Financial Derivatives and Risk Management
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Lecture 36
Option Spread Strategies

Spread strategies

There are many spread strategies similar to the vertical bull spread with calls. We have bear spread with calls, bull & bear spreads with puts. Using the same terminology as above, let us study a bear call spread.

The bear call spread consists of buying a call option and selling a call option on the same underlying with same maturities but with different strike prices. In this case, the short call A is at the lower exercise price K_A and the long call B is at the higher exercise price K_B. Both have the same maturity T.

Since the price of a call varies inversely as the exercise price and the vertical bear spread involves a short call at the lower strike (higher premium c_A) and a long call at the higher strike (lower premium c_B), it follows that the vertical bear spread entails an initial cash inflow (at t=0) at the time of setting up of the strategy. Now, we look at the payoff. We have:

$\Pi_{\text{bear spread}}(S_T)=-\max(0,S_T-K_A)+\max(0,S_T-K_B)=\max(0,K_A-S_T,K_A-K_B)$ and the profit $\pi_{\text{bull spread}}(S_T)=-\max(0,S_T-K_A)+\max(0,S_T-K_B)+c_B-c_A=\max(0,K_A-S_T,K_A-K_B)-c_B+c_A$

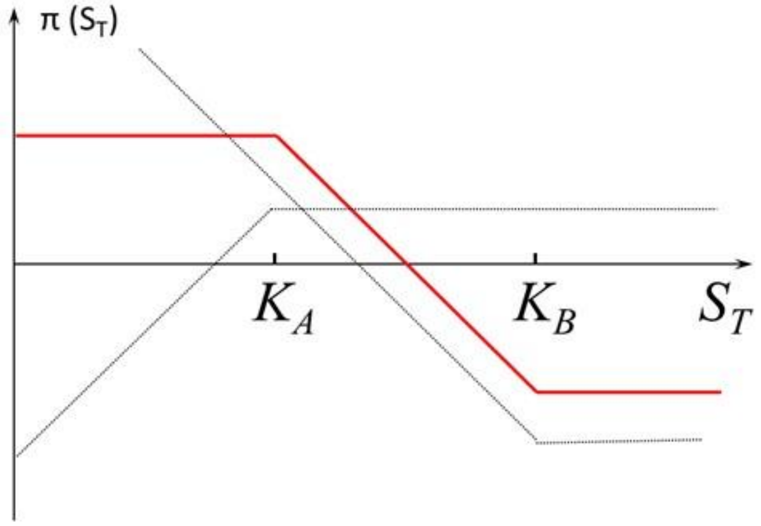
$$\pi_{BEAR\ SPREAD\ WITH\ CALLS}=\pi_{SHORT\ CALL\ A}+\pi_{LONG\ CALL\ B}$$

$$=\begin{cases} c_A-c_B \ \ if\ \ S_T<K_A<K_B \\ -(S_T-K_A)+(c_A-c_B)\ \ if\ \ K_A<S_T<K_B \\ (S_T-K_B)+(K_A-S_T)+(c_A-c_B)\ if\ \ K_A<K_B<S_T \end{cases}$$

$$=\begin{cases} c_A-c_B\,(>0\ ALWAYS)\ if\ \ S_T<K_A<K_B \\ -(S_T-K_A)+(c_A-c_B)\ \ if\ \ K_A<S_T<K_B \\ (K_A-K_B)+(c_A-c_B)(<0\ ALWAYS)\ if\ \ K_A<K_B<S_T \end{cases}$$

	t=0	t=T		
		$S_T<K_A$	$K_A<S_T<K_B$	$K_B<S_T$
SHORT CALL A	$+c_A$	0	$-(S_T-K_A)$	$-(S_T-K_A)$
LONG CALL B	$-c_B$	0	0	(S_T-K_B)
TOTAL	$(c_A-c_B)>0$	0	K_A-S_T	$K_A-K_B<0$

We can divide the entire spectrum of stock price into three segments viz.
$S_T<K_A<K_B$: Both calls will lapse;
$K_A<S_T<K_B$: Call A will be exercised and B will lapse;
$K_A<K_B<S_T$: Both calls will be exercised.



