Financial Derivatives and Risk Management
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Lecture 35
Options Strategies continued

Long Straddle

The long strangle strategy comprises of the following:

(i) A long call option on stock S with strike K_C and maturity T; and
(ii) A long put option on the same stock S with same strike K_P and same maturity T.

Let c, p be the premia on the call & put respectively. Now, two situations arise:

(i) $K_P<K_C$
(ii) $K_C<K_P$

(i) **$K_P<K_C$**

The payoff & profit on the strategy at option maturity t=T is:

$\Pi_{strangle}(K_P<K_C, S_T) = \max(0,S_T-K_C)+\max(K_P-S_T,0) = \max(K_P-S_T,0,S_T-K_C)$
$\pi_{strangle}(K_P<K_C,S_T) = \pi_{long\ call}(K_C,S_T)+\pi_{long\ put}(K_P,S_T)=\max(0,S_T-K_C)-c+\max(K_P-S_T,0)-p$
$= \max(K_P-S_T,0,S_T-K_C)-(c+p)$.

Clearly, maximum loss is at $K_P \le S_T \le K_C$ and is given by (c+p).

The BEPs are: $0 = \max(K_P-S_{BEP},0,S_{BEP}-K_C)-(c+p)$ or $S_{BEP}=K_P-(c+p)$, $K_C+(c+p)$.

$$\pi_{Strangle} = \pi_{Long\,Call} + \pi_{Long\ Put} = \begin{cases} (K_P - S_T) - (p+c) \ \ if \ \ S_T < K_P \\ 0 \ if \ \ K_P < S_T < K_C \\ (S_T - K_C) - (p+c) \ \ if \ \ S_T > K_C \end{cases}$$

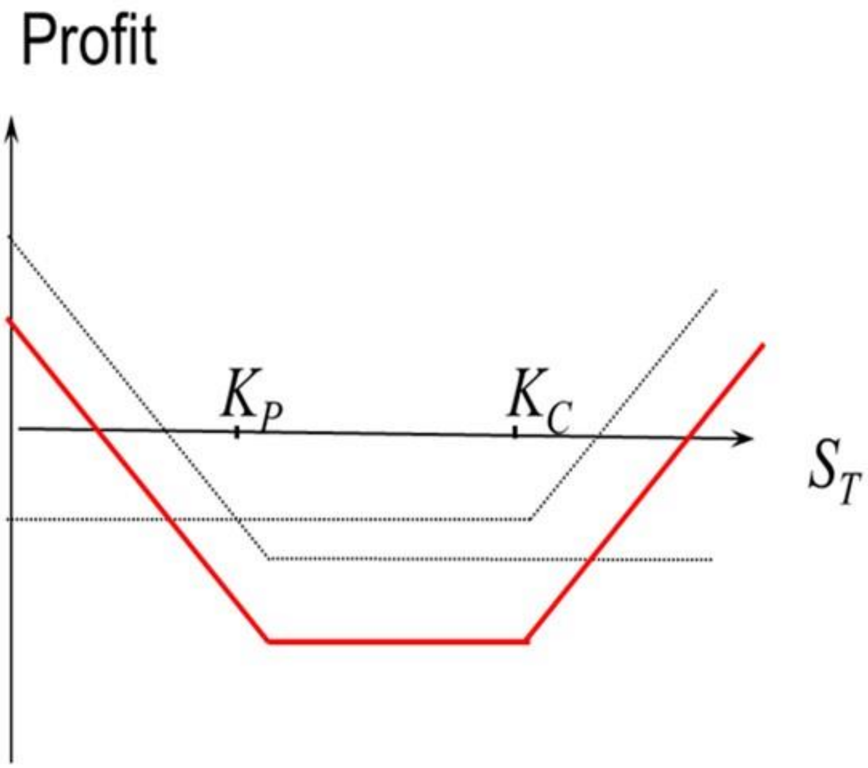
$$\pi^{MAX}_{STRANGLE} = \infty; \quad \pi^{MIN}_{STRANGLE} = p+c; S_T^{BEP} = K_P - (p+c), \ K_C + (p+c)$$

	t=0	t=T		
		$S_T<K_P$	$K_P<S_T<K_C$	$K_C<S_T$
LONG CALL	-c	0	0	S_T-K_C
LONG PUT	-p	K_P-S_T	0	0
TOTAL	-(c+p)	K_P-S_T	0	S_T-K_C

For a strangle:

$S_T<K_P<K_C$: Put will be exercised, call will lapse;

$K_P<S_T<K_C$: Both will lapse;
$K_P<K_C<S_T$: Call will be exercised, put will lapse.



