Financial Derivatives and Risk Management
Professor J.P.Singh
Department of Management Studies
Indian Institute of Technology Roorkee
Lecture 04
Forwards Pricing: Consumption Assets

Forward pricing of consumption assets

As mentioned in an earlier lecture, assets are classified as (i) investment assets and (ii) consumption assets in the context of forward pricing. Investment assets are assets held by significant numbers of people purely for investment purposes (e.g. gold, silver) whereas consumption assets are assets held primarily for consumption (e.g. copper, oil).

Now, the no-arbitrage forward pricing of investment assets was based on bidirectional arbitrage i.e. both cash & carry arbitrage and reverse cash & carry arbitrage were enabled and thereby an equilibrium price was arrived at. Precisely,

(i) If $F_0>S_0exp(rT)$ prevailed in the market at a particular instant, the arbitrageurs borrowed funds, acquired the asset spot and took short forward positions. On maturity, they delivered the asset, received the forward proceeds and repaid the loan with interest and ended up with some arbitrage profit (cash & carry arbitrage). This enhanced spot market demand for the asset while suppressing it in the forward markets, thereby causing the two prices to converge.

(ii) On the other hand, if $F_0<S_0exp(rT)$ prevailed in the market at a particular instant, the arbitrageurs borrowed the asset, sold it spot invested the proceeds and took long forward positions. On maturity, they liquidated their investment, received the asset, tendered the forward proceeds under the long forward and replenished the asset to the owner and ended up with some arbitrage profit (reverse cash & carry arbitrage). This enhanced forward market demand for the asset while suppressing it in the spot markets, thereby causing the two prices to converge.

It is, apparent from the above that in the case of reverse arbitrage, that during the life of the forward contract i.e. from t=0 to t=T, the arbitrageur is deprived of the physical possession of the asset.

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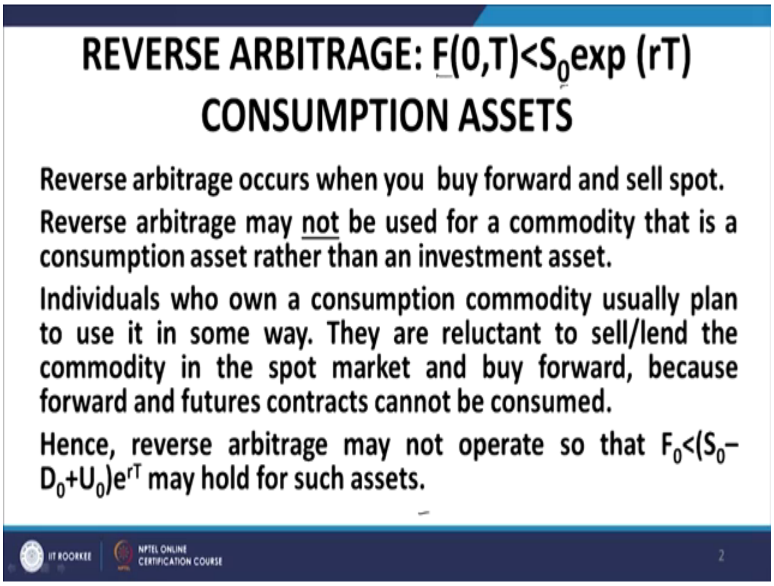

Now, in the case of investment assets, this does not, really, matter too much to the investor since he has acquired the asset merely for the purpose of earning returns. He is concerned with the returns from the investment, and does not get unduly perturbed if he temporarily loses physical possession of the asset in the process of earning returns.

However, when an entity is holding an asset for consumption, the situation is different. When one is holding an asset for consumption, the objective of acquiring the asset is to use it, to consume it, for some purpose e.g. in one's business. That being the case, the physical separation of the entity from a consumption asset becomes more significant than in the case of investment assets. So what is the implication of this in the context of forward pricing?

As a result of this reluctance of an entity to part with physical possession of a consumption asset, the process of reverse arbitrage (that involves selling the asset spot and buying it forward and hence, physical separation of the asset from the arbitrageur during the life of the forward contract) which can operate without any inhibition, without any impediment, in the case of investment assets may not so operate in the case of consumption assets. Market partcipants may not be willing to part with the physical possession of such assets because they may want to use it forthwith. Parting with these assets may adversely affect their regular business operations.

Thus, this reverse arbitrage phenomenon may be constrained for consumption assets. Now, reverse arbitrage occurs when $F_0<S_0\exp(rT)$. And this reverse arbitrage operates to converge the two sides of this inequality. Hence, if reverse arbitrage is impeded, the two sides will not converge. In other words, in the absence of reverse arbitrage the market will sustain the

differential $F_0<S_0\exp(rT)$. Thus, for consumption assets, the there may exists markets wherein the forward prices of such assets are lower than the no-arbitrage forward prices. This is logical as well, because people have a propensity to buy spot and hold on to such assets, rather than selling them and buying them forward. The spot demand for consumption assets is, therefore, enhanced whereas forward demand diminished causing a relative shift of prices in favour of spot prices.

So, that is the difference between the pricing of forward contracts on investment assets, and forward contracts and consumption asset. And that is what gives rise to the concept of convenience yield which we take up now.

