} = \Theta\right)\Bigg|_{T=T_0} .\Delta T + \left(\frac{\partial c}{\partial\sigma} = \nu\right)\Bigg|_{\sigma=\sigma_0} .\Delta\sigma$$

$$+\left(\frac{\partial c}{\partial r} = \rho\right)\Bigg|_{r=r_0} \Delta r + \left(\frac{\partial c}{\partial q} = \varphi\right)\Bigg|_{q=q_0} \Delta q$$

The above Taylor expansion enables us to define the fundamental option greeks as the various partial derivatives that are shown in the round brackets viz. $\frac{\partial c}{\partial S} = \Delta$, $\frac{\partial^2 c}{\partial S^2} = \Gamma$, $\frac{\partial c}{\partial T} = \Theta$, $\frac{\partial c}{\partial \sigma} = \nu$, $\frac{\partial c}{\partial r} = \rho$ and $\frac{\partial c}{\partial q} = \varphi$.

This left hand side represents the call price at a slightly displaced value of these parameters from the original value at which the call price was $c_0 \equiv c\left(S_0, T_0, \sigma_0, r_0, q_0\right)$. Consequent to a slight shift in the underlying parameters, the call price shifts to $c\left(S_0 + \Delta S, T_0 + \Delta T, \sigma_0 + \Delta\sigma, r_0 + \Delta r, q_0 + \Delta q\right)$.

We have defined the option greeks as the partial derivatives of the call price with respect to the underlying variables. Clearly, therefore, these option greeks represent the sensitivities of the call price to these variables viz. the stock price, the time to maturity or their time expired, the volatility of the stock, the risk-free rate and the yield around an infinitesimal neighbourhood of the point at which they are computed.

Recall that partial derivatives are simply the ratio of the changes in the value of a function f(x,y) to an infinitesimal change in the value of one of its argument, while keeping the other arguments unchanged i.e. $\frac{\partial f\left(x,y\right)}{\partial x}$ is the ratio of the change in value of $f\left(x,y\right)$ i.e. $\partial f\left(x,y\right)$ and the infinitesimal change in value of x i.e. ∂x causing the change $\partial f\left(x,y\right)$ while keeping y unchanged.

Other than Γ, all the greeks are first order derivatives. This is because we chose to truncate the Taylor series at that point. We assumed and accepted that the anticipated changes in these variables would be small enough for higher order Δ terms to be insignificant. However, it is not at all mandatory that we must truncate the Taylor expansion at the level of first order and ignore all the higher order terms. With the increase in computing power, indeed, higher accuracy in predicting derivative prices can be achieved by including higher terms of the Taylor expansion.

Although the set Delta, Gamma, Theta, Vega, Rho and Phi constitute the fundamental set of option greeks that are in regular use, the set of option greeks has been actually extended to include the second order partials with the increase in computing power e.g.

$$c\left(S_0 + \Delta S, T_0 + \Delta T, \sigma_0 + \Delta\sigma, r_0 + \Delta r, q_0 + \Delta q\right) = c_0 + \frac{\partial c}{\partial S}.\Delta S + \frac{1}{2}\frac{\partial^2 c}{\partial S^2}.\left(\Delta S\right)^2 + \frac{\partial c}{\partial T}.\Delta T$$
$$+ \frac{\partial c}{\partial \sigma}.\Delta\sigma + \frac{\partial c}{\partial r}dr + \frac{\partial c}{\partial q}dq + \frac{1}{2}\frac{\partial^2 c}{\partial T^2}.\left(\Delta T\right)^2 + \frac{1}{2}\frac{\partial^2 c}{\partial \sigma^2}.\left(\Delta\sigma\right)^2 + \frac{\partial^2 c}{\partial T \partial S}.\Delta T \Delta S$$
$$+ \frac{\partial^2 c}{\partial T \partial \sigma}.\Delta T \Delta\sigma + \frac{\partial^2 c}{\partial S \partial \sigma}.\Delta S \Delta\sigma + \ldots$$

Of course, all these derivatives need to be valued at the point of reference i.e. $\left(S_0, T_0, \sigma_0, r_0, q_0\right)$