Let us look at the pay offs. In this case, we split the spectrum of stock prices into three partitions $S_T<K_P$, $K_P<S_T<K_C$ and $S_T>K_C$. If $S_T<K_P$, the put option will be exercised while the call will lapse so that the payoff will be K_P-S_T. In case the stock price ends up in the range $K_P<S_T<K_C$, neither the put nor the cash gets exercised whence the payoff from the strategy is zero in this scenario. On the other hand, if $S_T>K_C$, the put will lapse unexercised while the call will be exercised yielding a payoff of S_T-K_C. Thus, the payoff is K_P-S_T if $S_T<K_P$, 0 if $K_P<S_T<K_C$ and S_T-K_C if $S_T>K_C$. The payoff is bounded on the one side since we must have $S_T>0$, while being unbounded on the other. Thus, the profit on the strangle is unbounded while the maximum loss equals the aggregate premia paid on the two options and occurs at $K_P<S_T<K_C$ when neither option gets exercised.

Investor perception & strangle

It is important to note that the strangle strategy is a variant of the straddle. In the straddle the strike prices of the put and call were the same. However, in the strangle they are different. Now, in the range $K_P<S_T<K_C$, the put will not be exercised because the stock price exceeds its strike price and the call will also not be exercised because the stock price is less than its exercise price. Thus, the strategy's payoff over this range is zero. Other than this, the payoff & hence, the investor perception is pretty much the same as in the case of a straddle.

(ii) **$K_C<K_P$**

The payoff & profit on the strategy at option maturity t=T is:

$\Pi_{strangle}(K_C<K_P, S_T)$ = max(0,S_T-K_C)+max(K_P-S_T,0) =max(K_P-S_T,(K_P-S_T)+(S_T-K_C),S_T-K_C) =max(K_P-S_T,K_P-K_C,S_T-K_C)
$\pi_{strangle}(K_C<K_P,S_T)$ =$\pi_{long\ call}(K_C,S_T)+\pi_{long\ put}(K_P,S_T)$=max(0,$S_T$-$K_C$)-c+max($K_P$-$S_T$,0)-p = max($K_P$-$S_T$,$K_P$-$K_C$,$S_T$-$K_C$)-(c+p).

Clearly, maximum loss is at $K_P \le S_T \le K_C$ and is given by K_P-K_C-(c+p).

The BEPs are: 0 =max(K_P-S_{BEP},K_P-K_C,S_{BEP}-K_C)-(c+p) or S_{BEP}=K_P-(c+p), K_C+(c+p).

$$\pi_{Strangle}=\pi_{Long\,Call}+\pi_{Long\ Put}=\begin{cases}(K_P-S_T)-(p+c)\ \ if\ \ S_T<K_C\\(K_P-S_T)+(S_T-K_C)\ if\ \ K_C<S_T<K_P\\(S_T-K_C)-(p+c)\ \ if\ \ S_T>K_P\end{cases}$$

$$=\begin{cases}(K_P-S_T)-(p+c)\ \ if\ \ S_T<K_C\\(K_P-K_C)\ if\ \ K_C<S_T<K_P\\(S_T-K_C)-(p+c)\ \ if\ \ S_T>K_P\end{cases}\qquad \pi_{STRANGLE}^{MAX}=\infty;\ \ \pi_{STRANGLE}^{MIN}=K_P-K_C-(p+c);$$

$$S_T^{BEP}=K_P-(p+c),\ \ K_C+(p+c)$$

	t=0	t=T		
		$S_T<K_C$	$K_C<S_T<K_P$	$K_P<S_T$
LONG CALL	-c	0	S_T-K_C	S_T-K_C
LONG PUT	-p	K_P-S_T	K_P-S_T	0
TOTAL	-(c+p)	K_P-S_T	K_P-K_C	S_T-K_C

For a strangle:

$S_T<K_C<K_P$: Put will be exercised, call will lapse;
$K_C<S_T<K_P$: Both will be exercised;
$K_C<K_P<S_T$: Call will be exercised, put will lapse.

