Financial Derivatives and Risk Management
Professor J.P.Singh
Department of Management Studies
Indian Institute of Technology Roorkee
Lecture 03: Forwards: Pricing & Arbitrage

Quick Recap

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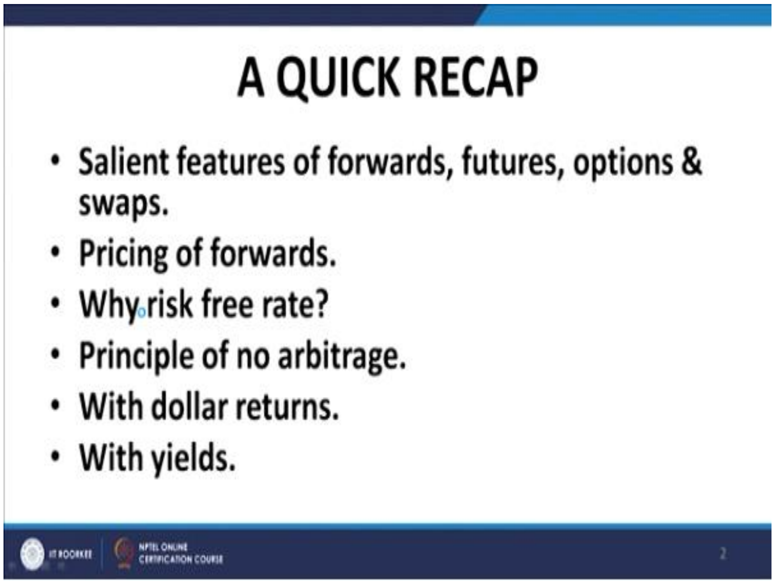

Welcome back, I shall continue from where I left off last time, but before that a brief recapitulation of what I discussed in the last class. In the last class I started with the introduction of the salient features of various derivatives: forwards, futures, options and swaps.

Forwards are essentially contracts which are entered into as of today, the negotiation, the price and other terms of delivery are fixed as of today, agreed as of today and the actual settlement of the contract that is the payment of the price and the receipt of the asset is at a future date.

In the case of futures, it is very similar to forwards except for the fact that futures are traded on recognized exchanges and because they are traded on recognized exchanges, it becomes necessary that they be standardized and the risk element be eliminated. The chances of default have to be eliminated so that the trading is uninhibited.

Options are slightly different from forwards and futures, in the sense that one of the parties to the option contract has a right, the discretion, to exercise the option or not to exercise the option i.e. to let the option lapse. The other party to the contract, however, is obliged, is mandated to honor his leg of the commitment, he has no discretion.

Swaps are contracts which involve an exchange of cash flows over a future period of time in installments as per a pre agreed formula, which is incorporated, which is embedded, in the swap contract.

So, these are the four fundamental types of derivatives, that are commonly available in the market. There are numerous variants of these derivatives, there are futures options, swaptions and variants of those, even options are of multiple varieties.

Then I went on to pricing of forward contracts. Here I invoked the concept of no-arbitrage,

I also discussed the relevance or the appropriateness of the risk free rate as the measure of time value of money in the context of pricing of forward contracts. The issue here is that at what point the time value of money or the interest rate comes in to the picture. It is relevant in context of the borrowing that we make at t=0 to buy the underlying asset in the spot market. We, then, keep the asset with us and thereafter, deliver it against the short forward position.

Now, at this point, we are assuming that the forward is default free. In other words, the cash proceeds that we are going to receive under the forward contract (on delivering the asset that **we already have with us**) are certain. Thus, the delivery of the asset under the forward contract is certain, as we already have the asset with us and the receipt of the cash against this delivery is also certain as the forward is assumed default-free. Since, we shall get the cash with certainty, it is natural to presume that the cash will be utilized for repayment of the borrowings that we made for buying the asset in the spot market. This is the reason why we use risk free rate as a benchmark for the pricing of forward contracts.

Then I discussed the pricing of forward contracts when we have dollar returns, that is returns in terms of amounts of currency, returns in terms of money. In this case, the mere fact of carrying of the asset which we purchased from the spot market results in a realization of certain amount of cash, as a results of which, the amount that we need to borrow to start with to buy the asset comes down by the present value of that amount.

In other words, the amount that we are going to receive as dividends during the life of the forward contract by holding the asset that we acquired in the spot market can be used to repay a part of the loan proceeds that we borrow for buying the asset in the spot market.

As a result of this, the effective cost of holding the asset when we purchase the asset from the spot market comes to S_0-D_0, where D_0 is the present value of the dividends that you receive during the life of the forward contracts by holding the underlying asset during that period.

Then I also discussed the issue of pricing of the forward contract, when instead of a dollar amount, instead of a rupee amount or a cash amount, we have a return on the asset that we acquired in the spot market, in terms of a percentage yield. In that case, when we need one unit of asset for delivery under the short forward position, we actually need to acquire less than that one unit by a factor of exp(-qT) because exp(-qT) number of units of asset at t=0 will grow to one unit at t=T if the continuously compounded yield rate is q..