We define $Inertia = \frac{\partial^2 c}{\partial T^2}$, $Volga = \frac{\partial^2 c}{\partial \sigma^2}$, $Charm = \frac{\partial^2 c}{\partial T \partial S}$, $Veta = \frac{\partial^2 c}{\partial T \partial \sigma}$, $Vanna = \frac{\partial^2 c}{\partial S \partial \sigma}$, $Vera = \frac{\partial^2 c}{\partial \sigma \partial r}$, $Colour = \frac{\partial^3 c}{\partial S^2 \partial t}$, $Ultima = \frac{\partial^3 c}{\partial \sigma^2}$ and $Zomma = \frac{\partial^3 c}{\partial S^2 \partial \sigma}$ whence we can write

$$\begin{aligned}
&c\left(S_0 + \Delta S, T_0 + \Delta T, \sigma_0 + \Delta \sigma, r_0 + \Delta r, q_0 + \Delta q\right) = c_0 + \Delta . \Delta S + \frac{1}{2}\Gamma . \left(\Delta S\right)^2 + \\
&\Theta . \Delta t + \nu . \Delta \sigma + \rho . \Delta r + \varphi . \Delta q + \frac{1}{2} Inertia . \left(\Delta t\right)^2 + \frac{1}{2} Volga . \left(\Delta \sigma\right)^2 \\
&+ Charm . \Delta t \Delta S + Veta . \Delta t \Delta \sigma + Vanna . \Delta S \Delta \sigma + \ldots
\end{aligned}$$

with these greeks being valued at the point of reference i.e. $\left(S_0, T_0, \sigma_0, r_0, q_0\right)$. The complete set of greeks, presently in use are tabulated below:

$Delta\ \Delta = \frac{\partial c}{\partial S}$	$Gamma\ \Gamma = \frac{\partial^2 c}{\partial S^2}$	$Colour = \frac{\partial^3 c}{\partial S^2 \partial t}$
$Theta\ \Theta = \frac{\partial c}{\partial t} = -\frac{\partial c}{\partial T}$	$Inertia = \frac{\partial^2 c}{\partial t^2}$	$Ultima = \frac{\partial^3 c}{\partial \sigma^2}$
$Vega\ \nu = \frac{\partial c}{\partial \sigma}$	$Charm = \frac{\partial^2 c}{\partial t \partial S} = \frac{\partial \Delta}{\partial t} = \frac{\partial \Theta}{\partial S}$	$Zomma = \frac{\partial^3 c}{\partial S^2 \partial \sigma}$
$Rho\ \ \rho = \frac{\partial c}{\partial r}$	$Veta = \frac{\partial^2 c}{\partial t \partial \sigma} = \frac{\partial \nu}{\partial t} = \frac{\partial \Theta}{\partial \sigma}$	
$Phi\ \ \varphi = \frac{\partial c}{\partial q}$	$Volga = \frac{\partial^2 c}{\partial \sigma^2}$	
	$Vanna = \frac{\partial^2 c}{\partial S \partial \sigma}$	
	$Vera = \frac{\partial^2 c}{\partial \sigma \partial r}$	

Nevertheless, the fundamental ones are Delta (Δ), Gamma (Γ), Theta (Θ), Vega (ν), Rho (ρ) and Phi (φ). The others are more of cosmetic nature but nevertheless they contribute to enhancing accuracy.

Delta

The option delta is the rate of change of the value of the derivative position with respect to its underlying price. It is given by the partial derivative $\Delta = \frac{\partial c}{\partial S}$ of the value of a given derivative position with respect to the price of the underlying asset. Thus, it represents the slope of the curve between the price of the underlying asset and the value of the derivative position. For example, for a call option, delta would represent the slope of the call price curve with respect to the price of the underlying asset.

It has been shown earlier that delta of a BS call is equal to $\mathbf{N}(d_1)$. Now, $\mathbf{N}(d_1)$ is the cumulative standard normal distribution function. Hence, we must have, $0 \le \mathbf{N}\left(d_1\right) = \Delta \le 1$. Thus, $\Delta_{call} \in (0,1)$.

The BS put delta, on the other hand is **N**(d_1)-1 so that its value lies between -1 and 0 i.e. $\Delta_{put} \in (-1,0)$.