Let us look at the pay offs. In this case, we split the spectrum of stock prices into three partitions $S_T<K_C$, $K_C<S_T<K_P$ and $S_T>K_P$. If $S_T<K_C$, the put option will be exercised while the call will lapse so that the payoff will be K_P-S_T. In case the stock price ends up in the range $K_C<S_T<K_P$, both the put and the call get exercised whence the payoff from the strategy is (K_P-S_T)+(S_T-K_C) =K_P-K_C>0 in this scenario. On the other hand, if $S_T>K_P$, the put will lapse unexercised while the call will be exercised yielding a payoff of S_T-K_C. Thus, the payoff is K_P-S_T if $S_T<K_P$, K_P-K_C if $K_C<S_T<K_P$ and S_T-K_C if $S_T>K_P$. The payoff is bounded on the one side since we must have $S_T>0$, while being unbounded on the other. Thus, the profit on the strangle is unbounded while the maximum loss equals the aggregate premia paid on the two options and occurs at $K_C<S_T<K_P$ when both options gets exercised.

In this case, in the region $K_C<S_T<K_P$, both the options get exercised and the profit gradient of the long call annuls that of the long put. Therefore, we have a constant payoff in this region. Contrast this with the previous case where we had a constant payoff in the intermediate region

due to non-exercise of both options. Here we are again having constant payoff, but because both the options are exercised. In this case, the maximum loss turns out to be K_P-K_C-(c+p). In the previous case it was simply (c+p).

It is interesting to note that in this case, the payoff at maturity of the strategy is invariably positive in all cases with the minimum being K_P-K_C>0. By no arbitrage the cost of setting up this strategy must be positive i.e. the setting up of the strategy must involve a cash outflow at t=0 which is, indeed the case with the cost being the total premia c+p.

We also have

$$\pi_{STRANGLE}^{MIN} = \left(K_P - K_C\right) - \left(p + c\right)$$
$$From\ \ put\text{-}call\ \ parity\ \left(K_C < K_P\right):$$
$$c + K_C e^{-rT} = p_C + S_0;\ c_P + K_P e^{-rT} = p + S_0$$
$$\left(c - c_P\right) - \left(K_P - K_C\right)e^{-rT} = p_C - p$$
$$\left(K_P - K_C\right)e^{-rT} - \left(p + c\right) = -\left(c_P + p_C\right) < 0$$

Thus, ignoring time value of money, we have K_P-K_C<c+p. This must be so because K_P-K_C represents the minimum profit. If it were that K_P-K_C>c+p, then the minimum possible profit would exceed the strategy's cost. In other words, the strategy would be guaranteeing a profit more than its cost in all possible scenarios. This would induce arbitrage and disequilibrium. In other words, the strategy has a loss K_P-K_C-(c+p)<0 in the region K_P<S_T<K_C. This is just like the earlier case where we had the loss of -(c+p)<0.

Investor perception & strangle

It is the same as in case (i)

Strips & straps

A strip consists of a long position in one call and two puts on the same underlying with the same strike price and expiration date.