For example, if you need 1 dollar to fulfil your short obligation under the forward contract for selling 1 dollar, then you do not need to buy 1 dollar in the spot market today, you need to buy somewhat less than 1 dollar i.e. exp(-qT) dollars, because that amount when invested in a dollar deposit will give you 1 dollar on maturity of the forward contract.

Forward pricing with carrying costs

Now, let us look at the other side of things, we have talked about pricing of forwards on assets that generate income during the life of the forward contracts. By buying the asset spot and holding the spot asset, you get a dollar return or you have a yield in terms of percentages.

We now examine the case when buying the asset spot and holding it entails a cost. For example, let us take the case of wheat, you buy wheat as the underlying commodity in the spot market. To store wheat, to be delivered under a forward contract, you have to keep it in a godown, for which you have to pay a certain amount of rent. Similarly, if the underlying is gold you would like to get it insured which may entail an upfront premium for the insurance.

So, for certain assets, when you buy the assets in the spot market and hold them for delivery against the short forward position, some carrying cost may need to be incurred. The treatment, in this case, is absolutely parallel to what we did in the case of income. The only difference is, where we are taking the dividend during the life of the forward contract as an inflow, this carrying cost is an expenditure and hence, an outflow. And therefore the present value of this carrying cost will be added to the spot price instead of being deducted.

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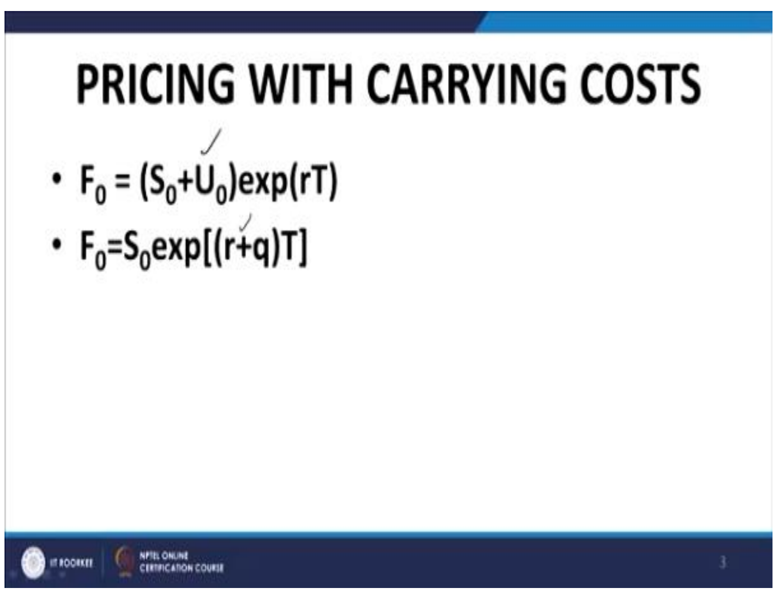

In the case of dividend, we deduct the present value of dividend D_0 from the spot price S_0. In this case, the present value of the carrying costs U_0 will be added to S_0.

And similar is a situation where, there is a loss, for example, due to evaporation of a formulation such that the instantaneous rate of evaporation is proportional to the instantaneous quantity or mass. in that case instead of the no arbitrage forward price being $F_0=S_0\exp[(r - q)T]$, it would be $F_0=S_0\exp[(r + q)T]$.

The rest of the reasoning is absolutely parallel, the process of no-arbitrage is absolutely similar.

Arbitrage, Risk & Return

Let us now go back to, revisit the concept of arbitrage, I want to take this up in greater detail because arbitrage forms the corner stone of the pricing theory in modern finance, no-arbitrage constitutes the fundamental postulate on the basis of which literally all pricing is done in modern finance.

The essence of arbitrary pricing is to construct two portfolios, the two portfolios have identical payoffs at a particular instant of time. They have identical risk characteristics and then because we know the price of one portfolio we argue that because the payoffs are same, the risk profile is same, therefore, the cost of the portfolios must also be the same. This is the fundamental principle of arbitrage but we need to look at it a bit more closely.