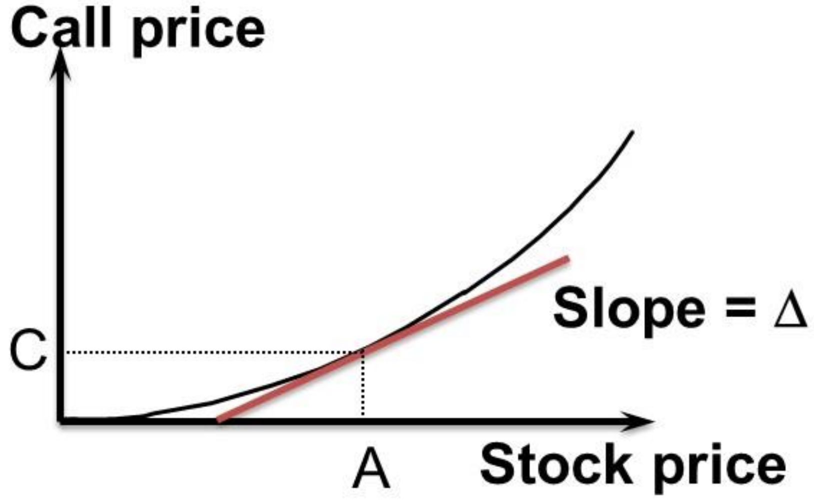
From put-call parity, it can be established that the sum of the absolute values of the delta of the call option and the delta of the put options is equal to 1. We have, from put-call parity,

$$c + Ke^{-rT} = p + S;\ \frac{\partial c}{\partial S} = \frac{\partial p}{\partial S} + 1$$

$$However,\ \frac{\partial c}{\partial S} \geq 0\ so\ that\ \frac{\partial c}{\partial S} = \left|\frac{\partial c}{\partial S}\right|\ and\ \frac{\partial p}{\partial S} \leq 0\ so\ that\ \frac{\partial p}{\partial S} = -\left|\frac{\partial p}{\partial S}\right| whence\ \left|\frac{\partial c}{\partial S}\right| + \left|\frac{\partial p}{\partial S}\right| = 1$$

$$\Delta = \frac{\partial c}{\partial S}$$

$$\Gamma = \frac{\partial^2 c}{\partial S^2}$$



The call value curve is shown in this diagram. Call value is not a linear function of the stock price. The slope of this curve at any given point gives us the value of Δ at that point.

Not only does the plot have a curvature, so that the slope (Δ) changes from point to point, but the curvature (Γ) changes from point to point along the curve.

In other words, Δ is different at different stock prices. It changes from point to point along this price curve. This change in the value of Δ is captured by the second derivative which is called the Γ.

Now, when the stock price registers an increase, the (i) probability of the call option finishing in the money on maturity also increases and (ii) the potential payoff from the ITM call which is proportional to stock price at maturity S_T is also likely to increase. It follows that the value of the call option also registers an increase. Thus, the value of delta is invariably positive.

The above is best elucidated by the following example:

A stock price follows a lognormal distribution with an expected rate of return μ of 14% and a volatility of 30% p.a. The stock pays dividends at a rate of 2% p.a. (with continuous compounding). The current price of the stock is INR 1,000. Calculate the probability that the stock price will exceed INR 1,250 at the end of six months from now.

Suppose, now that the stock price has spontaneously gone up to 1,100. Calculate the revised probability of the stock finishing at 1,250 at the end of six months. Assume the jump was spontaneous.

Solution

The stock price is distributed as follows:

$\ln S_T \xrightarrow{distribution} N\left[\ln S_0 + \left(\mu - q - \frac{1}{2}\sigma^2\right)T, \sigma^2 T\right]$. In the given problem S_0=1,000, μ=0.14, q=0.02, σ=0.30 and T=0.50 so that

$$\ln S_T \xrightarrow{distribution} N\left[\ln 1000 + \left(0.14 - 0.02 - \frac{1}{2}\times 0.30^2\right)\times 0.50, 0.30^2 \times 0.50\right]$$
$$= N\left[6.9078 + 0.0375, 0.045\right] = N\left[6.9453, 0.045\right]$$

We need to find P(S_T>1,250)=P(lnS_T>7.13)=P(Z>(7.13-6.9453)/0.21)=P(Z>0.88) =0.1894