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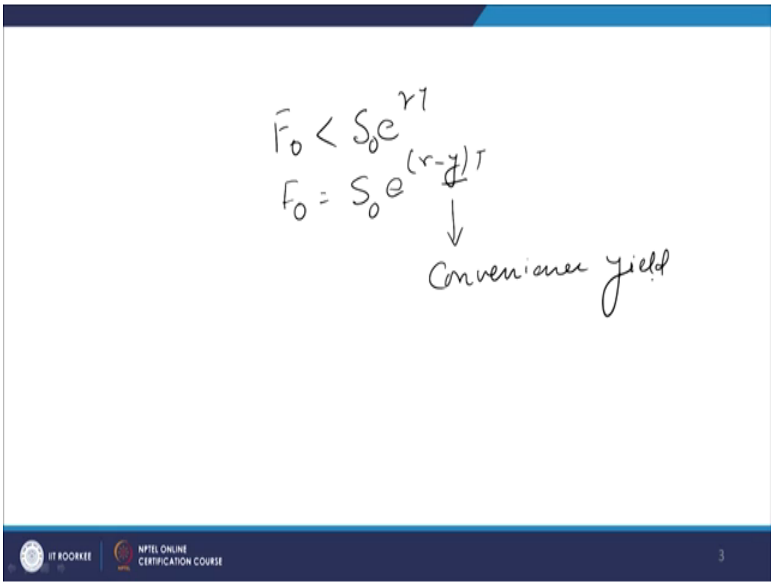

Convenience yield

As I mentioned earlier, there can be market situations that sustain $F_0<S_0\exp(rT)$ in the case of consumption assets due to impeded reverse arbitrage. The right hand side is larger than the left hand side. So, I can rescale the right hand side by putting the factor <1 such that it makes it equal to the left hand side. This means that I can introduce a positive y such that $F_0=S_0\exp[(r-y)T]$ because $F_0<S_0\exp(rT)$ and $\exp(-yT)<1$ and, therefore, y can be so chosen that $F_0=S_0\exp(rT)\exp(-yT)= S_0\exp[(r-y)T]$.

This y is called the convenience yield. The significance of this "convenience yield" arises from the inequality $F_0<S_0\exp(rT)$. In other words, the forward market price is less than what it should have been, if market participants were willing to part with consumption assets just like

investment assets. But because participants are not willing to physically part with consumption assets, this price differential may subsist and hence, this factor ‘y’ or ‘convenience yield’ comes into play.

Interpretation of convenience yield

To understand the interpretation of y, we note that F_0 is the actual forward price of the consumption asset (whatever it may be) in the market. We also abbreviate $F_{na}=S_0\exp(rT)$ as the theorectical no arbitrage forward price. Then, we have:

$$\exp\left(yT\right)=\frac{F_{na}}{F_0} \quad or \quad y=\frac{1}{T}\ln\frac{F_{na}}{F_0}$$

It is clear that y is a measure of the extent to which the actual forward price is lower than the no-arbitrage forward price. In other words, it is indicative of the extent to which reverse arbitrage process is being inoperative or impeded in the given market which is resulting in the actual forward price lying below the no-arbitrage forward price, because this differential is arising only due to the failure of reverse arbitrage in neutralizing the price differential.

But we know that the reverse arbitrage is being inhibited because of the reluctance of the investors to part with and to retain physical possession of consumption assets. Thus, convenience yield is a measure of the extent to which market participants are motivated to retain physical possession of the assets i.e. the extent of yield they are willing to forego, to relinquish, to give up, in their desire to retain physical possession of the assets.

But why are people not willing to part with these assets? It may be because they feel more secure by retaining physical possession of their assets. Thus, y is indicative of preference of the consumers to own and retain the asset and consume it rather than have a forward contract on it.

The convenience yield reflects the market's expectations concerning the availability of the commodity in the near future. The greater the possibility that shortages will occur, the higher the convenience yield. Again, if users of the commodity have high inventories and there is very little chance of shortages in the near future, then the convenience yield tends to be low. If inventories are low, shortages are more likely and the convenience yield is usually higher. So, the basic thing is the convenience yield gives a measure of why the investor wants to retain physical possession of the asset. The preference, the desire, the temptation to retain physical possession of the asset, rather than part with it and derive arbitrage profit therefrom determines the convenience yield.

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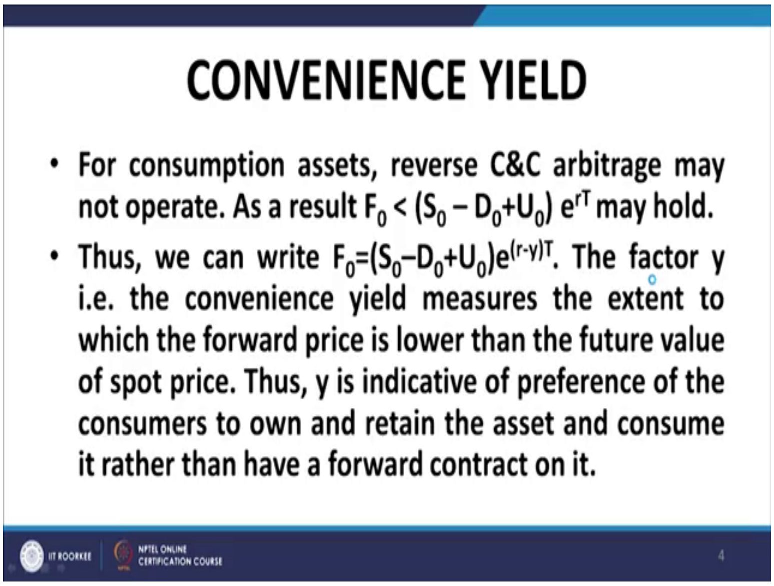

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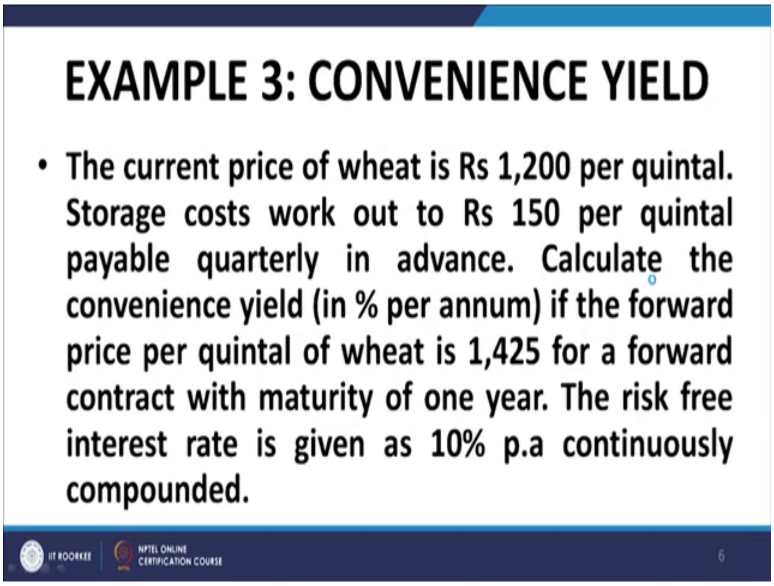