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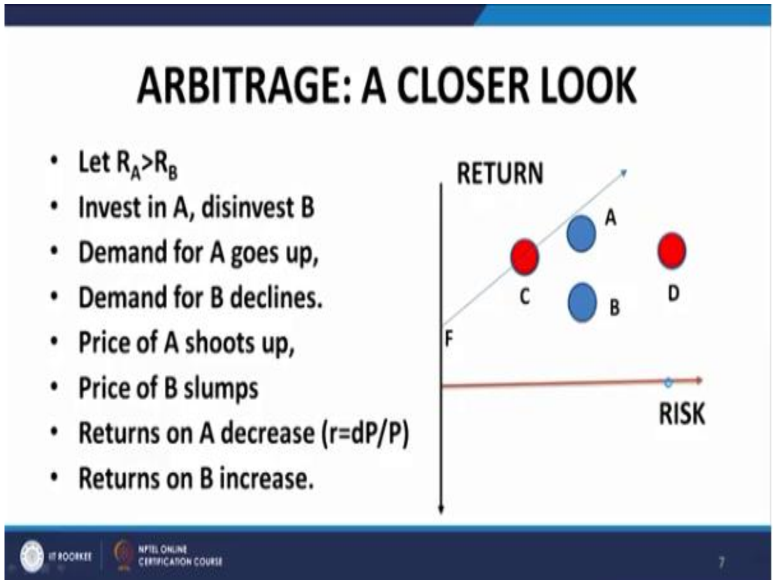


Let us look at the above diagram. Whenever we talk about analysis in finance, it is very convenient to do the analysis in two dimensional space, risk return space. Normally, we have risk along the X axis and return along the Y axis. We leave aside the issue of measurement of risk and return for the moment and assume that there exist unique unambiguous measures of risk and return that are acceptable to and employed by all market participants.

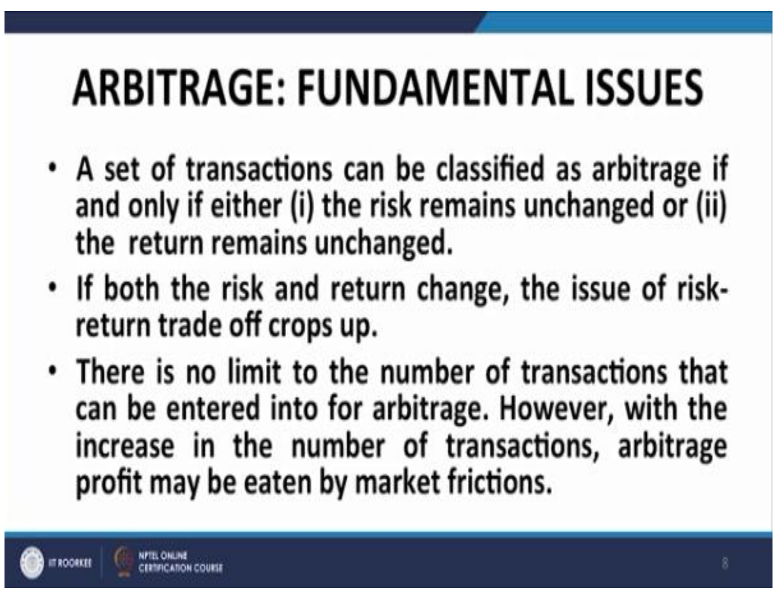
Consider asset A and B, asset A has a return of R_A, asset B has a return of R_B. It is very clear that $R_A > R_B$ but the risk of both assets is the same. Now, in this situation when you are defining an asset by only two characteristics viz. risk and return and one of the characteristic is identical, then it is the other characteristic that solely determines its market features. Here, the risk of A & B is the same and, thus, gets factored out $R_A > R_B$. It is this return differential (without any corresponding risk differential) that can be traded on to earn arbitrage profits. If the prices of A & B are the same to start with, then an arbitrageur can construct a riskless portfolio consisting of A long and B short and thereby earn a sure arbitrage profit. This is because A having a higher return than B shall yield a higher price at the end of the holding period compared to B.

This kind of situation cannot persist indefinitely in any market and, in fact, greater the market efficiency, greater is the rapidity with which this arbitrage profit dissipates and, in equilibrium, the prices of A & B so adjust themselves that the returns equalize (the arbitrage opportunities cause an increase in demand for A and a fall in demand for B which causes the price of A to rise and that of B to fall, resulting in reduction of returns on A and increase in returns on B. This process of realignment will continue until the returns converge).

The same will be the situation between C and D because they have the same return for different levels of risk. Again we can extract riskless profit by constructing a portfolio short in D and long in C and an asset E (that has some risk ($Risk_E > Risk_C$) and has a return greater that $R_C=R_D$ i.e. $R_E>R_C=R_D$). We adjust the composition of portfolio such that (i) there is no net investment in the portfolio i.e. the proceeds received from shorting D are exactly invested in C+E) and (ii) the $Risk_C+Risk_E = Risk_D$. Since C+E are long and D is short, the overall risk of the portfolio is zero. But as, $R_E>R_C=R_D$ and we are long in E, the portfolio will generate an overall positive return which can, thus, be arbitraged because it is riskfree and does not entail any initial investment.