In the second case, the ln of the stock will be distributed as:

$$\ln S_T \xrightarrow{distribution} N\left[\ln 1100 + \left(0.14 - 0.02 - \frac{1}{2}\times 0.30^2\right)\times 0.50, 0.30^2 \times 0.50\right]$$
$$= N\left[7.003 + 0.0375, 0.045\right] = N\left[7.0406, 0.045\right]$$

We need to find P(S_T>1,250)=P(lnS_T>7.13)=P(Z>(7.13-7.0406)/0.21)=P(Z>0.426) =0.3350

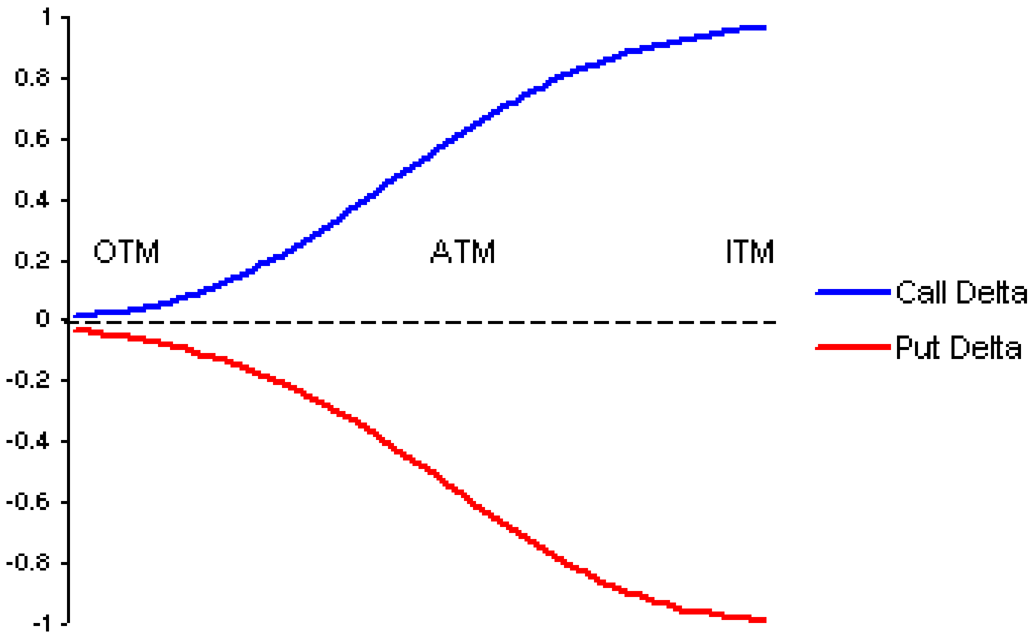
The positivity of Δ also follows from its Black Scholes value of **N**(d_1). Since **N**(z) is the cumulative standard normal distribution, it represents the probability the standard normal variate Z can take values between -∞ and z i.e. **N**(z)=P(Z<z) where Z is N(0,1). Obviously, it must be non-negative.

In fact, as the value of d_1 increases, **N**(d_1)=Δ must also increase since $\mathbf{N}(d_1) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d_1} e^{-\frac{1}{2}\omega^2} d\omega$. Now, from the expression for d_1 viz.

$$d_1 = \frac{\ln\frac{S^{-qT}}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln\frac{S}{K} + \left(r - q + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

it is obvious that, other things remaining unchanged, d_1 is a monotonically increasing function of ln S and hence, of S. Thus, Δ of a call is not only essentially positive, but it also increases with increase in stock price so that $\Gamma = \frac{\partial \Delta}{\partial S} > 0$.

Delta & stock price



(i) As a general rule, in-the-money options will move more than out-of-the-money options.

(ii) When the call option is deep out-the-money, it has a delta of 0. The call will not move much at all due to movements in the underlying asset. This is because the call is very likely not to be exercised and the payoff will not be realized. Hence, its rate of change of price is independent of the rate of change of price of stock.

(iii) When the call option is deep in-the-money, it has a delta of 1. The call will move point for point in the same direction as movements in the underlying asset. This is because the call is very likely to be exercised and the payoff on the call is S_T-K. Hence, its rate of change of price mimics the rate of change of price of stock.

