**Financial Derivatives & Risk Management**
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**Lecture 29**
**US T-Bond Futures: Conversion Factor; Options**

## Conversion factor of T-bond futures

In general, the exchanges allow a variety of deliverable bonds against any particular T-bond futures. Further, the right of choosing a particular deliverable bond from the spectrum of permitted deliverable bonds against a bond futures usually vests with the short party. For instance, against the T-bond futures traded on the Chicago Mercantile Exchange (CME), any government bond that has a maturity of more than 15 years on the first day of the delivery month and is not callable within 15 years can be delivered by the short party in satisfaction of its obligation under the futures.

Subject to the above restriction, the choice of the deliverable bond vests with the party making the delivery i.e. the short party. Obviously, at any point in time, there will be a number of deliverable bonds meeting the aforesaid eligibility criterion of 15 years and hence, a number of options will be available to the short party for meeting its obligations.

These multitude of bonds will naturally differ in terms of maturity, coupon rates and possibly YTMs. Therefore, there needs to be put in place a mechanism, a methodology for comparing these various bonds and thereby arriving at their worth for the final settlement of the futures i.e. for determining the cash payable by the long party against delivery of the chosen bond out of these eligible ones. In other words, the worth of each deliverable bond eligible for satisfying the delivery obligation against a futures needs to be benchmarked against a so called standard bond.

This mechanism is provided through the concept of conversion factor. The conversion factor provides the equivalence between a particular bond which is deliverable against a short position in a futures contract vis-à-vis a certain standard bond. This standard bond is deemed to have a YTM of 6% p.a. compounded semi-annually in the case of the CME T-bond futures contracts, although this figure would vary across exchanges.

The cash received by the short party against delivery on final settlement of a T-bond futures is given by the following:

Cash received by short party=Quoted price for the delivered bond +AI= MRSP*CF+AI (where MRSP: Most recent settlement price, CF: Conversion factor and AI: Accrued interest).

***The settlement price of the T-bond futures is quoted in terms of the standard bond.***

Depending on the conversion factor of the bond which the short party chooses to tender against a particular futures contract, the settlement value of the tendered bond is computed by multiplying the conversion factor with the most recent settlement price (which is quoted in terms of the standard bond). The accrued interest from the last coupon date till the delivery date is then added to arrive at the cash obligation of the long party.

**<u>Computation of conversion factor</u>**

The conversion factor for a bond is set equal to the quoted price the bond would have per dollar of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals 6% per annum (with semiannual compounding).

There are 4 steps in working out the conversion factor:

(i) Rounding down of maturity: The first step is the rounding down of the remaining maturity of the bond as on the first day of the delivery month. Whatever is the remaining maturity of the bond, ***it is to be rounded down*** i.e. rounded off to the ***next lower quarter***. Thus, if the remaining maturity of the bond is 18 years and 4 months, it is rounded down to 18 years 3 months; if it is 18 years and 8 months, rounding down is to 18 years 6 months. ***The rounding operation operates always on the lower side, invariably on the lower side and it rounds down to the next lower quarter.*** Recall all this rounding down of maturity needs to be done in respect of maturity remaining as on the first day of the delivery month.

(ii) Identifying coupon dates: On the basis of rounding down as per step (i), we have two situations, either the rounded down maturity of the bond is an integral multiple of a 6-month period or it is not so.

Case A: If, after rounding down, the bond maturity lasts for an exact number of 6-month periods, the first coupon is assumed to be paid in 6 months from the first day of the relevant delivery month. Recall that the relevant date is the first day of the delivery month. So the first coupon is assumed to be received at the end of 6-months from this first day of the delivery month and thereafter coupons are deemed to be received regularly at 6-monthly intervals until the rounded down maturity of the bond.

Case B: If, after rounding down, the bond maturity is not an exact number of 6-month periods (i.e., there are an extra 3 months), the first coupon is assumed to be paid after 3 months and accrued interest is subtracted.