$$Current\ Price\ of\ Wheat\left(S_0\right)=1200;\ Risk-free\ Rate\left(r\right)=0.10$$
$$Storage\ \mathbf{Costs}:150\ p.a.\ payable\ quarterly\ in\ advance$$
$$Maturity\ \left(T\right)=1.00\ year.$$
$$Storage\ \mathbf{cos}ts\ are\ payable\ quarterly\ in\ advance.$$
$$Hence,\ there\ will\ be\ 4\ payments\ of\ 150/4=37.50.$$
$$The\ storage\ payments\ will\ be\ at\ t=0,\ beginning\ of\ 4\ months,$$
$$beginning\ of\ 7\ months\ and\ beginning\ of\ 10\ month\ i.e.$$
$$at\ t=0,\ end\ of\ 3\ months,\ 6\ months\ and\ 9\ months.$$
$$i.e.\ at\ t=0,\ t=0.25,\ t=0.50\ and\ t=0.75\ year$$
$$P\mathbf{r}esent\ Value\ of\ Storage\ \mathbf{Cos}ts\left(U_0\right):$$
$$U_0=37.5+37.5e^{-0.10\times0.25}+37.5e^{-0.10\times0.50}+37.5e^{-0.10\times0.75}=144.54$$
$$Forward\ Price\ in\ absence\ of\ Convenience\ Yield:$$
$$F_0=\left(S_0+U_0\right)\mathbf{exp}\left(rT\right):\left(1200+144.54\right)e^{0.10\times1.00}=1485.95$$
$$Actual\ forward\ price:1425.$$
$$If\ F_{act}\ is\ the\ actual\ forward\ price\ then$$
$$F_{act}=\left(S_0+U_0\right)\mathbf{exp}\left[\left(r-y\right)T\right]$$
$$Theoretical\ forward\ price=F_0=\left(S_0+U_0\right)\mathbf{exp}\left(rT\right)$$
$$Hence,\ F_{act}=F_0\,\mathbf{exp}\left(-yT\right)\ or\ y=\frac{1}{T}\mathbf{ln}\left(\frac{F_0}{F_{act}}\right)$$
$$Hence,\ Convenience Yield:\mathbf{ln}\left(\frac{1485.95}{1425}\right)=4.19\%$$

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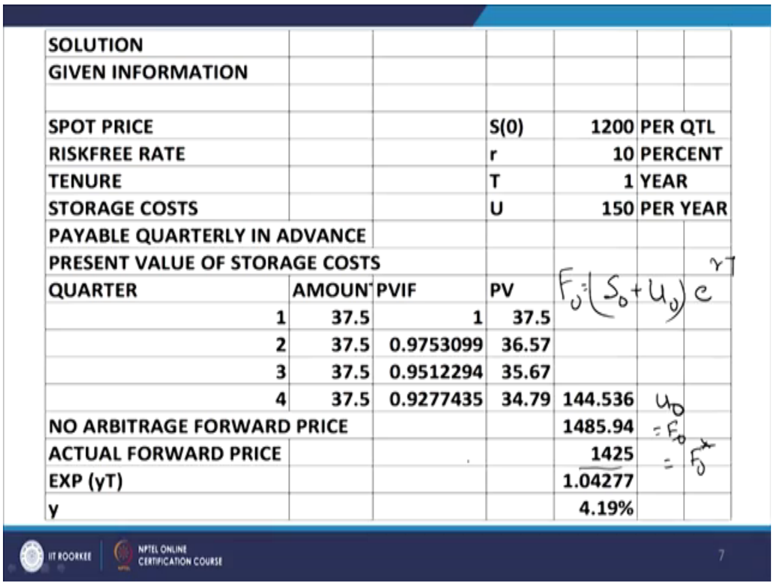

SOLUTION					
GIVEN INFORMATION					
SPOT PRICE			S(0)	1200	PER QTL
RISKFREE RATE			r	10	PERCENT
TENURE			T	1	YEAR
STORAGE COSTS			U	150	PER YEAR
PAYABLE QUARTERLY IN ADVANCE					
PRESENT VALUE OF STORAGE COSTS					
QUARTER	AMOUN'	PVIF	PV		
1	37.5	1	37.5		
2	37.5	0.9753099	36.57		
3	37.5	0.9512294	35.67		
4	37.5	0.9277435	34.79	144.536	
NO ARBITRAGE FORWARD PRICE				1485.94	
ACTUAL FORWARD PRICE				1425	
EXP (yT)				1.04277	
y				4.19%	

So this factor y of 4.19 percent is indicative of the desire, the temptation, the intention, the motivation of the of the market player to retain physical position of the asset. Because we want to keep the asset, we want to retain the physical possession of the asset, the forward price goes down.

Thus, the need for segregation in terms of investment assets and in terms of consumption assets for forward pricing arises from the adaptability of such assets to reverse arbitrage..

Valuing a forward contract

As we discussed earlier, the value of a forward contract at the point of inception is invariably zero, because the contract is entered into at the forward price at that time. The forward price at a given instant is defined as the very price that makes the value of the forward contract initiated at that instant equal to zero. Thus, if F_0 is the forward price at t=0, then a forward contract initiated at t=0 at that price i.e. F_0 will have zero value. Obviously, F_0 is based on the perception of the contracting parties (and, indeed, the market as a whole) as what they expect the price of the asset to be on the date of maturity of the forward i.e. at t=T (say S_T).