Consider a call that is deep out of the money e.g. one written at 50 with the current stock price hovering around, say 10. Suppose, now, that the stock price rises to 11. The question is whether this change will significantly affect the **expected** payoff from the call. For this purpose, we need to ascertain (i) whether this change significantly affects the probability of exercise of the call and (ii) whether it significantly influences the amount of payoff.

Now, in the given situation, it is very probable that the option will not be exercised on maturity since it is likely to finish OTM. Even if there is a small increase in the stock price, the probability of non-exercise is not likely to improve significantly, given the massive gap between the exercise price and the current stock price. It is still probable that the option will finish OTM. Because this perception of non-exercise is not going to change significantly, even the increase in the potential payoff, if any, is likely to be completely eclipsed, the value of the option is not likely to change much. As a result of which delta of OTM options is pretty close to zero.

What happens at the other end of the spectrum i.e. when the stock is very high? Option exercise at maturity is, now almost a certainty and a payoff of S_T-K is going to result. If the stock price increases/ decreases by a small amount, the probability of exercise is not likely to be affected, the option will, still most likely finish ITM and be exercised. The payoff will result.

However, the value of the payoff will change precisely by the change in the value of the stock price at maturity. The current value of the call will, therefore, change by the present value the change in maturity stock price. But this will be precisely the current change in stock price. Thus, the call price will change precisely by the change in current stock price i.e. $\Delta=1$. Let us take an example, Suppose the current stock price increases by 1 unit. Then, the expected increase in stock price at call maturity i.e. $t=T$ will be e^{rT}. Thus, the expected payoff from the call will increase by the same amount e^{rT} since the payoff from call is S_T-K (as the call is certain to be exercised). Thus, the increase in call's current value which is the present value of this expected increase in call payoff is $e^{-rT}\ e^{rT} =1=$ increase in stock price. Thus, in this situation $\Delta=1$.

As the stock price moves close to the exercise price, then any small change in stock price significantly affects the probability of the option ending ITM/OTM. Therefore, the realizability of a payoff at maturity as also the amount of payoff are both affected although uncertain. As such, in such situations Δ hovers around 0.50.

Delta of put options

(i) When the underlying price rises, the price of the option will decrease by Δ amount.

(ii) Put Δ will increase (move from a negative to zero) as the option moves further out-of-the-money.

(iii) When the put option is deep in-the-money and has a Δ of -1, then the put will move point for point in the opposite direction as movements in the underlying asset.

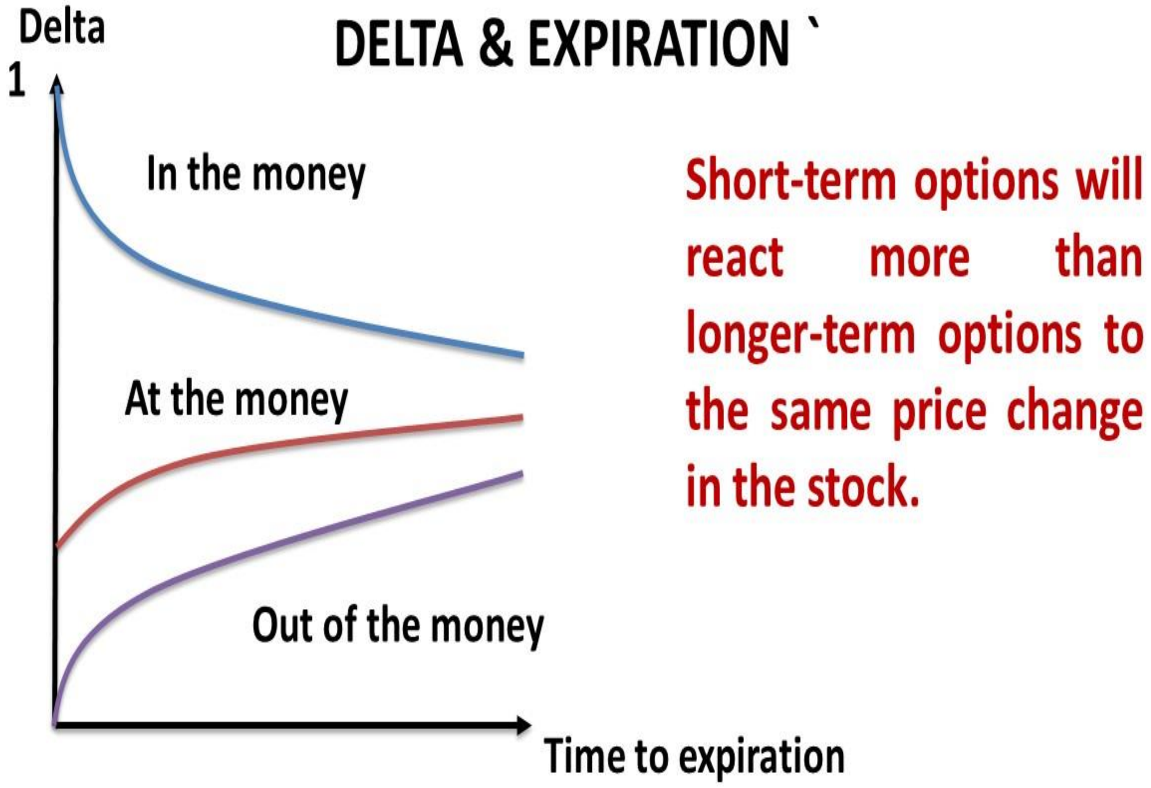
In the case of the put options, when the price of the underlying rises the put option tends to go out of the money and hence become cheaper. As the price of stock increases the value of the put option decreases since the payoff from a put at maturity is $\max(K-S_T,0)$. The payoff is inversely related to S_T. Hence, $\Delta<0$. It ranges in $\Delta\in(-1,0)$. Put Δ will increase from -1 to 0 as the option moves from ITM to OTM.