(iiia) Calculate the intrinsic value @ 6% p.a. compounded semi-annually: In case (A), the next step is simply to calculate the intrinsic value of the bond assuming a yield (discount rate) of 6% p.a. compounded semi-annually i.e. 3% for each semi-annual period. ***The intrinsic value is called on the basis of the coupons and principal repayments at the notional dates arrived at on the basis of rounding down and not on the basis of the coupon dates as per the original contract of issue of the bond.*** The intrinsic value so obtained is divided by the face value of the bond to obtain the conversion factor.

**<u>Example</u>**

Consider a 10% coupon bond with 20 years and 2 months to maturity. Calculate the conversion factor for this bond.

**<u>Solution</u>**

(i) Rounding down of maturity: Since the actual maturity of the bond is given as 20 years & 2 months, for the purposes of calculating the conversion factor, the bond's maturity is rounded down to exactly 20 years.

(ii) Identifying coupon dates: The first coupon payment is notionally assumed to be made after 6 months from the first day of the delivery month. Thereafter coupon payments shall be assumed to be made at the end of every 6-month period upto the rounded down maturity of 20 years. Thus, there will be assumed to be 40 coupon payments in all, each of USD 5.00 (Assuming face value of the bond as USD 100.00). At the end of the 20th year, the principal of USD 100.00 will be deemed to be redeemed.

(iii) The notional intrinsic value is calculated on the basis of the rounded down maturity and the above scheme of coupon payments and using a 6% p.a. compd s.a. yield rate. We have, the value of the bond is:

$$\sum_{k=1}^{40}\frac{5}{1.03^k}+\frac{100}{1.03^{40}}=146.23$$

Dividing this figure by the face value of USD 100.00 we get the conversion factor as 1.4623.

We are having a 10% coupon bond with 20 years and 2 months to maturity. The first step of rounding down gives us the maturity of the bond as 20 years. Because this maturity is an integral multiple of 6-months, the first coupon is deemed to be received at 6 months from today (the first day of the delivery month). The bond is a 10% bond, that means half yearly coupon of USD 5.00 on a face value of USD 100.00. The intrinsic value is worked out accordingly at a yield of 6% p.a. s/a. compd or 3% for each 6-month period.

(iiib) We, now, address case (B) where, after the rounding down, the notional maturity of the bond works out to be a non-integral multiple of the 6-month period i.e. the notional maturity is an odd number of quarters. Equivalently, the rounded down deemed maturity of the bonds consists of an integral multiple of 6-month periods plus an extra 3-month period i.e. an extra quarter. In this case, the first coupon is deemed to be 3-months from the relevant date and thereafter coupons are deemed paid at the end of every 6-month period over the rounded down maturity of the bond. The intrinsic value of the bond as on the relevant date (first day of delivery month) is worked out by discounting all the future deemed coupon and principal payments (including the one at 3-months) @ a yield of 6% p.a. compounded s.a. The formula used is:

$$V_{\text{int}rinsic,0}=\sum_{t=1}^{n}\frac{C}{(1+r)^v(1+r)^{t-1}}+\frac{M}{(1+r)^v(1+r)^{n-1}}=\frac{1}{(1+r)^v}\left[C+\frac{C}{1+r}+\ldots+\frac{C}{(1+r)^{n-1}}+\frac{M}{(1+r)^{n-1}}\right]$$

where:

$V_{intrinsic,0}$ =Intrinsic value of bond at t=0 (first day of delivery month)

C = Semi-annual coupon payment amount

M = Maturity value

r = Semi-annual required rate of return

n = Total number of semi-annual coupons remaining

v = Days between settlement of the trade and the next coupon divided by the number of days in the coupon period.

In our case, r=3% for each semi-annual period (which represents the time period between two coupons) and $\upsilon$=3 months =0.25 years

Actually, we work out the intrinsic value at t= 3 months (including therein the coupon deemed to be received as at that date). Having done that, we then work out the present value of this intrinsic value by using the same 3 percent rate. The corresponding quarterly rate is $(1.03)^{0.50}-1=1.4889\%$. This gives us the cash price at t=0 i.e. the first day of delivery month.