But, as time passes, perception of the market about the expected price of the asset on the date of maturity of the forward contract i.e.S_T changes. As a result of this, the forward price also changes. Now, because the forward price changes, the forward contract originally entered into, acquires a value. We want to determine the value of that forward contract.

Let us assume that an investor X takes a long position in a forward contract at t=0 with maturity at t=T at a forward price of F_0. This means that at t=T, X will receive one unit of the underlying and pay a price of F_0. We call this contract A.

Let us, now, assume that some time later on, say at t=t*, X decides to cancel his contract. We need to determine the implications of this cancellation to the contracting parties. Let at t=t*, F_{t^*} be the forward price of the underlying corresponding to a maturity on the same day as contract A i.e. T-t* days from now. Thus, if a forward contract (say contract B) is entered into at t=t* at price F_{t^*} with maturity T-t*,

(i) Contract B will have zero value at t=t* since it is instituted at t=t* at the forward price at t=t* i.e. $V_B(t^*)=0$.

(ii) Contract B matures on the same date as A i.e. at t=T. hence, it will involve transfer of one unit of underlying and payment of price F_{t^*}, both on the same date i.e. t=T.

Let us now assume that X constitutes a portfolio consisting of

(i) Long position in A;

(ii) Short position in B.

Then, the value of his portfolio at t=t* is $V_A(t^*)-V_B(t^*)= V_A(t^*)$ since $V_B(t^*)=0$.

Let us look at the payoff of his portfolio at maturity of the forwards i.e.at t=T.

(i) He receives one unit of underlying under the long forward A. But he delivers the same under his short forward position in B. Hence, the portfolio does not acquire any unit of the underlying asset.

(ii) He receives cash F_{t*} under the short forward B out of which he pays out F_0 against his long forward A. Hence, his net cash inflow is $F_{t*}-F_0$.

(iii) On the premise that both the forwards are default free, both the above cash-flows are certain.

Thus, the value of X's portfolio at t=t* must be the present value of this certain cash inflow Further, since this inflow is certain, it will be discounted at the risk-free rate and we have:

$V_A(t^*)=(F_{t*}-F_0)\exp[-r(T-t^*)]$

Now, if the no-arbitrage conditions hold, then, $F_{t*}=S_{t*} \exp[-r(T-t^*)]$ and F0=S)exp(rT) so that

$V_A(t^*)=S_{t*}-S_0\exp(-rt^*)$

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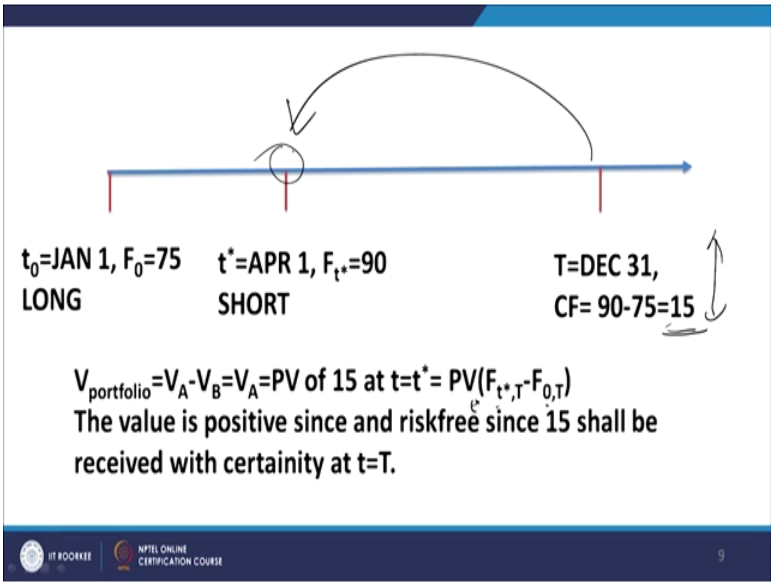

Let us see it on a timeline. We start at, say, 1st January which is t=0. Let the underlying asset be USD and the forward price of one USD at 1st January for delivery on 31st December be INR 75 i.e. F_0=75. Thus, if I want to buy USD, fixing the price today for delivery & payment at 31st December, I will get it for INR 75. In other words, I shall pay INR 75 on 31st December and get 1 USD on the same day and the agreement is that there will be no change in the price whatever happens in between. Let us call this forward contract A.

Now, say, today is 1st April, and I need to calculate the value of contract A. I find that, as of today (1st April) the forward price of 1 USD for delivery on 31st December (Contract B) is INR 90. Note here that delivery of this new forward (B) is not one year, delivery is on 31st December i.e. the same date as that of contract A. The period has come down from one year to 9 months. The date of delivery remains 31st December for both contracts.

Thus, the forward contract B (instituted on 1st April) envisages delivery of 1 USD against a payment of INR 90 on 31st December. In other words, on 1st January, if I wanted USD on 31st December, I got them for INR 75 each. On 1st April, if I want a dollar delivered on 31st December, the predetermined price or the price that is fixed as on April 1 is, INR 90 per dollar.

The question is: What is the value of contract A of 1st January as on 1st April?

How do the two contracts differ? The two contracts differ in terms of the cash flow that is going to occur on the date of maturity that is 31st December. Suppose, I had a long position in A (1st January contract), that is, I will receive 1 USD on 31st December for INR 75. Now, I take a short position in B (1 April contract), that is, I shall deliver 1 USD on 31st December and receive INR 90. Thus, I receive 1 USD under contract A and I deliver the 1 USD, so received, in the contract B. So the USD side is balanced.

What happens to the INR side? As far as INR is concerned, I receive INR 90 against B and pay INR 75 against A. I receive the USD and pay INR 75 against A, and I deliver the USD and receive INR 90 against B. In other words, I make a profit of INR 15. But this profit arises on 31st December. However, if we assume forwards to be default-free, then this profit is certain. So, the value of the strategy of going long in A and short in B will be the present value of this INR 15, discounted at the risk-free rate (because it is a certain cash-flow) back to April 1 (because 1st April is the date at which we want to ascertain the value of the.1st January contract i.e. contract A).