Δ of OTM puts is close to 0 as they are unlikely to be exercised while Δ of ITM puts is close to -1 since the payoff varies as the negative of the maturity stock price S_T.

Thus, as the put option moves from OTM to ITM, the Δ changes from 0 to -1 and vice versa.

In the BS framework $\Delta_{put} = \mathbf{N}(d_1)-1$.

Delta & Expiration



It may be noted that time to expiration increases along the X-axis. Thus, as the option approaches maturity, we need to move towards the left i.e. towards the origin in interpreting the above diagram. In actual fact, time to expiration decreases, so we have to look at this graph backwards along the X-axis while reading the timeline.

The Δ of ITM call approaches 1 as the option approaches maturity. Payoff of the an ITM call is s S_T-K. As we approach the call maturity date, there remains less time available for the stock price to fluctuate. Thus, the closer one is to maturity, the greater is the chance that the stock will sustain its current value S at call maturity and hence, greater is the chance of realizability of the payoff close to S-K. Thus, closer will be Δ to unity.

Clearly, the probability of the stock touching a particular value at maturity from a certain current value decreases as maturity approaches. In fact, the variance of ln S_T (in this model) is directly proportional to the term to maturity T, so that as maturity approaches, the variance and hence the amplitudes of fluctuations decline. Simply stated, there is greater certainty of the stock price finishing in any given interval.

Recall an earlier example wherein we were given that a stock price follows a lognormal distribution with an expected rate of return μ of 14% and a volatility of 30% p.a. The stock pays dividends at a rate of 2% p.a. (with continuous compounding). The current price of the stock is INR 1,000. We were required to calculate the probability that the stock price will exceed INR 1,250 at the end of six months from now. We found that

The stock price is distributed as follows:

$\ln S_T \xrightarrow{distribution} N\left[\ln S_0+\left(\mu-q-\frac{1}{2}\sigma^2\right)T,\sigma^2 T\right]$. In the given problem S_0=1,000, μ=0.14, q=0.02, σ=0.30 and T=0.25 so that

$$\ln S_T \xrightarrow{distribution} N\left[\ln 1000+\left(0.14-0.02-\frac{1}{2}\times 0.30^2\right)\times 0.50, 0.30^2\times 0.50\right]$$
$$=N\left[6.9078+0.0375, 0.045\right]=N\left[6.9453, 0.045\right]$$

We need to find P($S_{0.50}$>1,250)=P(ln$S_{0.50}$>7.13)=P(Z>(7.13-6.9453)/0.21)=P(Z>0.88) =0.1894