(iv) This step is required only in Case (B). We need to work out the quoted price of the bond. We have the cash price by step (iiib). But the cash price is the sum of the quoted price and the accrued interest i.e. the quoted price is the cash price less accrued interest. Hence, from the cash price (intrinsic value) obtained above, we need to deduct the accrued interest to arrive at the quoted price for the bond which when divided by the face value yields the conversion factor.

Pertinent to emphasize here that accrued interest is not deducted in Case (A) because, when the rounded down maturity is integral multiple of 6-month period, the next coupon is assumed at the end of 6 months. Hence, the intrinsic value calculated at t=0 does not contain any accrued interest as it is worked out immediately after the payment of an instalment of interest. Thus, there is no accrued interest in the calculated price hence, the question of deducting interest does not arise.

Contrast this with the situation in Case (B). We have worked out the intrinsic value of the bond at t=0 by discounting the cash flows at t=3 months and beyond. But the cash flow that arises at t=3 months is the coupon that covers the interest NOT for the period of 3 months i.e. t=0 to t=3 months but for the period of 6 months from t=-3 months to t=+3 months. Stated otherwise, the cash price at t=0 (calculated by discounting coupons & principal) includes the interest component for t=-3 months to t=0 as well. Since this period has already elapsed, the interest pertaining to this period (accrued interest) is deducted, in practice to work out the quoted value of the bond which forms the basis of the conversion factor.

**Example**

Consider an 8% coupon bond with 18 years and 4 months to maturity. Calculate the conversion factor for this bond.

**Solution**

(i) Rounding down of maturity: For the purposes of calculating the conversion factor, the bond is assumed to have exactly 18 years and 3 months to maturity.

(ii) Identifying coupon dates: Since the rounded down maturity is not an integral multiple of 6-month period i.e. there is an extra quarter, the first coupon is assumed to arise at t=3 months and thereafter at 6-monthly intervals. Hence, there will be 37 coupon payments in all. The principal (equal to face value) will be repaid together with the final coupon.

(iii) Calculating intrinsic value: Discounting all the payments back to a point in time to the relevant date at 6% per annum (compounded semiannually) gives a value of:

$$V_{intrinsic,0} = \frac{1}{1.03^{0.50}}\left(4+\sum_{k=1}^{36}\frac{4}{1.03^{k}}+\frac{100}{1.03^{36}}\right)=123.9840$$

(iv) Subtracting the accrued interest of 2.0, this becomes $121.99. The conversion factor is therefore 1.2199.

The given bond is an 8% with 18 years and 4 months to maturity. Thus, the coupon payment will be USD 4.00 on a face value of USD 100.00 on each coupon date. 18 years and 4 months will be rounded down to 18 years and 3 months, 18 full years that is 36, 6 monthly periods plus an extra 3-month period. So, the first coupon date is deemed to be at 3 months from now. Thereafter, coupons will be paid every 6 months over the remaining rounded down life of the bond of 18 years.

We have one coupon of USD 4.00 at t=3 months and 36 subsequent coupons and the principal at the end. The present value of all these flows at 3% semi-annual is 123.99.

The discounting for 3-months from t=3 months to t=0 is at the effective rate $(1.03)^{0.50}$-1= 1.4889%. We are discounting here not on the basis of simple interest, but on the basis of a compounding factor. In other words, we are using $(1.03)^{0.50}$-1= 1.4889% for working out the effective rate for 3 months and not (1+0.03*0.50)-1= 1.50%. This is as per the market practice, convention.

Now, the look at the first interest payment at t=3 months of USD 4.00. This interest of USD 4.00 relates to a period of 6 months. Thus, it covers the interest from t=-3 months to t=+3 months. In other words, this interest includes the interest for the period from t=-3 months to t=0 as well i.e. it includes the accrued interest for the period from t=-3 months to t=0. This amount is USD 2.00. So, USD 2.00 needs to be deducted from the cash price (intrinsic value) at t=0 to arrive at the quoted price (actual worth) of this bond at t=0, which is, therefore, 121.99. dividing by the face value of USD 100.00 (assumed), we arrive at the conversion factor of 1.2199.