Further, this INR 15 is the difference in the forward prices at 1st April and 1st January.

So, $V_A - V_B$ ('-' sign because B is short) $= PV(F_B - F_A)$.

But V_B is the value of a forward contract on the date of its inception at the forward price on that date. It must be zero. It may be noted that we are doing the valuation on the very date of inception of contract B. Hence, $V_A = PV (F_B - F_A)$

Thus, value of the contract A which was entered into on 1st January is equal to the present value of the difference in the forward prices, the present value being worked out at the date of valuation using the riskfree rate.

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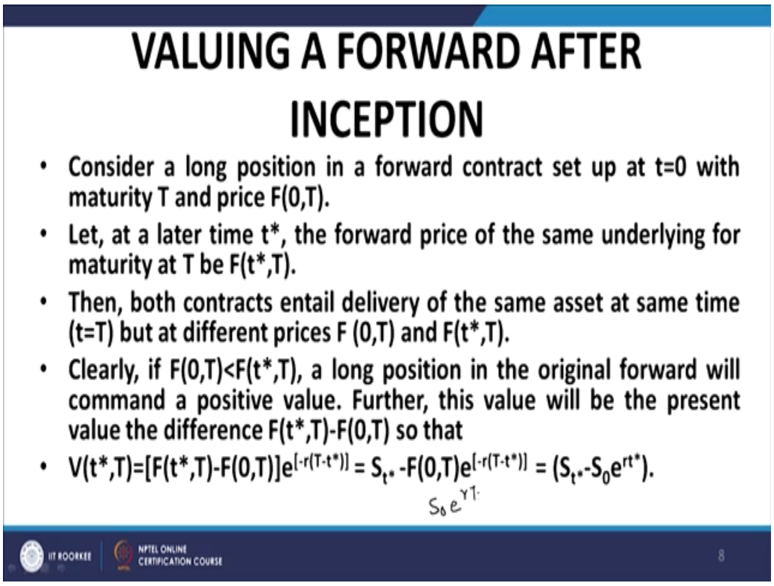

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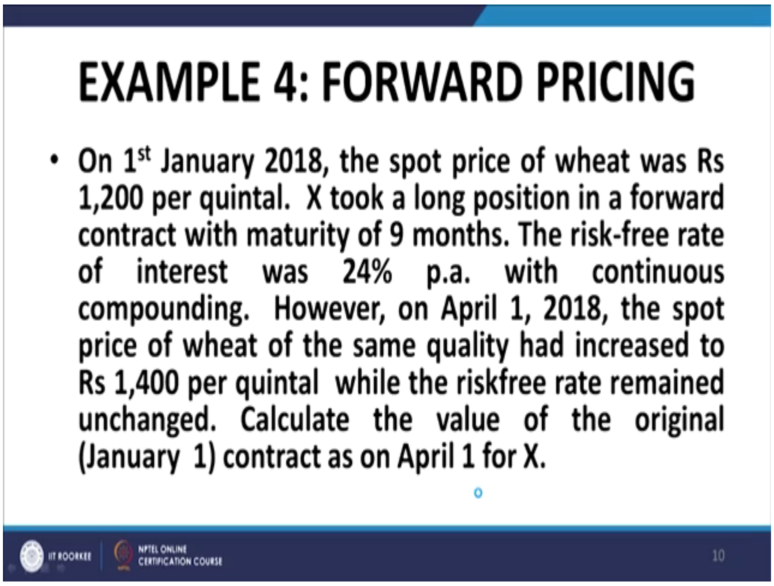

The example is solved below:

$$Spot\ price\ on\ Jan\ 1,\ 2018\ S_0 = 1200.00$$
$$Risk-free\ Rate(r) = 0.24$$
$$Maturity\ of\ forward\ (T) = 0.75\ year.$$

$$Now,\ on\ April\ 1,\ 2018:\ Spot\ price\ S_1 = 1400.00$$

$$Value\ of\ Long\ forward\ of\ Jan\ 1\ on\ Apr\ 1 = \left(S_1 - S_0 e^{0.24\times 3/12}\right)$$
$$= 1400 - 1274 = 125.80$$

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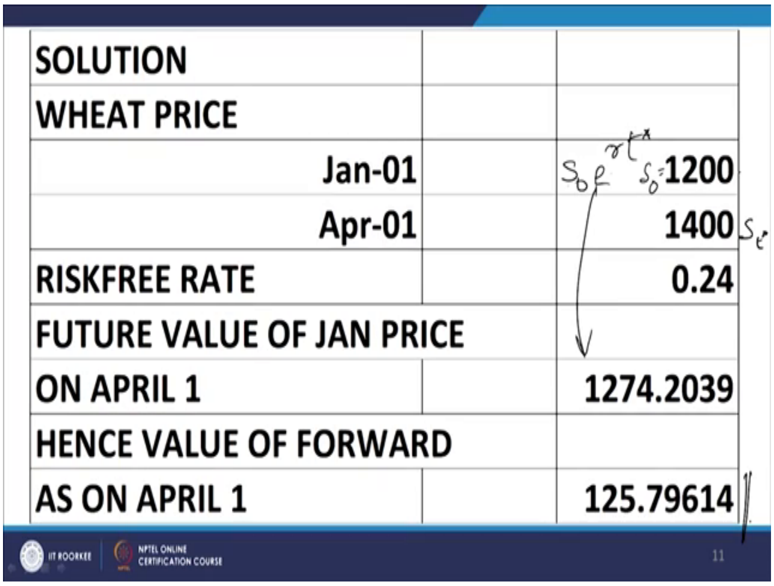