Now, suppose we are required to find the probability of the stock price touching 1,250 at the end of 3 months from now. We have

$$\ln S_T \xrightarrow{distribution} N\left[\ln 1000+\left(0.14-0.02-\frac{1}{2}\times 0.30^2\right)\times 0.25, 0.30^2\times 0.25\right]$$
$$=N\left[6.9078+0.01875, 0.0225\right]=N\left[6.9266, 0.0225\right]$$

We need P($S_{0.25}$>1,250)=P(ln$S_{0.25}$>7.13)=P(Z>(7.13-6.9266)/0.15)=P(Z>1.356) =0.0875 showing that as maturity becomes closer, the probability of hitting a target price given a current price diminishes. In other words, the certainty around a particular current price increases.

For at the money calls, the call value fluctuates significantly with changes in stock prices as the options get closer to maturity. For ATM calls, we have $\ln\left(\frac{S}{K}\right)=\ln(1)=0$ so that $d_1=\frac{\left(r-q+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}=\frac{\left(r-q+\frac{1}{2}\sigma^2\right)}{\sigma}\sqrt{T}\propto\sqrt{T}$. Thus, as maturity approaches T and hence, d_1 and so Δ=**N**(d_1) also decreases.

Similarly, for OTM options, as maturity approaches, there is lesser possibility of any current favourable price movements carrying a positive impact on the option moneyness at maturity, the Δ becomes more and more rigid towards 0.

In all the types of options, ITM, ATM & OTM, the slope of the Δ curve increases in magnitude as the call approaches maturity, although the slope is positive for ITM options and negative for ATM & OTM options. It means that the price of the call will change more in a given time interval of same magnitude when this interval is closer to maturity.

<u>Curvature of the call price curve & delta, Delta hedging</u>

(i) Delta indicates the number of shares of stock required to mimic the price behavior of the option. Thus, delta is also called the Hedge Ratio.

(ii) Delta neutrality means the combined deltas of the options involved in a strategy net out to zero.

(iii) Delta neutral portfolios are insensitive to price changes of the underlying asset within an INFINITESIMAL region around the point at which Delta neutrality is attained.

(iv) However, since every change in the underlying price changes the Delta, it is necessary to continuously rebalance the portfolio to achieve sustained Delta neutrality after every stock move.

(v) Nevertheless, a gamma (derivative of delta) near zero means that the option position is robust to changes in underlying prices and immune to price change over a significant range of values and hence, portfolio rebalancing need not be so frequent.

Δ is the change in value of a call option due to a small change in the price of the underlying asset. Therefore, if one constructs a portfolio of one call and -Δ units of the stock, the price change in the call will be neutralized by the price change in the stock position and one gets a risk-neutral portfolio. Let us call it Δ risk-free portfolio.

However, since the curve c=f(S) is nonlinear, it has a curvature. Because of this curvature, the slope i.e. the Δ changes from point to point along the curve. Therefore, with every price change the Δ also changes. Hence, if a portfolio is delta neutral at a particular point on the curve, as soon as the stock price makes a shift, the delta value required for neutrality becomes different. Therefore, the portfolio loses its delta neutrality. Thus, although we can construct a Δ risk-free portfolio by using one unit of the derivative and -Δ units of the stock, it would remain Δ-neutral over a very small (infinitesimal) range of stock prices, every time the stock price registers a change, the portfolio will lose its delta neutrality (as its delta will change) and will have to be rebalanced.

Thus, theoretically, it would be necessary to rebalance the Δ-neutral portfolio after every stock price move to sustain the neutrality. This is an impracticable exercise. The bottomline is that if one has a Δ-neutral portfolio or a Δ risk-free portfolio, the Δ risk-free portfolio remains risk-free only for infinitesimal changes in the stock price and not for large changes in stock price. Large changes in stock price will not be neutralized inter se, between the price of the derivative and the price of the stock. That is very important, that is where all the problems creep in.

Thus, to sustain Δ-neutrality over sustained regions of stock price movements, it is necessary to rebalance the portfolio frequently. How frequently would depend on the rate at which Δ changes with respect to the stock price i.e. on the value of Γ which is simply the rate of change of delta with respect to the stock price.