Thus, the important thing to note here is that the first coupon that we are discounting relates to the period of 3 months that has already expired plus the period of 3 months that is yet to expire. Therefore, we need to deduct the accrued interest for the period that has already expired (which is of 3 months) from the cash price to obtain the quoted price.

***The discounted value or intrinsic value always yields the cash price of a security i.e. the price inclusive of accrued interest.***

**Cheapest to deliver**

Having explained the concept of conversion factor and knowing that there can be multiple delivery options for the short party against a particular T bond futures contract, it sometimes happens that some particular bond delivery turns out to be cheapest for the short party. This brings us to the concept of cheapest to deliver bond.

As we know, at any given time during the delivery month, there are many bonds that can be delivered in the Treasury bond futures contract. It is the short party that enjoys the right to choose which of the available bonds is "cheapest" to deliver. The relative "cheapness" of a bond delivery option is evaluated on the basis of the following:

The short party on tendering delivery receives=(Most recent settlement price x Conversion factor) + Accrued interest

The cost of purchasing the bond for meeting the delivery obligation is= Quoted bond price + Accrued interest

Hence, the cheapest to deliver bond will be the one for which Quoted bond price -(Most recent settlement price x Conversion factor) is least.

Because the settlement prices quoted in terms of delivery of the standard bond, the short party actually receives the settlement price scaled by the conversion factor, this conversion factor being the equivalent number of standard bonds represented in the tendered bond. Of course we also have the accrued interest for the period which has elapsed since the last coupon date.

Now, in order to deliver a particular bond, the short party will purchase the bond from the market. But unlike futures, the bonds themselves are not quoted in terms of any standard bonds. They are quoted in terms of their respective worths/yields. Bonds of a particular issue have their prices determined by their own demand & supply that relate to their cardinals like credit risk, coupon rates & patterns, maturities, yields etc.

So the short party has to pay the quoted bond price for the bond chosen by him for tendering delivery (with accrued interest). No issue of conversion factor arises here. Because the short party has a choice, he selects that option for which the amount he pays less the amount he receives is the least i.e. for which quoted bond price less (most recent settlement price * conversion factor) is the least.

A number of factors determine the cheapest-to-deliver bond. When bond yields are in excess of 6%, the conversion factor system tends to favor the delivery of low-coupon long-maturity bonds. When yields are less than 6%, the system tends to favor the delivery of high-coupon short-maturity bonds. Also, when the yield curve is upward sloping, there is a tendency for bonds with a long time to maturity to be favored, whereas when it is downward-sloping, there is a tendency for bonds with a short time to maturity to be delivered.

**Example**

The short party has decided to deliver and is trying to choose between the three bonds in the table below. Assume the most recent settlement price is 93.25. Which is the CTD bond of the following?

**Solution**

MRSP:93.25

| Bond | Quoted bond price ($) | Conversion factor | MRSP X CF | Difference |
|---|---|---|---|---|
| 1 | 99.50 | 1.0382 | 96.81 | 2.69 |

| | | | | |
|---|---|---|---|---|
| 2 | 143.50 | 1.5188 | 141.63 | 1.87 |
| 3 | 119.75 | 1.2615 | 117.63 | 2.12 |

## Concluding remarks on interest rate futures

## Futures maturity vs underlying maturity


The maturity of the futures relates to the period 0 to T. At the point t=T, the underlying instrument is delivered by the short party to the long party. This underlying interest rate instrument e.g. a T-bill or Eurodollar deposit or a T-bond has a maturity of its own, say at t=N. At t=N. the underlying instrument (e.g. T-bill etc) matures for payment and the party who holds the instrument on that date i.e. t=N, gets credited with the redemption proceeds.

For example, consider a T-bill futures. It is a futures agreement. It entails the delivery of the underlying asset (a 90-day T-bill) at a predetermined price ($F_0$) at a predetermined date (T). Thus, at t=T, the long party pays an amount $F_0$ and the short party tenders a T-bill with a remaining 90-day maturity. On the maturity of this 90-day T-bill i.e. at t=T+90, the 90 T-bill will become due for redemption and its face value will be credited to the holder on that date.