Let us discuss the payoff. We can split the entire stock price spectrum on maturity into the partitions $(0,K_A)$, (K_A,K_B) and (K_B+). Now, if the stock ends up at maturity below K_A, neither call gets exercised and there is zero payoff from the strategy. If S_T is between the two strikes, the short call gets exercised yielding a negative payoff of $–(S_T-K_A)=K_A-S_T$. Obviously the party in favour of whom call A is written will exercise that call because he is getting a positive payoff out of this since $S_T>K_A$. The payoff of the spread holder decreases by one unit for every unit increase in stock price in this region. However, if the stock ends up beyond K_B, then the spread holder will exercise call B with a positive payoff of S_T-K_B. The holder of call A will, of course continue to exercise the call A. Since, the spread holder is short in A, he has to pay this amount (K_A-S_T) to holder of A as the option writer while he recoups S_T-K_B on exercising call B if $S_T>K_B$. Hence, his net payoff if $S_T>K_B$ is K_A-K_B.

Thus, we find that as the value of S_T decreases (so long that $K_A<S_T<K_B$), the payoff from the strategy increases (in fact, by the same amount). Therefore, this strategy is called bearish vertical spread. The investor who is setting up this strategy will make a profit if the stock price goes down (so long that $K_A<S_T<K_B$).

The maximum payoff is 0 and occurs when $S_T<K_A$ while the minimum payoff is K_A-K_B which is essentially negative, so that the payoff is invariably non-positive. However, the cashflow at setting up this strategy is c_A-c_B, which is clearly positive since $K_B>K_A$ by choice and we have an inverse relation between call strikes and premia whence $c_B<c_A$. Recall that this strategy involves shorting option A and therefore receiving c_A while being long in B for which entails paying c_B, so that there is a cash inflow for setting up this strategy. Interestingly, from put-call parity for the two options:

$c_A+K_Ae^{-rT}=p_A+S_0$, $c_B+K_Be^{-rT}=p_B+S_0$ so that $(K_B-K_A)e^{-rT}-(c_A-c_B)=p_B-p_A\geq 0$ (since the premia of puts varies directly with the strike price) whence $(K_B-K_A)e^{-rT}\geq (c_A-c_B)$. In the event that time value of money is ignored: $-(K_A-K_B)\geq (c_A-c_B)$ or $-(K_A-K_B) - (c_A-c_B)\geq 0$.

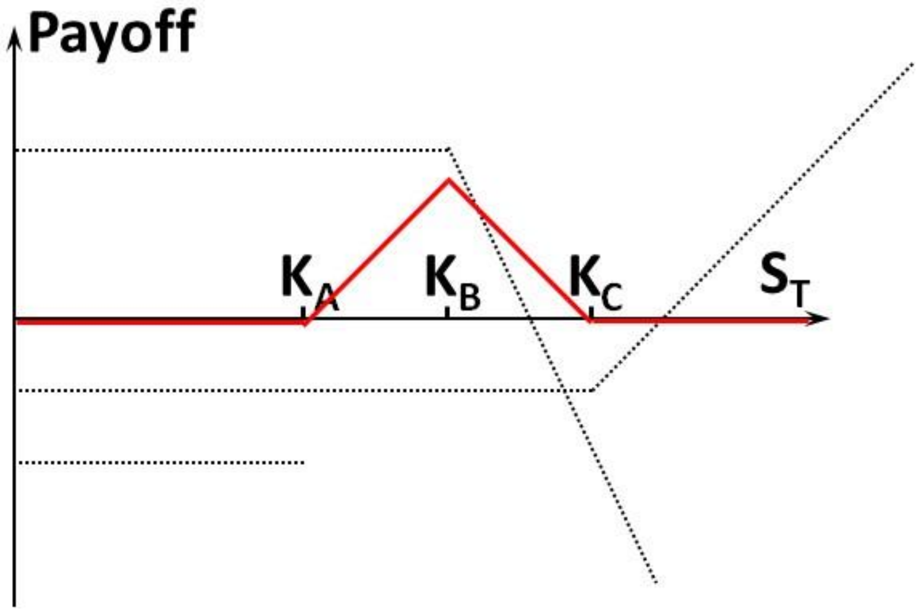
Thus, the maximum outflow on the strategy exceeds the cash inflow of setting it up, which is justified since there are stock prices where the outflow is zero or lesser than the cash inflow for setting the strategy.