So, if Γ is small, i.e. Δ is changing very slowly due to changes in stock price, the required frequency of rebalancing would be much less. However, if Γ is large, then frequent rebalancing to counter the rapid changes in Δ with changes in stock price would be required.

Delta positive strategies

(i) Long Call
(ii) Short Put
(iii) Bullish Call Spread
(iv) Bullish Put Spread
(v) Covered Call Write

Delta negative strategies

(i) Long Put

(ii) Short Call
(iii) Bearish Put Spread
(iv) Bearish Call Spread
(v) Covered Put Write

Delta neutral strategies

(i) Iron Condor
(ii) Butterfly
(iii) Short Straddle
(iv) Short Strangle
(v) Long Straddle
(vi) Long Strangle
(vii) Long Calendar Spread

Iron condor

An iron condor consists of the following:

(i) Long OTM put (X) at K_1
(ii) Short OTM put (Y) at K_2
(iii) Short OTM call (A) K_3
(iv) Long OTM call (B) K_4

$K_1 < K_2 < K_3 < K_4$

Initial investment

Let us first look at the initial investment.

(i) Let us, first look at the put spread. We have long OTM put X at K_1 and short OTM put Y at K_2 with $K_1 < K_2$. Now, for put options, the premium varies directly with the exercise price i.e. higher the exercise price, higher the premium since put is a right to sell the asset at the exercise price. Since, the long put (purchased) is at a lower exercise price and the short put (sold) it at a higher exercise price, there will be a net cash inflow from this combination of put options when the spread is created.

(ii) Now, the call spread. We have short OTM call A at K_3 and long OTM call B at K_4 with $K_3 < K_4$. For call options, the premium varies inversely with the exercise price i.e. higher the exercise price, lower the premium since call is a right to buy the asset at the exercise price. Since, the long call (purchased) is at a higher exercise price and the short call (sold) it at a lower exercise price, there will be a net cash inflow from this combination of call options at the creation of the strategy.

(iii) Thus, both the put spread and the call spread result in a cash inflow at inception so that the iron condor strategy will generate a cash inflow at inception.

Payoff of iron condor

Let us now look at the payoff at maturity of the condor. As usual, we divide the stock price spectrum as $0<S_T<K_1$; $K_1<S_T<K_2$; $K_2<S_T<K_3$; $K_3<S_T<K_4$; $K_4<S_T$. Then,

S_T	$(0, K_1)$	(K_1, K_2)	(K_2, K_3)	(K_3, K_4)	$(>K_4)$
$X(\text{Long Put})$	$K_1 - S_T$				
$Y(\text{Short Put})$	$S_T - K_2$	$S_T - K_2$			
$A(\text{Short Call})$				$K_3 - S_T$	$K_3 - S_T$
$B(\text{Long Call})$					$S_T - K_4$
$TOTAL$	$K_1 - K_2 < 0;$	$S_T - K_2 < 0;$	0	$K_3 - S_T < 0;$	$K_3 - K_4 < 0$

(i) Now, long put option X has a strike of K_1, so it will be exercised if $0<S_T<K_1$ and will generate a payoff of K_1-S_T.

(ii) Short put Y has strike K_2 and will be exercised if $S_T<K_2$. It will generate a payoff of S_T-K_2.

(iii) Short call A has strike K_3 and will be exercised if $S_T>K_3$ generating a payoff of K_3-S_T.

(iv) Long call B has strike K_4 and will be exercised if $S_T>K_4$ generating a payoff of S_T-K_4.

If we aggregate all these payoffs, we find that in every scenario except when $K_2<S_T<K_3$, the payoff is negative. And even when $K_2<S_T<K_3$, the payoff is zero. Thus, in the best case scenario the payoff is zero, otherwise it is always negative. However, we had a positive cash flow at the inception of this strategy.

Thus, if the stock price ends up as $K_2<S_T<K_3$ at options maturity, the investor makes a zero loss at maturity but retains the positive cashflow of $-P_X+P_Y+P_A-P_B$ i.e. the net premium from the strategy which is equal to the difference between the premia of the two calls and the premia of the two puts that constituted the strategy at the time of inception.

So, the iron condor at maturity does not give a positive payoff but it does give a positive inflow at the time of creation of the strategy.