Financial Derivatives and Risk Management
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Lecture 30: Options: Basic Theory

Terminology in context of option contracts

European options vs American options

European options are exercisable at a particular point in time called the option maturity or the exercise date. American options are exercisable within a period, up to a particular point in time called the option maturity.

Underlying asset

The asset that forms the substratum of the options contract i.e. that can be bought (call) or sold (put) at the predetermined contracted price on the maturity date is called the underlying asset. The option contract is said to be written on the underlying asset.

Strike price or exercise price

The strike/exercise price is the price at which the call holder is entitled to purchase one unit of the underlying asset under the option contract. This price is predetermined and is an integral constituent of the option contract. It is contained in the option contract, fixed at the time of the negotiation and finalization of the contract and cannot thereafter be varied over the life of the option. To that extent, it is intrinsic to a particular option contract.

Maturity/exercise date

The maturity or exercise date of an option is, in the case of a European option, the date on which the option can be exercised by the option holder i.e. the date on which the right embedded in the option becomes exercisable and, in the case of an American option, the date upto which the option can be exercised. Important to emphasize here that American options can be exercised at any time during a time span whereas European options can be exercised only at a specific maturity date.

Option holder and option writer

The option holder is the party to the option contract who has acquired the right in the option contract i.e. who is entitled to exercise the right embedded in the option contract. The option writer is the other party to the option contract. The option writer has no right under the option contract. He has an obligation. He has the obligation to meet his commitment under the option contract, if the option holder exercises the option. For example, if the call holder exercises the call, the call writer is obligated to sell the underlying asset at the exercise price to the option buyer for the exercise price.

Of course, if the option holder lets the option lapse, then the writer goes scot free and pockets the premium that he received for writing the option from the option buyer.

The option holder is also said to be long in the option and the writer, short in the option.

Intrinsic value of options

The intrinsic value of an option is its value assuming it were exercised immediately. Equivalently, intrinsic value of an option is the amount by which it is in the money. The intrinsic value of an out of the money option is zero.

(i) For a call option $IV_{call}(t) = \max(0, S_t - K)$
(ii) For a put option $IV_{put}(t) = \max(0, K - S_t)$

The intrinsic value of the option relates to the moneyness of the option. In other words, it is the payoff that could possibly be earned if the option was exercised forthwith. If it is a call option and the current underlying price S_t exceeds the strike price K, then one can buy the asset at the exercise price K under the option and sell it in the market at S_t, thereby earning a payoff of S_t-K which, then constitutes the intrinsic value of the option. Similarly, the intrinsic value of a put is K-S_t if $S_t<K$. However, since the worth of a right can never be negative and the holding of an option confers a right, the intrinsic value is invariable bounded from below by 0. Thus, $IV_{long\ call}=\max(S_t-K,0)$ etc.

Time value of option

The time value of an option is the total value of the option, less the intrinsic value.

Time value= MV –IV

Time value arises from (i) the uncertainty of future price movements of the underlying and (ii) the unwinding of the discount rate between now and the expiry date.

The second component of the value of the option is called time value of the option.

It may so happen that at a particular point in time, $S_t<K$ so that the call option may have no intrinsic value. So, one would not exercise the option. However, that does not preclude the situation that as on maturity of the option at i.e. t=T, the option ends up in the money. There is always a probability that on maturity of the option, we may have $S_T>K$ notwithstanding that at an earlier date $S_t<K$. Howsoever small this probability may be, it enjoys a certain price. This chance of earning a profit on call expiry is worth some market value and this market value arises from the fact that the underlying evolves as a random process, a stochastic process, in an unpredictable manner. It could therefore end up in excess of K whence the call will yield profits on exercise. The discounted expected value of this profit will add to and be embedded in the option price as assessed by the collective wisdom of the market. This gives rise to time value of the option. So time value of the

option arises from the unpredictability of the price of the underlying over the remaining life of this option contract. Same logic holds for put options.

In the case of a European option, because the option cannot be exercised before the expiry date, so it is possible for the time value to be negative. However, if the time value of an American option is negative, it is logical to exercise it forthwith.

Moneyness of options

Moneyness of an option is the relative position of the current price S_T of the underlying asset vis a vis the exercise price K.