Butterfly call spread

The butterfly call spread consists of the following positions:

(i) Long call A (K_A,T, c_A);
(ii) Long call C (K_C,T, c_C);
(iii) Short two calls B (K_B,T, c_B)

with $K_A < K_B < K_C$; $K_C - K_B = K_B - K_A$ or $K_C + K_A = 2K_B$



We can divide the entire spectrum of stock price into four segments viz.

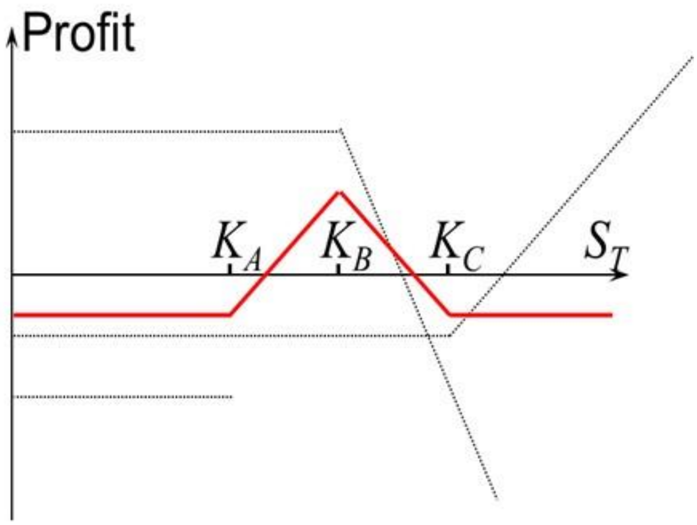
$S_T<K_A<K_B<K_C$: All calls will lapse;
$K_A<S_T<K_B<K_C$: Call A will be exercised, B & C will lapse;
$K_A<K_B<S_T<K_C$: A & B will be exercised, C will lapse;
$K_A<K_B<K_C<S_T$: All calls will be exercised.

	t=0	t=T			
		$S_T<K_A$	$K_A<S_T<K_B$	$K_B<S_T<K_C$	$K_C<S_T$
LONG CALL A	$-c_A$	0	S_T-K_A	S_T-K_A	S_T-K_A
2* SHORT CALLS B	$2c_B$	0	0	$-2(S_T-K_B)$	$-2(S_T-K_B)$
LONG CALL C	$-c_C$	0	0	0	S_T-K_C
TOTAL		0	$S_T-K_A>0$	$-S_T+(2K_B-K_A)$	0

$(2c_B\text{-}c_A\text{-}c_C)$				**$=K_C\text{-}S_T>0$**	

$$\pi_{BUTTERFLY\ CALL\ SPREAD} = \pi_{LONG\ CALL\ A} + 2\pi_{SHORT\ CALL\ B} + \pi_{LONG\ CALL\ C}$$

$$= \begin{cases} c_{2B,\text{-}A,\text{-}C} \quad if \;\; S_T < K_A < K_B < K_C \\ \left(S_T - K_A\right) + c_{2B,\text{-}A,\text{-}C} \quad if \;\; K_A < S_T < K_B < K_C \\ \begin{bmatrix} \left(S_T - K_A\right) + 2\left(K_B - S_T\right) + c_{2B,\text{-}A,\text{-}C} \\ = -S_T + K_C + c_{2B,\text{-}A,\text{-}C} \sin ce \;\; 2K_B = K_A + K_C \end{bmatrix} \quad if \;\; K_A < K_B < S_T < K_C \\ \begin{bmatrix} \left(S_T - K_A\right) + 2\left(K_B - S_T\right) + \left(S_T - K_C\right) + c_{2B,\text{-}A,\text{-}C} \\ = c_{2B,\text{-}A,\text{-}C} \end{bmatrix} \quad if \;\; K_A < K_B < K_C < S_T \end{cases}$$



In the case of a butterfly call spread, we have three calls involved. We buy a call at K_A with a premium c_A and a call C with strike K_C costing c_C. We also short two calls at strike K_B carrying premium c_B. All the calls have the same maturity T and the strikes are such that K_C-K_B=K_B-K_A.