So the bottomline is that a set of transactions would be classified as arbitrage if it involves two assets have the same risk but differential returns or vice versa. Can we have arbitrage between two assets that have differential expected returns accompanied by differential risks is a non-trivial question. Why? For this we need to delve deeper into investment theory. Let us assume that we have an investment horizon of one year (for simplicity) and are considering investing in an asset W. The price at which the asset is being traded today $P_{w,0}$ is, obviously known to us. We try to build up, on certain rational basis (e.g. economy, industry, company analysis), the spectrum of possible values (prices $P_{w,1}$) that the asset W could take at the end of our investment horizon i.e. at t=1 year from now. Because there would be several possible values for $P_{W,1}$ i.e. we have a spectrum of possible values for this $P_{W,1}$ and we do not know for certainty which value $P_{W,1}$ is going to take, we try to work out the respective probabilities for each projected value of $P_{W,1}$ i.e. we obtain the probability distribution for $P_{W,}$. Hence, we can work out the expected price $E(P_{W,1})$ and, therefore, the expected return. Since, the uncertainty in the attainment of this price or any target price is related to the amplitude of fluctuations, we use some direct or indirect measure of dispersion of the spectrum of values of $P_{W,1}$ around a target value as a measure of risk. Notwithstanding the above simplistic explanation, the manner in which return and risk are actually measured are non-trivial issues, sufficiently technical and ambiguous to warrant an independent elucidation. I shall, therefore sidestep these issues of risk and return measurement for the moment.

Having explained the technicalities of arbitrage, I come back to the main point. To explain the nuances, I consider assets A & C. A has higher risk and higher expected return than C. In order to examine the possibility of arbitrage between A & C, we need some mechanism, some risk-return trade-off function by which we can compare which of the two is better. Now, in the earlier cases, we had situations, where one variable was neutralized being equal for both securities, so uniletral comparison of the other variable facilitated the choice. However, here we have two variables and both are different. Hence, we need a functional relationship (a risk-return trade-off) by which we can move between the two variables (risk & expected return). It is only then that we can compare the two assets A & C. Now, there is no simple way of mathematically representing this trade-off function. There are different approaches based on varying premises. A universal unequivocal mechanism does not exist. Hence, we do not classify such transactions as arbitrage. (Refer Slide Time: 22:21)



Now, if somebody is going to take and implement arbitrage, he is using his own money. Thus, there is no restriction as such on the various methods or the various steps or various levels at which he can do arbitrage.

For example, let us say you start with INR, convert spot to USD, invest USD for 3 months in USD deposit, take a 3m short forward position in USD against GBP, borrow GBP against the short USD forward vs GBP and sell GBP spot to get INR.

So you can have n transactions, you can also simply play the spot market: Convert INR to USD, USD to CAD, CAD to GBP and then GBP back to INR, if you can make a profit out of this. You can play spot vs forward markets, like the above example. But the only issue is, as the number of steps, the number of transactions in the arbitrage cycle increases because of market frictions at every level, the chance of profit arising out of the entire cycle diminishes. This is so because market frictions are going to eat into that arbitrage profit, more transaction costs, brokers' commission, bid-ask spreads, borrowing and lending spreads etc. There are so many market frictions in practice and all these things would aggregate in eating away the arbitrage profit, as the number of steps, as the number of transactions in the arbitrage cycle increases. Theoretically, hypothetically, there is no restriction to that, you just have to identify the arbitrage opportunity and take advantage of that.

Arbitrage: Some Examples

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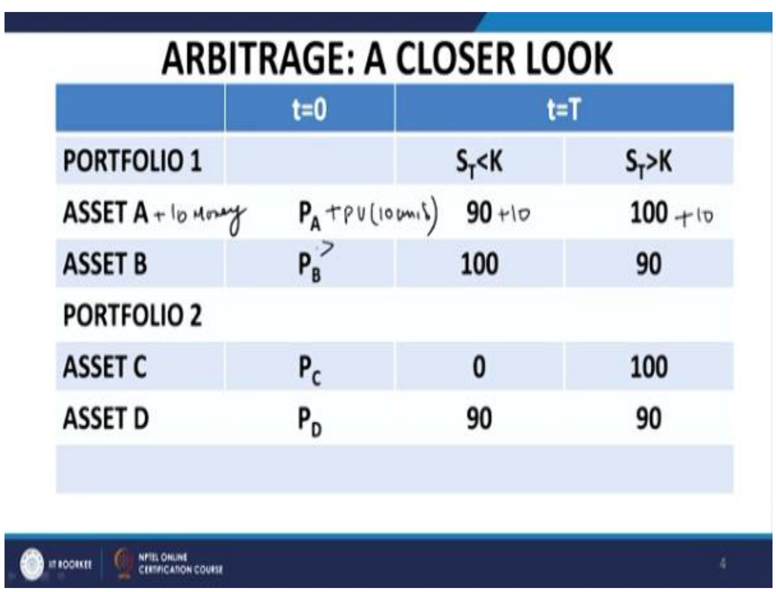

Now, I have taken a very simple case, where we have just the 2 states of nature at the end of our investment horizon. Let us say that the value of an index S_T can be less than a predetermined value K, or it can be greater than K. These are just the 2 possible situations. K is partitioning the entire spectrum of index values into 2 parts i.e. $S_T<K$ and $S_T>K$.