In particular, if the option would have positive intrinsic value if it were to expiring at the current underlying price, it is said to be in the money. If it would have zero intrinsic value if expiring at the current underlying price, it is said to be out of the money. If the current underlying price and strike price are equal, it is said to be at the money. The following therefore follows:

Condition	**Call**	**Put**
$S_t > K$	**In the money**	**Out of money**
$S_t < K$	**Out of money**	**In the money**
$S_t = K$	**At the money**	**At the money**

Thus, if the current underlying price exceeds the strike price, a call option is said to be in the money, whereas a put option will be out of the money. Because if $S_t > K$, the call will yield a positive payoff if exercised forthwith, while a put will yield a zero payoff. The converse is the case if $S_t < K$, then the call option goes out of the money and the put option comes into the money.

Moneyness is often measured in terms of the ratio of spot price of underlying to the option's strike. or its reciprocal, depending on convention.

Notation

c: European call option price
p: European put option price
S_0: Stock/underlying price today (t=0)
S_t: Stock/underlying price at arbitrary time t
K: Strike price
T: Life of option
σ: Volatility of stock price
C: American call option price
P: American put option price
S_T: Stock price at option maturity (t=T)

D_0: Present value of dividends paid during life of option
D_T: Future value of dividends paid during life of option at option maturity
r: Risk-free rate for maturity T with cont. comp.

The long European call

A long European call is the position of the party who is the holder of the European call option. A European call option is a contract whereby one party i.e. the party who has bought the option (the option holder) has the right under the contract to buy from the other party to the contract (the option writer), the underlying asset (S) at a certain predetermined price (the strike price K, also specified in the contract) at a particular day (maturity T, also specified in the contract).

Now, let us say that on the maturity of the call T, the market price of this of the underlying asset is S_T. Now, there are two possibilities, (i) $S_T \leq K$; or (ii) $S_T > K$.

(i) If $S_T \leq K$ i.e. the market price on the maturity of the call is lower than the strike price K, the call holder will obviously NOT exercise the call, for it would be more economical for him to buy the asset from the market at a price that is lower than the price at which he can acquire the same asset under the call. Therefore, in such a situation, the call will lapse unexercised and the payoff from it will be 0. This is because the option holder has the discretion and not the obligation to exercise the call, he may buy by exercising the call or he may let the call lapse.

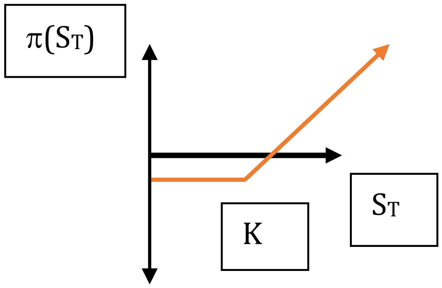
(ii) If $S_T > K$, on the other hand i.e. the market price is higher than the strike price or the price at which the underlying can be bought by exercising the call, the option holder will exercise the call, buy the asset under the call at K and then sell it in the market at S_T, thereby achieving a payoff of S_T-K.

Thus, the payoff from the call is 0 if $S_T \leq K$ and S_T-K, if $S_T > K$ i.e. $\Pi_{\text{long call}}(S_T)=\max(0,S_T-K)$. Because the cost of acquisition of the call is already incurred at t=0, the profit on a long call is $\pi_{\text{long call}}(S_T)=\max(0,S_T-K)-c$.

Of course, we are ignoring time value of money here. Actually, the cash outflow on account of premium occurs earlier at the time of buying the call i.e. t=0, while the payoff occurs at maturity i.e. t=T. So, ideally we should take the future value of c whence the profit function is $\pi_{\text{long call}}(S_T)=\max(0,S_T-K)-ce^{rT}$.

$$\pi_{long\ call} = \begin{cases} -c \ if \ S_T \leq K \\ \left(S_T - K\right) - c \ if \ S_T \geq K \end{cases}$$

$$\pi_{long\ call}^{\max} = \infty; \quad \pi_{long\ \ call}^{\min} = -c; \quad S_T^{bep} = K + c$$



It is very clear from the payoff function that because S_T is unbounded, the payoff & the profit are also unbounded. The loss is confined to the amount of premium paid upfront for buying the call. For the breakeven point, we have $0=\pi_{long\ call}(S_T)=\max(0,S_T-K)-c$ or $S_{BEP}=K+c$. The diagram is presented here. Up to the point $S_T=K$, because the option will not be exercised, the profit will be -c. Once $S_T>K$, the call would be exercised and profit will start accruing. Furthermore, for $S_T>K$, $\pi_{long\ call}(S_T)=S_T-K-c$ i.e. the coefficient of S_T is unity, the slope of the profit line is 45 degrees. Therefore, for unit increase in S_T, there's a unit increase in the profit as well.