Now in this case, we need to split the stock price spectrum at maturity into four regions viz (i) (0,K_A); (ii) (K_A,K_B); (iii) (K_B,K_C) and (iv) (K_C+).

If the stock price finishes in (0,K_A), none of the calls will be exercised and the net payoff will be 0. If it ends up in (K_A,K_B), only call A will be exercised resulting in a positive payoff of S_T-K_A. If the stock price at maturity lies in (K_B,K_C), calls A & B will be exercised. Call A will generate a payoff of S_T-K_A while the two calls B, being short will entail negative payoff of -2(S_T-K_B). The net payoff will be S_T-K_A-2(S_T-K_B)=2K_B-K_A-S_T=K_C-S_T>0 since S_T<K_C. Finally, if the maturity stock price exceeds K_C, all the calls will be exercised and the net payoff will be S_T-K_A-2(S_T-K_B)+S_T-K_C=2K_B-K_A-K_C=0. Hence, the payoffs at maturity in all possible scenarios are essentially non-negative. Whatever may be the respective probabilities, the expected payoff must therefore be positive. This means that the cost of setting of this strategy must be non-negative i.e. the cash flow for setting up this strategy at t=0 must be non-positive. Thus, we must have, as a corollary: $2c_B - c_A - c_C \leq 0$. The equality may hold, for example, if the probability distribution of the stock price and the strike prices are such that the stock price is certainly likely to finish up in $(0, K_A)$ so that all the calls are deep out of the money or if the stock price is certain to finish in $(K_C +)$ so that all calls are deep in the money etc. But these are not really realistic situations.

The above was an analysis of the long butterfly spread. The short spread can be analyzed similarly. Butterfly spreads can also be constructed using puts instead of calls.

Box spread

A box spread is a combination of

(i) Bull call spread consisting of:
Long call A (K_A,T, c_A) and a short call B (K_B,T, c_B) with $K_A < K_B$;
(ii) a bear put spread
Short put X (K_A,T, p_X) and a long put Y(K_B,T, p_Y)

All the options are European.

PAYOFF FROM BOX SPREAD ON MATURITY

STOCK VALUE	*OPTION A*	*OPTION B*	*OPTION X*	*OPTION Y*	*TOTAL PAYOFF*
$0 < S_T < K_A$	0	0	$-(K_A - S_T)$	$(K_B - S_T)$	$(K_B - K_A)$
$K_A < S_T < K_B$	$(S_T - K_A)$	0	0	$(K_B - S_T)$	$(K_B - K_A)$
$K_B < S_T$	$(S_T - K_A)$	$-(S_T - K_B)$	0	0	$(K_B - K_A)$

$$COST\ OF\ BOX\ SPREAD = c_B - c_A + p_X - p_Y\ (outflow)$$

$$HENCE, WE\ MUST\ HAVE\ \ c_A - c_B - p_X + p_Y = (K_B - K_A)e^{-rT}$$

	t=0	**t=T**		
	CASHFLOW	$S_T < K_A$	$K_A < S_T < K_B$	$K_B < S_T$
LONG CALL A	$-c_A$	**0**	$S_T - K_A$	$S_T - K_A$
SHORT CALL B	c_B	**0**	**0**	$-(S_T - K_B)$
SHORT PUT X	p_X	$-(K_A - S_T)$	**0**	**0**
LONG PUT Y	$-p_Y$	$K_B - S_T$	$K_B - S_T$	**0**
TOTAL	$(c_B - c_A) +$ $(p_X - p_Y)$	$K_B - K_A$	$K_B - K_A$	$K_B - K_A > 0$

As usual, we split the price spectrum into various partitions viz $(0,K_A)$, (K_A,K_B), (K_B+). If the stock price ends up at maturity in the first partition $(0,K_A)$, both the puts X & Y will be exercised. The short put Y will yield payoff of $-(K_A-S_T)$ while the long put yields (K_B-S_T) whence the net payoff is $K_B-K_A>0$. If the maturity stock price ends up between the two strikes, long call A gets exercised and long put Y gets exercised with respective payoffs of S_T-K_A and K_B-S_T resulting in a net payoff of K_B-K_A. Finally, if the stock price at maturity exceeds K_B, the long call A & the short call B both get exercised with a net payoff K_B-K_A again.