Let us look at the first portfolio, Portfolio 1, we have got 2 assets, Asset A and Asset B whose payoff at t=T are functions of S_T.

Asset A: $f_A(S_T<K)=90$; $f_A(S_T>K)=100$; Asset B: $f_B(S_T<K)=100$; $f_A(S_T>K)=90$

The issue is: Can we compare A & B purely on this information?

We cannot. Why? Because we do not know the relative chance, the relative probabilities of S_T finishing less than K and of S_T finishing greater than K and hence, of the relative realizabilities of the payoffs of 90 and 100 for asset A and of 100 and 90 for asset B. Hence, we cannot prime facie say which is better. To illustrate, let us assume that $Prob(S_T<K)=0.99$ and $P(S_T>K)=0.01$. In that case, because B's payoff is more in the region $S_T<K$, it would be preferred.

The story does not end there. Suppose we add a constant payoff of 10 to asset A. Now, the situation is:

Asset (A+10): $f_{A+10}(S_T<K)=100$; $f_{A+10}(S_T>K)=110$; Asset B: $f_B(S_T<K)=100$; $f_A(S_T>K)=90$

It is, now, clear that the payoff of asset (A+10) ≥ the payoff of asset B under both scenarios. Hence, the price of (A+10) must be greater than B and we get the following arbitrage condition:

Price A+PV(10) >Price B. Similarly, we have: Price B+PV(10) > Price A.

Now, consider portfolio 2.

Asset C: $f_C(S_T<K)=0$; $f_C(S_T>K)=100$; Asset D: $f_D(S_T<K)=90$; $f_D(S_T>K)=90$

Here, the aforesaid argument is amplified even further. Although asset D has a total payoff of 180 units, while asset C has a total payoff of only 100 units, we still cannot prime facie decide for one. What if Prob(S_T<K)=0.00001 and P(S_T>K)=0.99999.

Now, this becomes intriguing. D gives a payoff of 90 to 0 of C, but the chance of this happening is 0.00001. C gives a payoff of 100 to 90 of D which is almost certain i.e. with probability 0.99999. The choice is certainly not simple. Let us work out the expected value of the payoffs in this case. We have, $E(f_c)$=99.999; $E(f_D)$=90.00. Hence, if we use expected value as the decision variable, the choice would be C rather than D. Seems paradoxical.

Therefore, the probability distribution of the index would mean that this 100 is more valuable to me than this 90 which is available in asset D, therefore, probably I may think of paying more for asset C.

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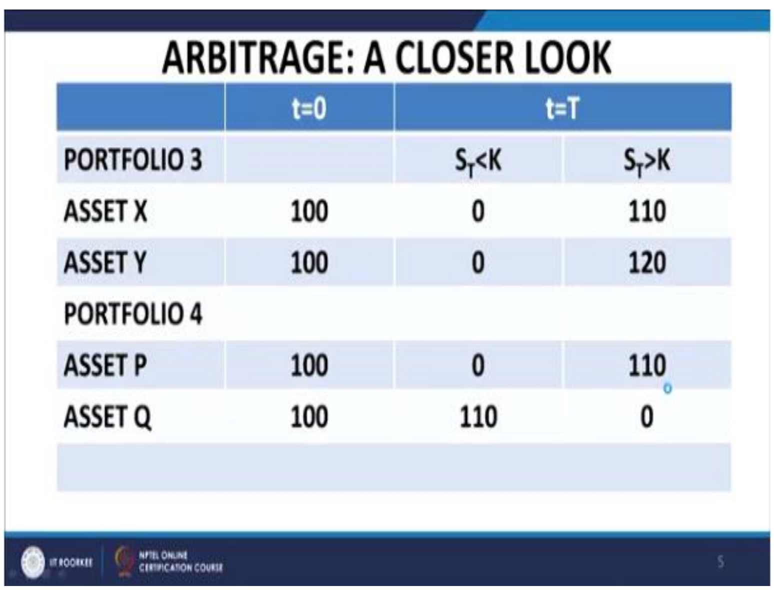

Portfolio 3 portrays a different scenario.

Asset X: $f_X(S_T<K)=0$; $f_X(S_T>K)=110$;   Asset Y: $f_Y(S_T<K)=0$; $f_Y(S_T>K)=120$

It is, prime facie, obvious that payoffs of asset Y are ≥ payoffs of asset X in both states whence, asset Y is definitely superior to asset X. It immediately follows that the price of Y should be more than X. Since, the current price of both X & Y is the same, arbitrage opportunities do exist in this case.

So, the issue here is arbitrage should only be invoked when there exists a clear cut demarcation between the payoff profiles of the two assets.

Portfolio 4 can be analysed similarly as portfolio 1. We do not have explicit arbitrage opportunities here.