Now, it is very clear from the above discussion that if the price of the underlying goes up above the threshold K, the call holder makes a profit and if the price goes down below K, he does not make a profit. Thus, an investor opting for a long call is clearly bullish about the underlying asset. In other words, he expects the price of the underlying to rise by the time the call reaches maturity. Higher the price of the underlying asset, higher is the investor's profit. Therefore, he is strongly bullish about the prospects of the underlying asset. If his perception turns out to be wrong, then he loses out on the option premium.

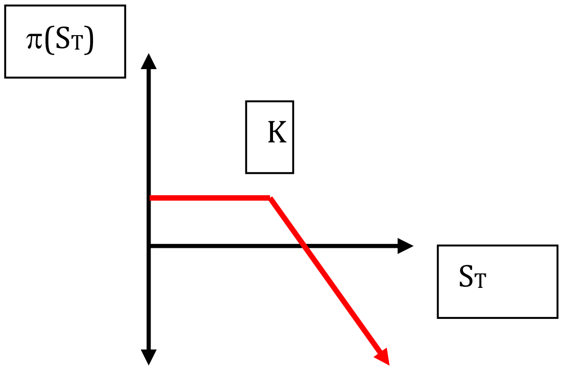
Short European call

The other party to a long call position is said to be short in the call. Thus, while the long call position is the option holder, the short call position is the option writer.

Consider the case, now, of the party who has written the call option. The call contract is a zero sum game. In other words, the payoff of the long party is the mirror image of the payoff of the short party. Thus, $\Pi_{short\ call}(S_T)=-\max(0,S_T-K)=\min(0,K-S_T)$. Because the short party has already received the call premium at t=0, the profit on a short call is $\pi_{short\ call}(S_T)=\min(0,K-S_T)+c$.

$$\pi_{short\ call}=\begin{cases}+c\ if\ S_T\le K\\(K-S_T)+c\ if\ S_T\ge K\end{cases}$$

$$\pi^{\max}_{short\ call}=+c;\ \pi^{\min}_{short\ call}=-\infty;\ S_T^{bep}=K+c$$



If S_T<K, then the long party does not exercise the call. The short party goes scot free and retains the premium received at the time of writing the call. In other words, his profit is +c and his payoff is obviously zero, because the option is not exercised by the other (long) party.

If S_T>K, then he has to honor his obligation because the long party exercises the call. The long party makes a profit by buying the asset from the short party at K and selling it in the market at S_T (S_T>K). The short party has to deliver the asset worth S_T for a lower price K and, thus incurs a negative payoff of K-S_T. Of course, he has got the premium. So, he gets a net profit of K-S_T+c.

The critical parameters of this strategy are: maximum profit is +c, maximum loss is unbounded and the breakeven underlying price is K+c.

<u>Relation between strike prices and premia</u>

Consider two calls A & B with strikes K_A & K_B ($K_A<K_B$) on the same underlying S with the same maturity T priced at c_A & c_B respectively. Thus, A gives the holder the right to buy one unit of S from the writer at T by paying K_A. B gives the same right but by paying a higher price K_B. Recall $K_A<K_B$. It clearly follows that one unit of S can be acquired under A by paying a smaller price than it can be under B. Thus, call A entitles the holder to buy the underlying at a lower price, call B at a higher price. Therefore, call A will naturally be more valuable than call B and command a higher price in the market i.e. we have $K_A<K_B \Rightarrow c_A>c_B$. Therefore, there is an inverse relation between the strike price and the premium or market price for calls.

The converse is the case of a put option, although with the same rationale since puts provide right to sell at the strike prices. For puts, $K_A<K_B \Rightarrow p_A<p_B$. Therefore, there is a direct relation between the strike price and the premium or market price for puts.

<u>The long European put</u>

A long European put is the position of the party who is the holder of the European put option. A European put option is a contract whereby one party i.e. the party who has bought the option (the option holder) has the right under the contract to sell to the other party to the contract (the option writer), the underlying asset (S) at a certain predetermined price (the strike price K, also specified in the contract) at a particular day (maturity T, also specified in the contract).

Now, let us say that on the maturity of the put T, the market price of this of the underlying asset is S_T. Now, there are two possibilities, (i) $S_T \leq K$; or (ii) $S_T > K$.