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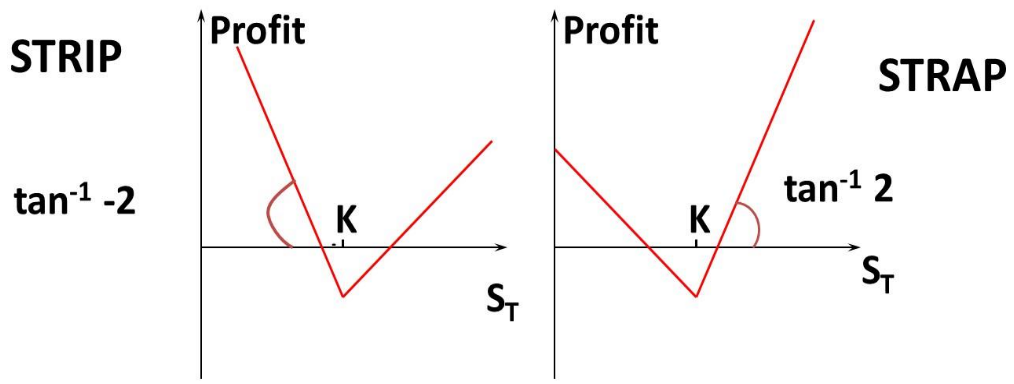
In a strip the investor is betting that there will be a big stock price move and considers a decrease in the stock price to be more likely than an increase. The converse is the case with straps.

$$\pi_{Strip} = \pi_{Long\ Call} + 2\pi_{Long\ Put}$$
$$= \begin{cases} 2\left(K - S_T\right) - \left(2p + c\right)\ \ if\ \ S_T < K \\ \left(S_T - K\right) - \left(2p + c\right)\ \ if\ \ K < S_T \end{cases}$$
$$\pi_{STRIP}^{MAX} = \infty; \qquad \pi_{STRIP}^{MIN} = -\left(2p + c\right);\ S_T^{BEP} = K - \left(p + \frac{1}{2}c\right),\ K + \left(2p + c\right)$$

$$\pi_{Strap} = 2\pi_{Long\ Call} + \pi_{Long\ Put}$$
$$= \begin{cases} (K - S_T) - (2p + c) \ \ if \ \ S_T < K \\ 2(S_T - K) - (2p + c) \ \ if \ \ K < S_T \end{cases}$$
$$\pi_{STRIP}^{MAX} = \infty; \quad \pi_{STRIP}^{MIN} = -(p + 2c); \ S_T^{BEP} = K - (p + 2c), \ K + \left(\frac{1}{2}p + c\right)$$



The factor 2 needs to be noted since we have two puts in a strip and two calls in a strap. Thus, the profit on a strip changes by 2 units per unit change in stock price in the region where the puts are exercised but because there is only one call, the profit changes by 1 unit per unit change in stock price in the region where the call is exercised. In strips/straps these two regions are mutually exclusive, they do not overlap. Similar pattern holds for the strap.

The investor perception

If you look at the profit diagram, you can immediately infer the investor perception. The investor perception is clearly that stock price at maturity of the option is likely to fluctuate significantly. But here he has a directional preference. In the case of straddle, he was neutral to the direction of the movement of the stock price. He carried no bias about the direction of price jumps of the underlying. He had given equal weightage to the increase/decrease in stock price. The probability of either was approximately equal.

For example, in the case of a strap, the investor has longed two calls and one put. So he is laying great emphasis on the stock price increasing in value rather than the contrary. His perception is clearly that there is a greater chance that the stock price would move up. Therefore, he desires to bet in favour of that perception and takes up a strategy such that he makes a larger profit per unit increase in the stock price beyond a threshold level (K). In this strategy, one-unit increase in the stock price realizes two units increase in profits if the stock price exceeds the threshold K. On the other hand, one-unit decrease in the stock price, in the region below K results in only one-unit increase in his profit if the stock moves in the downward direction. Thus, the investor has a biased perception and plays the market in line with his perception. His perception favours an upward bias i.e. he feels there are greater chances of the stock going up than down.

In the case of the strip, it is the other way round.

Now the important thing is, these are illustrative strategies where the investor takes positions in 2 calls and a put or vice versa. But there is nothing sacrosanct about these. One could as well formulate strategies with any number of calls & puts in order to make the strategy representative of one's perceptions. How many bets the investor is willing to take on his perception of particular directional movements of the stock price. Obviously, the possible profit is a function of these numbers but so is the cost of strategy setting.

Spread strategies

Spread strategies involve combination of long & short positions in options of the same type. Spreads are usually classified as:

(i) Vertical spreads
(ii) Horizontal spreads
(iii) Diagonal spreads

Vertical spreads

Vertical spreads comprise of long & short positions in options of same type on the same underlying with same maturity but with different exercise prices.