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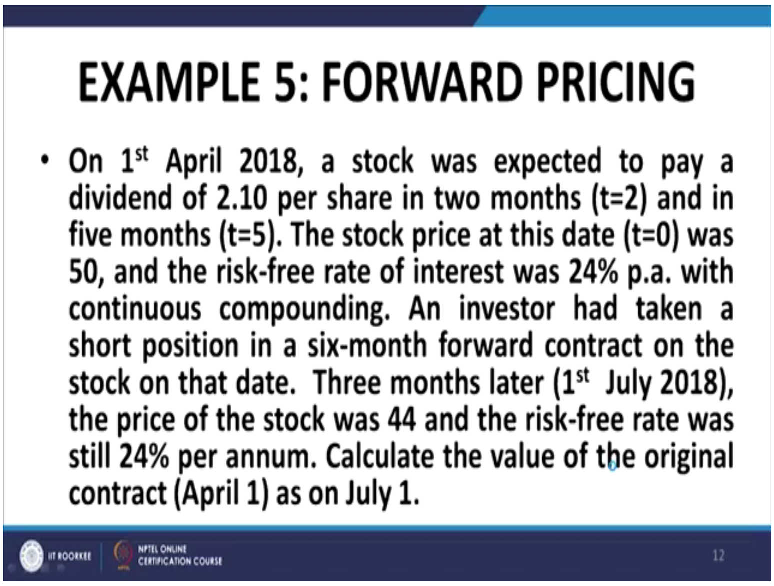

This is a slightly more involved question. The solution is as follows:

$Spot\ \ price\ on\ \ Apr\ 1,\ 2018 = 50.00$
$Dividends\ of\ \ 2.10\ \ after\ 2\ months\ (June\ 1)\ and\ 5\ months\ (Sept\ 1)$
$Risk - \ free\ Rate(r) = 0.24$
$Maturity\ of\ \ \ forward\ \ (T) = 0.50\ year.$
$Present\ Value\ of\ Dividends(D_0): 2.10e^{-0.24\times 2/12} + 2.10e^{-0.24\times 5/12}$
$= 2.0177 + 1.9002 = 3.9179$
$Hence,\ Effective\ Spot\ Price: S_0^* = 50.00 - 3.9179 = 46.0821$

$Now,\ on\ July\ 1,\ Spot\ \ price = 44.00$
$Remaining\ \ Dividend\ \ is\ at\ Sept\ 1\ i.e.\ after\ 2\ months.$
$Present\ value\ of\ \ this\ dividend\ \ as\ on\ July\ 1 = 2.10e^{-0.24\times 2/12} = 2.0177$
$Hence,\ effective\ stock\ \ price\ on\ July\ 1: S_1^* = 44.00 - 2.0177 = 41.9823$

$Value\ of\ \ SHORT\ \ forward\ of\ \ Apr 1\ on\ July\ 1 = -\left(S_1^* - S_0^* e^{0.24\times 3/12}\right)$
$= -\left(41.9823 - 46.0821e^{0.24\times 3/24}\right) = -41.9823 + 48.9317 = 6.9494$

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SOLUTION					
TIME		0.0000	2.0000	5.0000	6.0000
STOCK PRICE		50.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.1000	0.1200
DISCOUNT FACTOR			-0.0400	-0.1000	
			0.9608	0.9048	
DIVIDEND			2.1000	2.1000	
PV OF DIVIDEND			2.0177	1.9002	1.1275
		3.9178			
NET STOCK PRICE AT T=0		46.0822			
FORWARD PRICE AT T=6		51.9575			

Now, the important thing is when you are talking about 1st July one dividend payment has already taken place. So, we are left with only the second dividend payment when we recalculate the forward price on July 1. The first dividend payment had taken place before July 1 i.e. on June 1 and is, therefore, not relevant when we calculate the forward price on July 1.

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USING THE SAME METHODOLOGY WE CALCULATE THE FORWARD PRICE OF THE NEW CONTRACT					
TIME		3.0000	5.0000	6.0000	
STOCK PRICE		44.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.0600	
DISCOUNT FACTOR			-0.0400	-0.0600	
			0.9608	0.9418	
DIVIDEND			2.1000	0.0000	
PV OF DIVIDEND			2.0177	0.0000	1.0618
		2.0177			
NET STOCK PRICE AT T=0		41.9823			
FORWARD PRICE AT T=6		44.5784			
DIFFERENCE IN FORWARD PRICES		7.3791			
PV OF THIS DIFFERENCE		6.9494			

Hence, on July 1, we shall discount only one dividend payment i.e. the second payment or payment on September 1 (t=5 months from April 1). When we calculate the forward price on July 1, this dividend will be discounted for 2 months. The total period of the original forward was 6 months i.e. maturity was on September 30.

Also, on July 1, the period remaining in the forward contract is 3 months (July 1 to September 30).

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SOLUTION					
TIME		0.0000	2.0000	5.0000	6.0000
STOCK PRICE		50.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.1000	0.1200
DISCOUNT FACTOR			-0.0400	-0.1000	
			0.9608	0.9048	
DIVIDEND			2.1000	2.1000	
PV OF DIVIDEND			2.0177	1.9002	1.1275
		3.9178			
NET STOCK PRICE AT T=0		46.0822			
FORWARD PRICE AT T=6	1/4/2018	51.9575			

The earlier forward price price i.e. on April 1 was 51.9575.