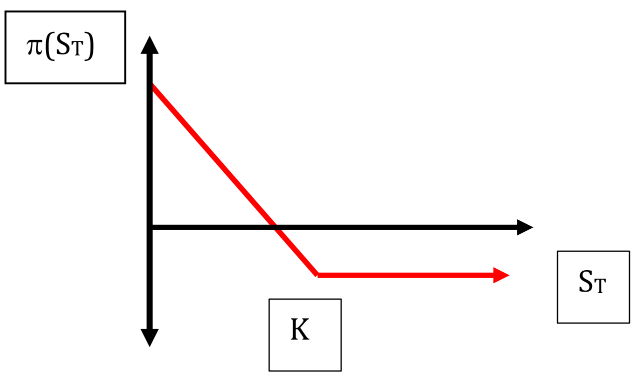
(i) If $S_T \leq K$ i.e. the market price on the maturity of the put is lower than the strike price K, the put holder will obviously exercise the put, for he would derive a positive payoff therefrom. Because the market price is lower than the strike price or the price at which the underlying can be sold by exercising the put, the option holder will buy the underlying from the market at S_T, exercise the put and sell the asset under the put at K thereby achieving a positive payoff of $K-S_T$.

(ii) If $S_T > K$, it would be more economical for the put holder to sell the asset in the market at a price that is higher than the price at which he can sell the same asset under the put. Therefore, in such a situation, the put will lapse unexercised and the payoff from it will be 0. This is because the option holder has the discretion and not the obligation to exercise the put, he may sell by exercising the put or he may let the put lapse.

Thus, the payoff from the put is $K-S_T$ if $S_T<K$ and 0, if $S_T \geq K$ i.e. $\Pi_{long\ put}(S_T)=\max(K-S_T,0)$. Because the cost of acquisition of the put is already incurred at t=0, the profit on a long put is $\pi_{long\ put}(S_T)=\max(K-S_T,0)-p$.

$$\pi_{long\ put} = \begin{cases} (K-S_T)-p \text{ if } S_T < K \\ -p \text{ if } S_T \geq K \end{cases}$$

$$\pi_{long\ put}^{\max} = K - p;\ \ \pi_{long\ put}^{\min} = -p;\ \ S_T^{bep} = K - p$$



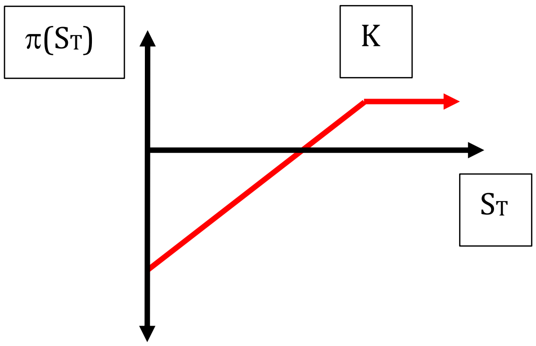
Because S_T cannot take negative values, the maximum payoff & profit are both bounded. The loss is confined to the amount of premium paid upfront for buying the put. For the breakeven point, we have $0=\pi_{long\ put}(S_T)=\max(K-S_T,0)-p$ or $S_{BEP}=K-p$. The diagram is presented here. For all values $S_T<K$, upto the point $S_T=K$, because the put will be exercised, $\pi_{long\ put}(S_T<K)=K-S_T-p$, the profit will accrue @ one unit for every unit decrease in the underlying price. Once $S_T>K$, the put would not be exercised and the loss will be confined to the put premium paid.

Clearly, the profit on the long put increases with fall in the price of the underlying, once it falls below the threshold K. Further, for maturity underlying price above K, the put ends worthless. Thus, an investor opting for a long put is clearly bearish about the underlying asset. In other words, he expects the price of the underlying to fall by the time the put reaches maturity to below the strike price. Further, lower the price of the underlying asset, higher is the investor's profit. Therefore, he is strongly bearish about the prospects of the underlying asset. If his perception turns out to be wrong, then he loses out on the option premium.

Short European put

$$\pi_{short\ put} = \begin{cases} -(K - S_T) + p \ if \ S_T < K \\ +p \ if \ S_T \geq K \end{cases}$$

$$\pi_{short\ put}^{\max} = p;\ \pi_{short\ put}^{\min} = -(K-p);\ S_T^{bep} = K - p$$



In the short put, the maximum profit is going to be confined only to the amount of premium on the short put, the premium that he receives on the put. Clearly, the put writer's perception would be that he is not optimistic about the underlying's price going down. But he is not overtly optimistic of the price going up either. Had he been highly bullish, he would rather have preferred a long call strategy than writing this put.

Similarly, if you recall the short call situation, the perception of the call writer is clearly being bearish about the underlying's prospects. A maturity price of the underlying below the strike price would enable the call writer to pocket the call premium. But again, this bearish perception is not overwhelmingly enough to induce the call writer to opt for an active long put strategy.