Horizontal spreads

Horizontal spreads comprise of long & short positions in options of same type on the same underlying with same exercise prices but different maturities.

Diagonal spreads

Diagonal spreads comprise of long & short positions in options of same type on the same underlying with different exercise prices but different maturities.

Common vertical spreads

(i) Bullish vertical spreads using calls
(ii) Bearish vertical spreads using calls
(iii) Bullish vertical spreads using puts
(iv) Bearish vertical spreads using puts
(v) Butterfly spreads with calls
(vi) Butterfly spreads with puts
(vii) Box spread

The collective definition of the spread strategy is that they contain long and short positions options of the same type on the same underlying. From the analysis perspective, the vertical spreads are the simplest to analyse, although the principles that are adopted in the case of vertical spreads carry forward to the analysis of the other types of spread strategies.

Bullish vertical spread

This strategy consists of buying a call option and selling a call option on the same underlying with same maturities but with different strike prices. The long call A is at the lower exercise price K_A and the short call is at the higher exercise price K_B. Both have the same maturity T.

Since the price of a call varies inversely as the exercise price and the vertical bull spread involves a long call at the lower strike (higher premium c_A) and a short call at the higher strike (lower premium c_B), it follows that the vertical bull spread entails an initial cash outflow (at t=0) to set up the strategy. Now, we look at the payoff. We have:

$\Pi_{bull\ spread}(S_T)=\max(0,S_T-K_A)-\max(0,S_T-K_B)=\max(0,S_T-K_A,K_B-K_A)$ and the profit

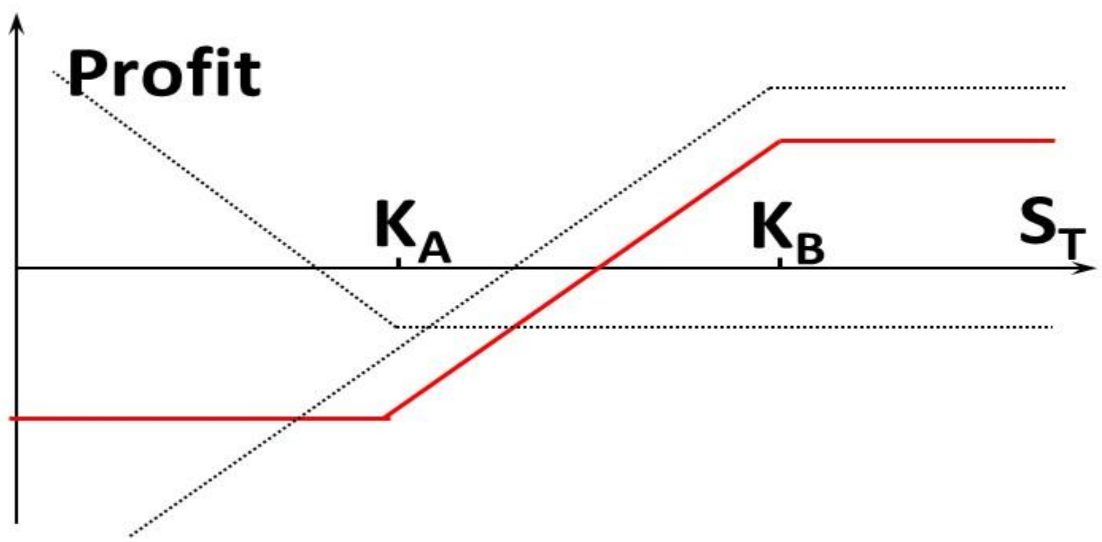
$\pi_{bull\ spread}(S_T)=\max(0,S_T-K_A)-\max(0,S_T-K_B)+c_B-c_A=\max(0,S_T-K_A,K_B-K_A)+c_B-c_A$

$$\pi_{BULLISH\ SPREAD\ WITH\ CALLS}=\pi_{LONG\ CALL\ A}+\pi_{SHORT\ CALL\ B}$$

$$=\begin{cases} c_B-c_A \quad if\ \ S_T<K_A<K_B \\ (S_T-K_A)+(c_B-c_A) \quad if\ \ K_A<S_T<K_B \\ (S_T-K_A)+(K_B-S_T)+(c_B-c_A)\ if\ \ K_A<K_B<S_T \end{cases}$$