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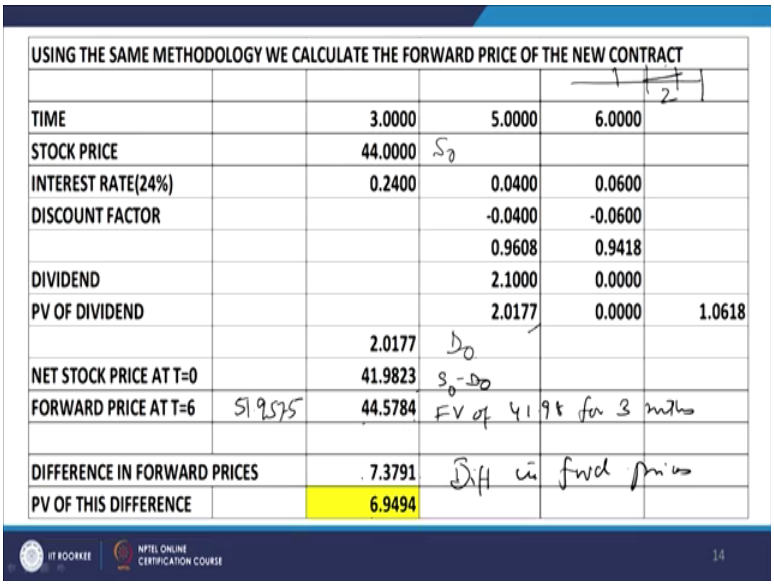

USING THE SAME METHODOLOGY WE CALCULATE THE FORWARD PRICE OF THE NEW CONTRACT

TIME		3.0000	5.0000	6.0000	
STOCK PRICE		44.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.0600	
DISCOUNT FACTOR			-0.0400	-0.0600	
			0.9608	0.9418	
DIVIDEND			2.1000	0.0000	
PV OF DIVIDEND			2.0177	0.0000	1.0618
		2.0177			
NET STOCK PRICE AT T=0		41.9823			
FORWARD PRICE AT T=6		44.5784			
DIFFERENCE IN FORWARD PRICES		7.3791			
PV OF THIS DIFFERENCE		6.9494			

This was the original price, original price that was on 1st April.

The forward price as on July 1 is 44.4784. So, this is the difference is 7.3791. The value of this contract is the present value (as on July 1 being the date of valuation) of the difference. So, this is discounted for three months and we get 6.9494.

Now a topic which is slightly digressive but which is very much relevant is the concept of spot and forward interest rates. All of us are familiar with spot rate.

Spot & forward interest rates

In analogy with spot prices being current market prices or, more precisely, market prices that envisage immediate delivery of the traded product, spot rates are the rates which are available in the markets for deposits made as of today. These rates may vary across deposits of different maturities i.e. the rate for a 6-month deposit initiated today may be different from the rate on a 3-year deposit initiated today. Thus, we have a spectrum of spot rates corresponding t different maturities. Rates are usually quoted for 6- monthly intervals. This phenomenon of the spot rates being a function of or varying with the envisaged maturity of the underlying loan is called the "term structure of interest rates".

Technically speaking, the spot rates of a desired maturity are defined as the yield-to-maturity (YTM) of zero-coupon bond of the same maturity.

A zero-coupon bond is a bond that is usually sold at a discount to face value, carries no coupon payment during its life and is redeemed at maturity at its face value. We have,

$P_0=F(1+S_{0T})^{-T}$ where P_0 is the zero coupon bond's current market price, F is its redemption value, T the maturity of the bond and S_{0T} is the spot rate for a maturity of T.

Forward interest rate

Just as a forward contract is a contract instituted and agreed upon at t=0 but actually settled at a future date (t=T) by delivery, a forward interest rate f(0,T,N) on a loan is an interest rate that is agreed upon at t=0 but pertains, or is applicable, to a loan that is initiated at a future date (t=T). The loan extends to and hence this forward rate applies to the period from t=T to t=N. where t=N is the date of maturity of this forward loan. For example, we can have a forward rate of interest (agreed upon today i.e. at t=0) that applies to a loan that is going to be disbursed after one year from now for a period of 0.50 years i.e. f(0,1,1.50).

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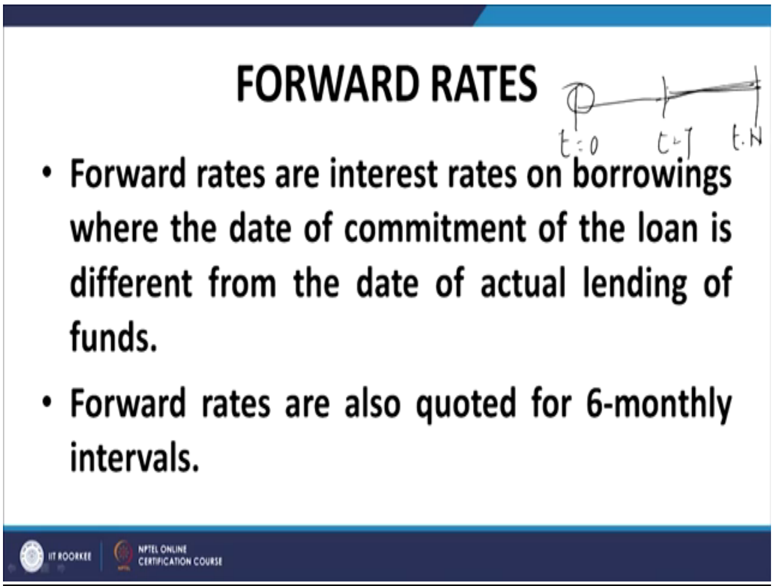

Forward rate is a rate that pertains to a loan transaction occurring in the future but the rate is negotiated is finalised as of today. So, it is obviously a forward contract but in this case the underlying instrument is not a commodity, it is not a share, it is no stock, it is an interest rate and that interest rate will cover a particular period of time from t=T to t=N.