Cash & carry arbitrage

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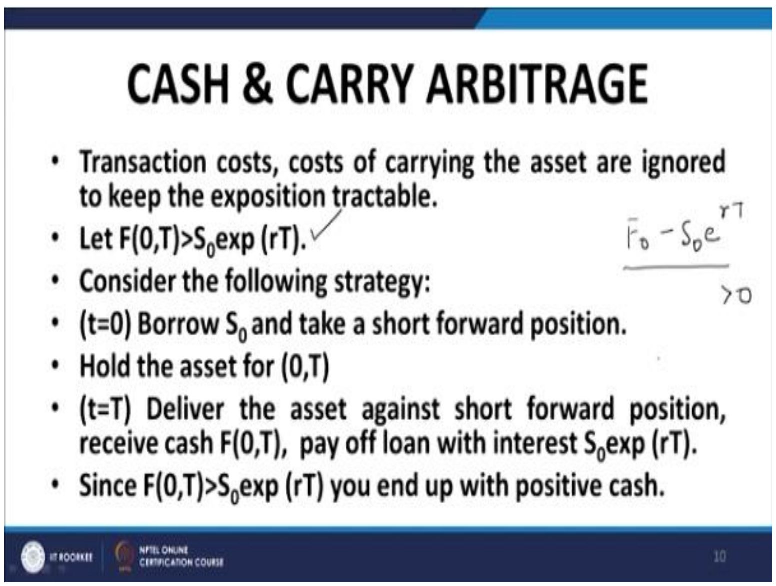

Let us see what happens if the condition of no arbitrage is violated, for example, if the actual forward price F_{act} is higher than the theoretical no arbitrage price $F_0=S_0exp(rT)$.

Obviously, because this equality is being violated, therefore, there will be arbitrage opportunities. The question is – how can one take advantage of this arbitrage situation? The first thing to note here is that since F_{act} is greater than the no-arbitrage forward price F_0, logic says that you sell at F_{act} and you buy at S_0. In other words, you buy the asset in the spot market and take a short position in the forward contract.

So what are the various steps:

(i) You start with borrowing an amount of S_0, buy one unit of the asset in the spot market and at the same time take a short forward position for one unit. (ii) You carry the asset until the maturity of the forward contract, deliver the asset against the short forward position, receive the amount of cash F_{act} and repay the loantogther with interest $S_0\exp(rT)$. So, now you have the cash $F_{act} - S_0\exp(rT) > 0$. In other words, by this arbitrage cycle, you will have made a profit.

Now consider the case when $F_{act} < F_0$. Since, the actual forward price is lower, you will buy in the forward market. Thus, you take a long forward position and at the same time borrow (short) the asset, sell it in the spot market and invest the proceeds in a riskfree deposit until the forward's maturity. At maturity, you will liquidate the investment, receive the amount $F_0 = S_0\exp(rT)$, pay the amount F_{act} against the long forward, receive the asset and replenish it to its owner from whom it was borrowed in the first place. The profit is $S_0\exp(rT) - F_{act}$.

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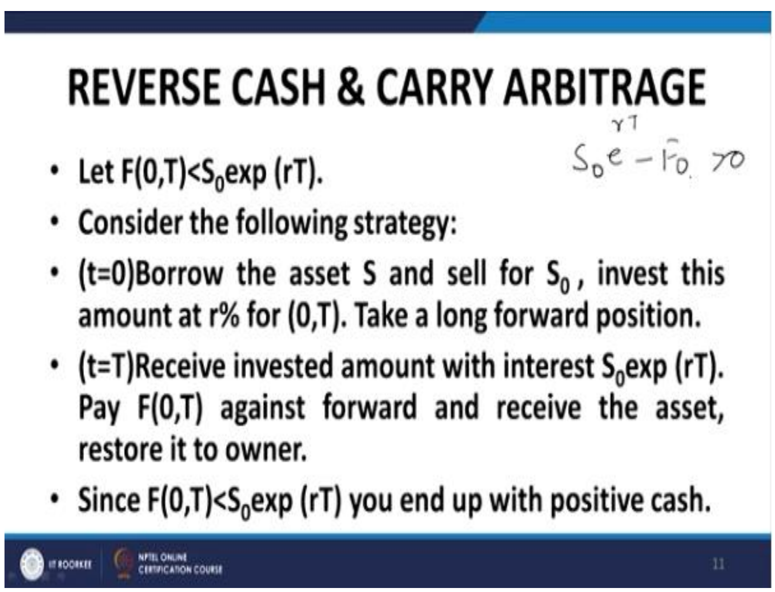

This process is called reverse cash and carry arbitrage, the previous process where you were buying the asset from the spot market, holding it and then giving it away against the forward position was called cash and carry arbitrage.

So, in the cash and carry arbitrage situation the actual forward price is higher than the no-arbitrage forward price in the reverse cash and carry arbitrage the actual forward price is less than the no-arbitrage forward price.