$$=\begin{cases} c_B-c_A(<0\ ALWAYS)\ if\ \ S_T<K_A<K_B \\ (S_T-K_A)+(c_B-c_A) \quad if\ \ K_A<S_T<K_B \\ (K_B-K_A)+(c_B-c_A)(>0\ ALWAYS)\ if\ \ K_A<K_B<S_T \end{cases}$$

	t=0	t=T		
		$S_T<K_A$	$K_A<S_T<K_B$	$K_B<S_T$
LONG CALL A	$-c_A$	0	S_T-K_A	S_T-K_A
SHORT CALL B	c_B	0	0	$-(S_T-K_B)$
TOTAL	$(c_B-c_A)<0$	0	S_T-K_A	$K_B-K_A>0$



We can divide the entire spectrum of stock price into three segments viz.

$S_T<K_A<K_B$: Both calls will lapse;
$K_A<S_T<K_B$: Call A will be exercised and B will lapse;
$K_A<K_B<S_T$: Both calls will be exercised.

Let us discuss the payoff. We can split the entire stock price spectrum on maturity into the partitions $(0,K_A)$, (K_A,K_B) and (K_B+). Now, if the stock ends up at maturity below K_A, neither call gets exercised and there is zero payoff from the strategy. If S_T is between the two strikes, the long call gets exercised yielding a positive payoff of S_T-K_A. Obviously the party in favour of whom call B is written will not exercise that call because he is not get any payoff out of this since $S_T<K_B$. The payoff of the spread holder increases by one unit for every unit increase in stock price in this region. However, if the stock ends up beyond K_B, then call B will also be exercised. The holder of call B will find it profitable to exercise call B and he will earn a payoff S_T-K_B. Since, the spread holder is short in B, he has to pay this amount to holder of B as the option writer. Now, if stock price goes beyond K_B, call A will continue to be exercised because $K_B>K_A$ so that $S_T>K_B$ implies $S_T>K_A$. Hence, he pays out this amount S_T-K_B out of his payin from option A of S_T-K_A. He, therefore, ends up with a net inflow of $K_B-K_A>0$ if the stock price at maturity exceeds K_B.

Thus, we find that as the value of S_T increases (so long that $K_A<S_T<K_B$), the payoff from the strategy also increases (in fact, by the same amount). Therefore, this strategy is called bullish vertical spread. The investor who is setting up this strategy will make a profit if the stock price goes up (so long that $K_A<S_T<K_B$).