Now the net payoff makes very interesting reading. There are two features:

(i) the payoff is K_B-K_A in all scenarios. It is constant and therefore, independent of the stock price at maturity. It is the same across the entire spectrum of possible maturity stock prices.

(ii) the payoff $K_B-K_A>0$.

To reiterate, the payoff is the same, is independent of the price of the underlying at maturity and is positive. These are three important characteristics. It means that in whatever the stock price evolves on maturity, the strategist is getting the same payoff. In other words, it is a risk free payoff. The stock price spectrum represents the risk. If variations in the stock prices do not impact the payoff of our strategy, it means our strategy is risk-free. Because our strategy is independent of the price of the stock at maturity, it must be classified as risk-free.

That being the case, the investment that goes into setting up the strategy must generate a riskfree rate of return. Putting it the other way round, the present value of the payoff at the riskfree rate must equal the cost of setting up the box spread. Hence, we must have $(K_B-K_A)e^{-rT}=c_A-c_B-p_X+p_Y$ which, indeed is implied by put-call parity:

$c_A+K_Ae^{-rT}=p_X+S_0$; $c_B+K_Be^{-rT}=p_Y+S_0$ whence the above result immediately follows.

Furthermore, since the payoff from the box spread is essentially positive, it must entail a negative cash flow at inception i.e. $-c_A+c_B+p_X-p_Y<0$. This can be justified by another argument. Since $K_A<K_B$, the long call A would cost more than the premium received on the short call B so $c_B-c_A<0$. For the same reason the premium received on the put X will be less than that paid on the put Y so that $p_X-p_Y<0$. Hence the result.

Box spread arbitrage

Now, what happen if $(K_B-K_A)e^{-rT}\neq c_A-c_B-p_X+p_Y$ i.e. what happens if the present value at the riskfree rate of the payoff from the box spread is not equal to the cost of setting up of the strategy? What happens if the cost of setting up of the box spread is lesser i.e. $(K_B-K_A)e^{-rT}\geq c_A-c_B-p_X+p_Y$?

Obviously, arbitrage will come into play. If the market price of the box spread is low, it is profitable to buy the box. This involves buying a call with strike price K_A buying a put with strike price K_B, selling a call with strike price K_B, and selling a put with strike price K_A. This combination is called a long box spread. The actual procedure will involve borrowing the amount $(c_A-c_B-p_X+p_Y)$ required to finance the box spread at the riskfree rate, taking appropriate positions in the constituents of the box spread and holding the strategy upto maturity of the options. The net flow realized at maturity in that case (K_B-K_A) will exceed the repayment of the borrowings resulting in a riskfree arbitrage profit. We see that from:

$(K_B-K_A)-(c_A-c_B-p_X+p_Y)e^{rT}=e^{rT}[(K_B-K_A)e^{-rT}-(c_A-c_B-p_X+p_Y)]>0.$

So if the box spread is being traded at a lower than parity price in the market, arbitrage profit can be earned by borrowing riskfree and investing in a long box spread.

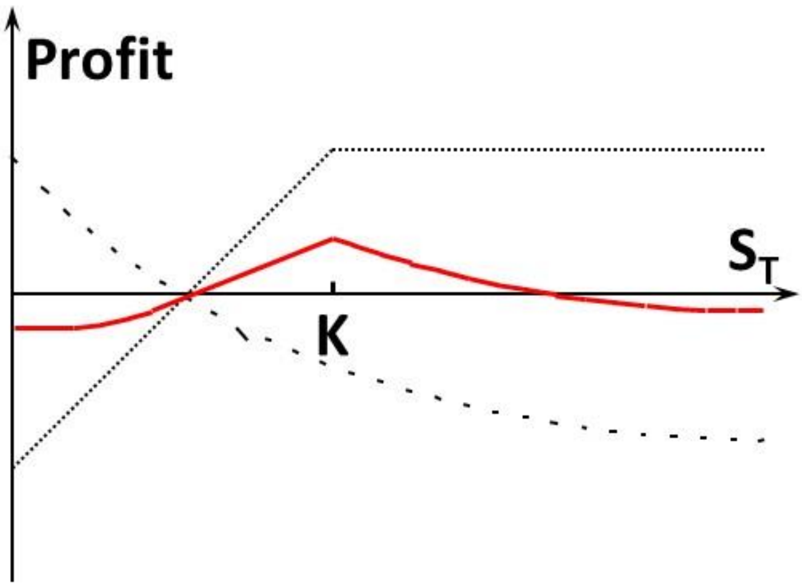
If the box spread is trading at above parity price, arbitrage profits can be obtained by shorting the box spread by reversing all the above trades and investing at the riskfree rate.