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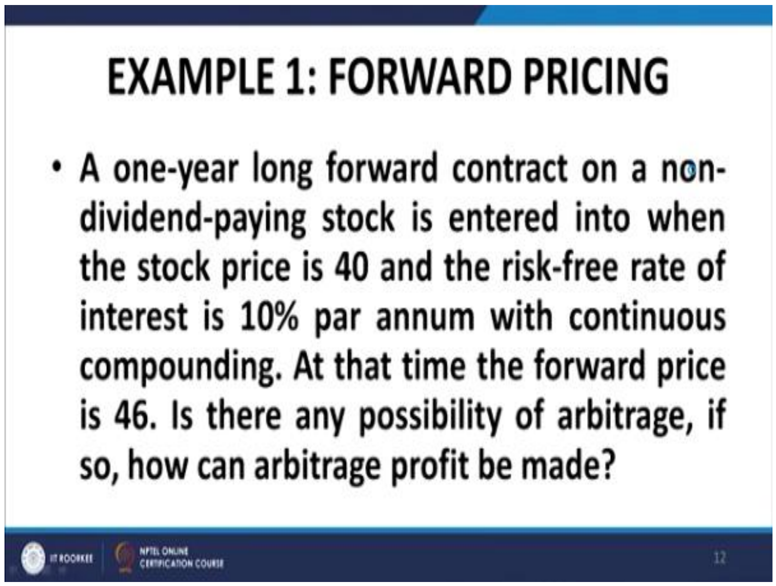

In this example, a one year forward contract on a non-dividend paying stock is entered into when the stock price is 40 and the risk-free rate of interest is 10 percent per annum with continuous compounding. At that time the actual forward price is 46. The question is whether any possibility of arbitrage exists?

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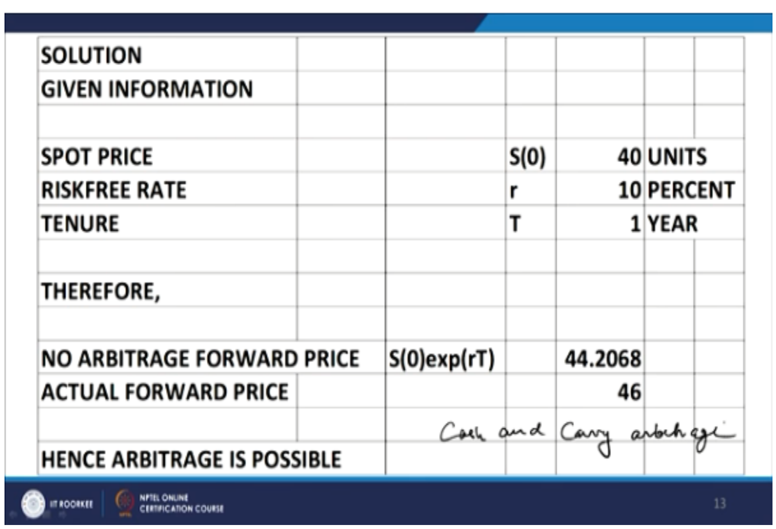

SOLUTION				
GIVEN INFORMATION				
SPOT PRICE		S(0)	40	UNITS
RISKFREE RATE		r	10	PERCENT
TENURE		T	1	YEAR
THEREFORE,				
NO ARBITRAGE FORWARD PRICE	S(0)exp(rT)		44.2068	
ACTUAL FORWARD PRICE			46	
	Cash and		Carry	arbitrage
HENCE ARBITRAGE IS POSSIBLE				

Let us look at the solution: S_0 is 40, r is 10% and T=1 year, so the no-arbitrage forward price is $S_0\exp(rT)$= 44.21. Against that the actual forward price is 46. Therefore, there is a clear cut arbitrage opportunity. Further, because the actual forward price is higher than the no-arbitrage forward price, cash and carry arbitrage will yield positive profit.

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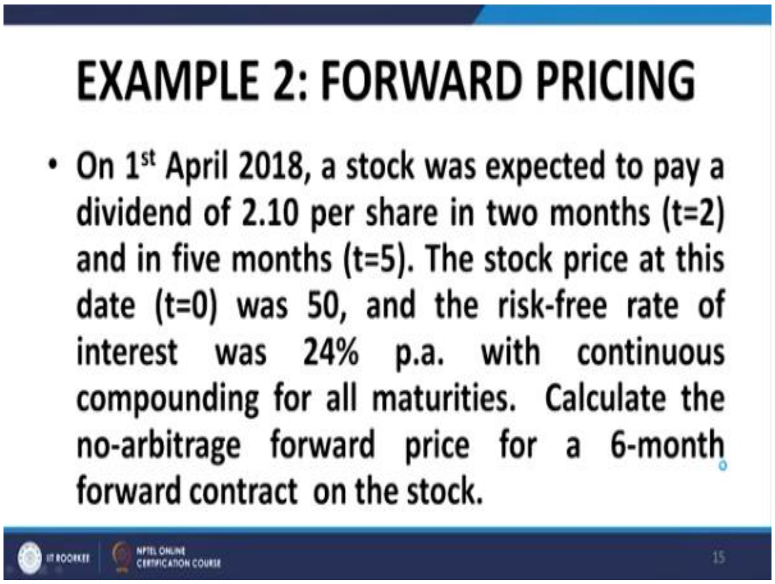