Relationship between spot & forward rates

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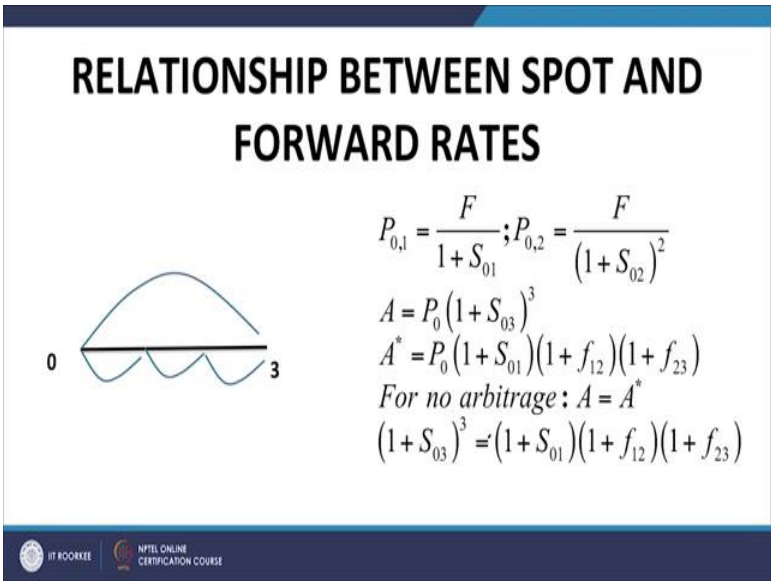

We can again invoke the principle of no arbitrage for determining a relationship between the spot rate and the forward rate. Consider the simplest case, when I want to make a deposit for two years of, say, INR 100. The following possibilities exist:

(i) I make the deposit in one go for the entire period of two years. The amount that I shall receive at maturity i.e. at t=2 years will be: $A_1=100(1+S_{02})^2$

(ii) I make the deposit in stages. First I make the deposit for one year and then reinvest the amount that I receive at the end of one year for a further period of one year. In this case, the amount that I shall receive at the end of two year period will be $A_2=100(1+S_{01})(1+S_{12})$ where S_{12} is the spot rate that will prevail in the market at the end of one year from now i.e. at t=1 years. Obviously, S_{12} is not known at t=0. Furthermore, not only this, interest rates are known to evolve with some randomness. Therefore, it is also impossible to predict with absolute certainty what the value of S_{12} will be, sitting at t=0. It, therefore, follows that by adopting this strategy, I am exposing myself to some amount of market risk arising from the uncertainty associated with the prediction of S_{12}. I shall only know S_{12} precisely when I get to t=1 years, and then I shall have no choice but to make the investment at whatever value S12 takes. The bottomline is that the amount that I shall receive at the end of two years A_2 cannot be predicted certainly, if I follow this strategy. Since, this amount is random, depending on what value S_{12} takes at t=1 year, I cannot formulate any arbitrage strategy between strategy (i)_ and strategy (ii).

(iii) The third possible strategy is that I follow the strategy (ii) i.e. make the investment for one year and roll it over for another year but, in this case, I fix the rate (at t=0) at which the rollover or the reinvestment will be made at the end of the first year. In other words, the rate at which the reinvestment is to be made is also fixed beforehand at t=0 by taking a forward rate agreement i.e. by agreeing to a forward rate, say f(0,1,2). Now, in this case since we have already fixed the reinvestment rate at t=0, i.e. all the interest rates are known at t=0, we can precisely work out the final proceeds arising from this strategy i.e. $A_3=100(1+S_{01})(1+f(0,1,2))$.

BNow, since both strategies (i) and (iii) provide values of the final receipts with certainty, i.e. there is no risk differential between these two strategies, their reurns must be the same. Because we start with the same initial amount, the maturity proceeds must also be the same so that $A_1=100(1+S_{02})^2=100(1+S_{01})(1+f(0,1,2))=A_3$ so that

$$f(0,1,2)=\frac{(1+S_{02})^2}{(1+S_{01})}-1$$

This no arbitrage relationship can be easily generalized to any number of years and we have:

$$\begin{aligned}(1+S_{0T})^T &= (1+S_{0T-1})^{T-1}\left[1+f(0,T-1,T)\right]\\ &= (1+S_{0T-2})^{T-2}\left[1+f(0,T-2,T-1)\right]\left[1+f(0,T-1,T)\right]\\ &= (1+S_{01})\left[1+f(0,1,2)\right]\left[1+f(0,2,3)\right]...\left[1+f(0,T-2,T-1)\right]\left[1+f(0,T-1,T)\right]\end{aligned}$$

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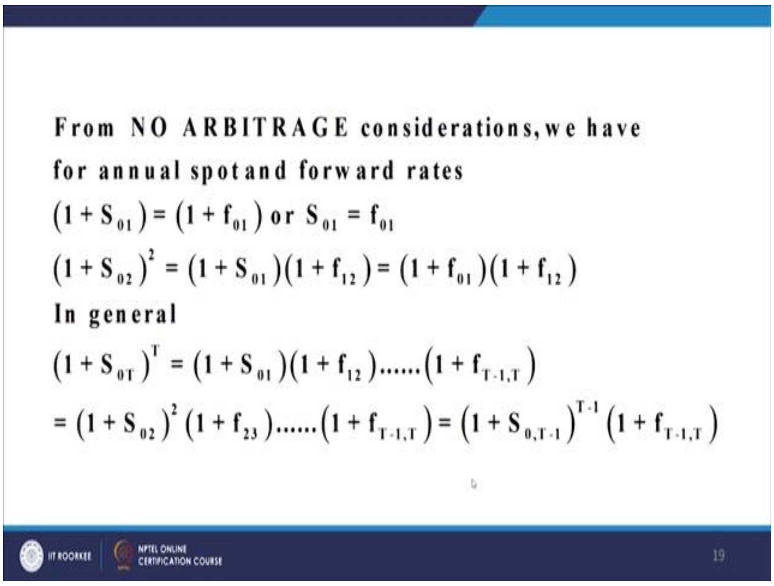

Now, we have a forward rate curve depicting, as of today, the forward rates projecting for different maturities. If the actual rates evolve according to the forward rate curve, then your pattern of investment does not does not make any difference. In other words

(i) if you buy long term bonds, that is, you invest today for maturity over the desired period of time, let us say t=T years i.e., you make an investment in one go, in a bond of maturity T or

(ii) if you make investments at the end of every year or every half year at the relevant rates which are assumed to follow the forward rate curve;

In both cases, you will receive the same return over your entire holding period. The proof is quite simple:

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