It is important to realize that a box-spread arbitrage works with European options ONLY.

Horizontal/calendar spread

A horizontal spread consists of long and short positions in two options A and B of the same type on the same underlying S with the same strike price K but with different maturities T_A & T_B respectively.

In the long calendar call spread, we short the near maturity call A and long the farther maturity call B i.e. $T_A<T_B$. The strikes of both the options is the same K and their respective prices are c_A & c_B. Clearly, since the maturity of B exceeds that of A, it carries more time value and hence, enjoys a higher premium so that $c_A<c_B$. Thus, it requires a cash outflow at t=0 to set up a long calendar call spread. Longer the maturity of an option, the costlier it is because it covers uncertainties of the stock price over a longer period and therefore has more time value and hence is costlier.



We now, look at the payoff of this strategy at the time point of maturity of the short maturity option A i.e. T_A. The option B continues to be live and is unexercised.

Now let us do the analysis. If $S_{T_A} << K$, the option A expires out of the money and hence, yields no payoff. The intrinsic value of option B is also zero. Besides, there is little chance of the option B ending up in the money at its maturity. Hence, B will carry insignificant time value. Thus, option A expires giving nothing while option B also has insignificant market value. Thus, the net payoff from the strategy if $S_{T_A} << K$ will be very small.. The investor therefore incurs a loss that is close to the cost of setting up the spread initially.

A somewhat similar situation prevails $S_{T_A} >> K$. In this case the option A expires in the money and yields a payoff of $S_{T_A} - K$. The intrinsic value of option B is also $S_{T_A} - K$. Besides, there is little chance of the option B ending up out of the money at its maturity. Hence, B will carry insignificant time value. Thus, option A expires giving $S_{T_A} - K$ while option B also has market value of a similar amount. Again, because we are short in A and long in B, the net payoff from the strategy will be insignificant. Thus, the net payoff from the strategy if $S_{T_A} >> K$ will also be very small. Again, the investor makes a net loss that is close to the cost of setting up the spread initially.

Let us now examine the case $S_{T_A} \approx K$ i.e. the stock price at maturity of the short maturity option is close to the strike price K. In this case, the payoff from the option A which matures at this point will be negligible. However, while the intrinsic value of B will also be small, its time value could be significant. Because the stock price is currently close to the strike price, even small variations in the stock price could have large impact on the moneyness (payoff) of the option at maturity. Thus, option B will command a significant market value at T_A. As a result, this spread strategy in this scenario will yield a relatively large payoff, since we are long in the option B which has a large market value while the short option A will mandate a small (relatively insignificant) outflow.

This is called the long calendar spread, a calendar spread where the shorter maturity option is short, the longer maturity option is long. The payoff diagram and profit diagrams are similar to those of the butterfly spread where we have a peak at intermediate points and nearly 0 payoffs at the two extremes of the stock prices.

We can also construct a calendar spreads using put options.

<u>Example</u>

"A box spread comprises four options. Two can be combined to create a long forward position and two can be combined to create a short forward position." Explain this statement.

<u>Solution</u>

Bull Call Spread: $[A(K_A,T,c_A)-B(K_B,T,c_B)]$
+Bear Put Spread: $[X(K_A,T,p_A)+Y(K_B,T,p_B)]$
=Long Forward: $[A(K_A,T,c_A)-X(K_A,T,p_A)]+$
Short Forward: $-[B(K_B,T,c_B)-Y(K_B,T,p_B)]$

LONG CALL +SHORT PUT

	t=0	t=T	
		S_T <K	S_T >K
LONG CALL	**-c**	**0**	S_T -K
SHORT PUT	**+p**	-(K-S_T)	**0**
TOTAL	**-(c-p)**	S_T -K	S_T -K