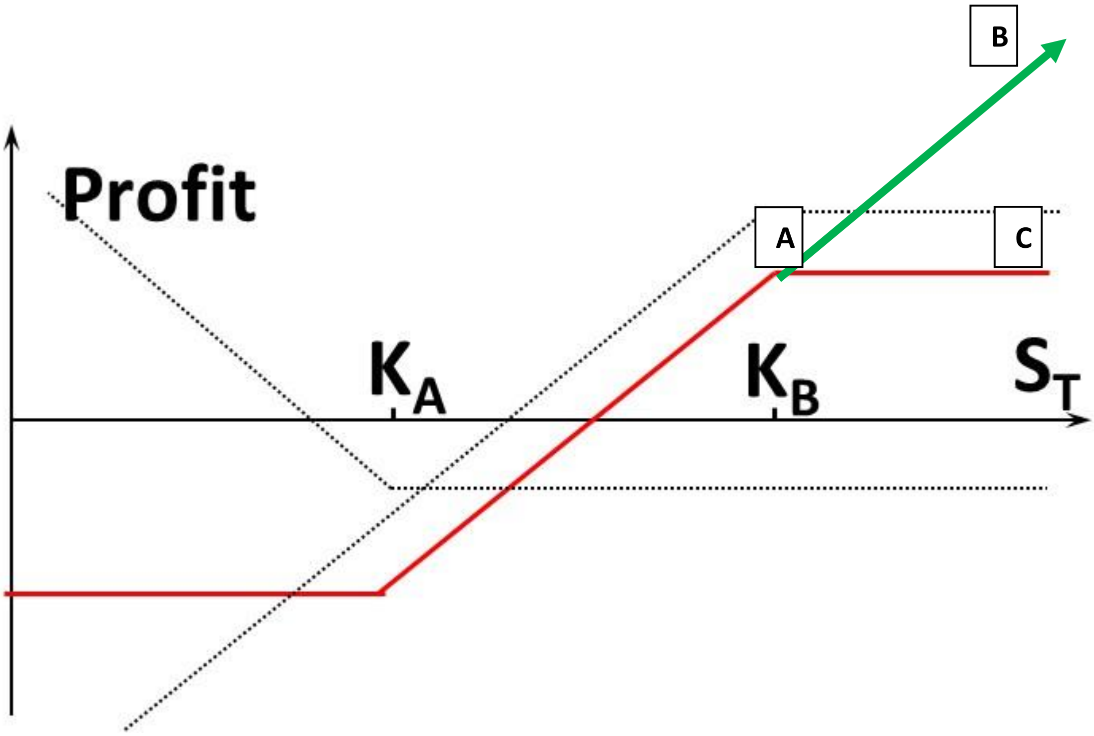
The maximum payoff is K_B-K_A, which is clearly positive since $K_B>K_A$ by choice. We also have, because of the inverse relation between call strikes and premia $c_B<c_A$ so that there is a cash outflow for setting up this strategy. Interestingly, from put-call parity for the two options:

$c_A+K_Ae^{-rT}=p_A+S_0$, $c_B+K_Be^{-rT}=p_B+S_0$ so that $(K_B-K_A)e^{-rT}-(c_A-c_B)=p_B-p_A\geq 0$ (since the premia of puts varies directly with the strike price) whence $(K_B-K_A)e^{-rT}\geq(c_A-c_B)$. In the event that time value of money is ignored: $(K_B-K_A)\geq(c_A-c_B)$ or $(K_B-K_A)-(c_A-c_B)\geq 0$. Thus, the maximum profit on the strategy exceeds the cost of setting it up, which is justified since there are stock prices where the profit is zero and lesser than the cost.

Further, it can be zero only if both puts are well out of the money i.e. both calls are well in the money.

Vertical bull spread vs long call

This is an interesting question. The question is the issue of choice between a long call and the vertical bull spread. Why would an investor not take up a long call and prefer the vertical bull spread? In the long call, upside profit is unbounded. In the bull spread, the upside profit is bounded by the difference of the strike prices (red line AC). Thus, what makes an investor sacrifice the region of profit indicated by the green line AB.



The answer is that the investor sacrifices this AB of profit because he perceives that the stock price is not likely to go beyond this value A at option maturity. Now, if the stock price is not likely to go beyond A as per the investor's perception, it means the investor does not expect to realize the profit along AB. Thus, this additional profit AB carries little expected value for the investor. So what does he do?

If some other market player feels that stock price could well go up beyond A and is willing to bet on it, he would be willing to buy a call with a strike at point A i.e. K_B. This is because, once the stock price exceeds K_B (which this other market player feels possible and, indeed probable) this market player could earn profit S_T-K_B by exercising this call. ***Thus, while on the one hand, our investor would be willing to write a call at K_B because he does not expect the stock price to reach K_B and hence does not expect the call to be exercised. On the other hand, this market player would be willing to buy this call at K_B, because he does expect the stock price to exceed K_B.***

Investing as per their respective perceptions, the original investor writes and the market player buys the call at K_B. The original investor gets the premium which reduces his investment cost while retaining the profit expectancy as per his perceptions. The market player also increases his profit expectancy since he expects the stock price to reach and exceed K_B whence he starts getting positive payoff on the call.

So that is the reason that these spread strategies become popular. It is all in the interplay of perceptions of various investors. How they feel about the stock price behaviour at the time of maturity of these options.

Of course, if the investor's perception turns out to be wrong and the maturity stock price exceeds K_B, he loses out on the extra profit along AB.