The solution is given below:

$$Current\ Price\ of\ Stock\left(S_0\right)=50;\ Risk-free\ Rate\left(r\right)=0.24$$
$$Dividends: 2.10\ at\ t=2\ months\ and\ 2.10\ at\ t=5\ months$$
$$Maturity\ \left(T\right)=0.50\ year.$$
$$Present\ Value\ of\ Dividends\left(D_0\right): 2.10e^{-0.24\times2/12}+2.10e^{-0.24\times5/12}$$
$$=2.0177+1.9002=3.9179$$
$$F_0=\left(S_0-D_0\right)\exp\left(rT\right):\left(50.0000-3.9179\right)e^{0.24\times0.50}=51.9574$$

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SOLUTION		$F_0 = (S_0 - D_0) e^{rT}$			
TIME		0.0000	2.0000	5.0000	6.0000
STOCK PRICE		50.0000			
INTEREST RATE(24%)		0.2400	0.0400	0.1000	0.1200
DISCOUNT FACTOR			-0.0400	-0.1000	-0.1200
PVIFs			0.9608	0.9048	
DIVIDEND			2.1000	2.1000	
PV OF DIVIDEND			2.0177	1.9002	
		3.9178			
NET STOCK PRICE AT T=0	X(0)=S(0)-D(0)	46.0822			
FVIF					1.1275
FORWARD PRICE AT T=6	X(0)*FVIF	51.9575			

What is the formula for no-arbitrage forward price? It is $F_0=(S_0-D_0)\exp(rT)$. What is D_0? D_0 is the present value of all the dividends that you receive during the entire life of the forward contract. There are 2 dividend payments in this problem during the forward's life, at the end of the second month and at the end of the fifth month, and so we need to work out the aggregate present value of bothe these dividend payments.

The important thing here is we are having two different dividend payments during the forward's life and each has to be discounted to be brought to t=0. The total of these two present values is D_0.

Now, you will recall that when I started talking about pricing of forward contracts, I had bifurcated assets into investment assets and consumption assets.

So far I have been talking about pricing of forwards on investment assets. Investment assets were those assets that were held for the purpose of investment like gold, silver, stocks, currency and so on. Consumption assets were those assets which were held for consumptions like copper, coal, wheat, barley, livestock etc. At that point I said that I will talk about the significance of this classification at a later point in time. It is here that I take up this issue.

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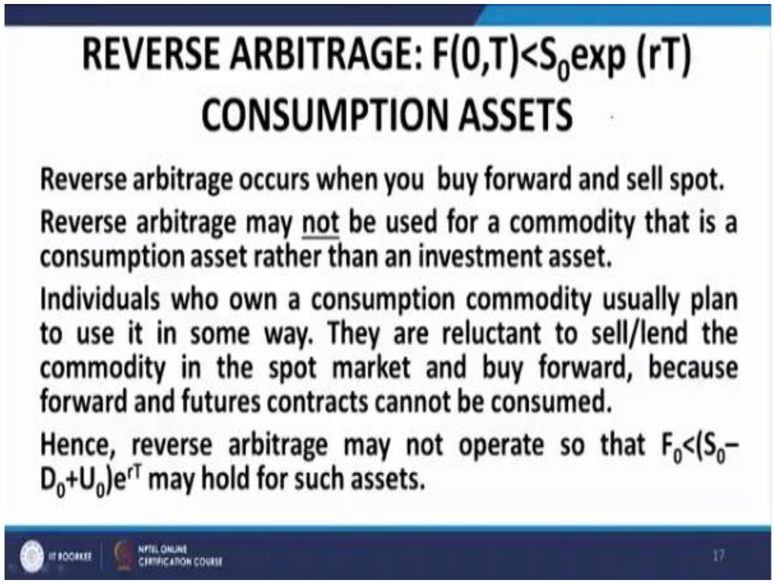

To understand the implications of this classification, we need to revisit the mechanism of reverse arbitrage. The process will involve your acquiring the asset by borrowing, selling it in the spot market and buying the asset in the forward market (taking a long forward position in the asset). You replenish the asset to the owner from whom you borrowed the asset when you receive it against the long forward position.

In case, you own the asset, you simply sell it spot and acquire it through a forward contract.

The important thing here is, the thing to understand is, that you get dispossessed of the physical possession of the asset between today and the maturity of the forward contract. You have sold the asset, invested the cash proceeds and you recover the asset only at the maturity of the forward contract.

So how is this classification of investment assets and consumption assets relevant in the context of forward pricing is what I shall take up next.