So, the important point is that this period (T,N) is usually 90, 91 or 92 days as the case may be, depending on the delivery options permitted by the relevant exchange. In US delivery under the T-bill futures may be met by tendering 90, 91, 92 days T-bill as mandated in the relevant futures contract. In India, T-bill futures delivery is satisfied by tendering 91-day T-bill. However, the period (0,T) need not be 90, 91 or 92 days. Indeed, Eurodollar futures are traded with maturities (0.T) as far forward as 10 years although the underlying in all cases is a 90 day Eurodollar deposit. In other words, quarterly Eurodollar contracts are available for trading over a period of 10 years i.e. 40 quarters. So, as of today one can trade and lock the forward rate for a deposit that is to be initiated 10 years from now. Indeed, the periods (0,T) and (T,N) are mutually independent albeit both are determined by the terms of the futures contract.

## Trade price vs quote price of interest rate futures

The "quoted" or "Index" price $Q_f = 100-y_f$ where $y_f$ is the quoted discount yield on the futures contract. The actual futures traded value at which the contract is traded per face value F of nominal value is $P_{trade} = F\left(1-\frac{y_f}{100}\frac{90}{360}\right)$. Now, the number 90/360=0.25 is a factor that is specified by the exchanges at which the futures are traded. It is a specification by the relevant exchange.

But for academic interest, this number need not necessarily be 90/360. For example, let us look at an extreme case. Let this number be 0. If this quantity is 0, It follows that the trade price will be independent of the yield over the life of the futures. The futures will be traded throughout their life at the face value of the underlying and there would be no change in price on a day-to day basis and hence, no marking to market and no volatility. However, on the date of maturity of the futures, final settlement mandates that the futures price be marked to the spot price of the underlying T-bill. If the maturity price must match the spot price but the price does not change over the remaining life of the futures (upto to maturity) then it means, there will be only one ‘’marking to market’’ and that will occur at the date of maturity where the face value F at which the contract is initiated will simply jump discontinuously to the spot price of the underlying prevailing on the date of maturity.

Let us, now, look at a more general case. We have, $P_{trade} = F\left(1 - \frac{y_f}{100}\theta\right)$ so that $\Delta P = P_{1,trade} - P_{0,trade} = F\frac{y_{f_0} - y_{f_1}}{100}\theta = -F\frac{\Delta y}{100}\theta$. Clearly, if θ is increased, the change in price corresponding to a given change in yield increases. Therefore, the marking to market increase, for given a change in the yield rates between yesterday and today, the corresponding change in the futures prices increases. Thus, MTM transfers will increase and so will the volatility. However, because the final settlement price is exogenous, it is dictated by the spot yield or the spot price of the underlying on the date of maturity, it follows that the aggregate MTM from the date of contract creation to (and including) the final settlement shall remain unaltered and shall remain at $S_T$-$F_0$. Therefore, while the aggregate will remain unchanged, the individual positive and negative transfers will increase in amplitude and so the volatility will increase. Putting it another way, θ dictates the pattern or amplitudes or volatility of margin transfers due to MTMs. If θ=0, the entire amount $S_T$-$F_0$ is transferred in one go as on the date of maturity. If θ=0.25, MTMs will follow a certain pattern, if θ is increased then the volatility of the transfers in the margin account will increase and vice versa.

Besides, if you want to retain the same rate of changes in prices, you can rescale the tick size commensurate with a different θ i.e. you rescale the measure of y so that $y_1\theta_1=y_2\theta_2$ etc. For example, if you have a tick size of 1 basis point, it translates to 25.00 US dollars for a futures on US T-bill of face value USD 1,000,000 with θ=0.25. You have equally well have the same scheme of things with a tick size of 0.50 basis points with θ=0.50. You will still have tick size (in terms of price change) of USD 25.00. So, by rescaling both y and θ one can implement the same scheme of price management as earlier.

**Option contracts**

***An option is a contract whereby the holder of the contract acquires a RIGHT to buy/sell a certain asset by/on a certain date for a certain price. Accordingly, the writer of the option has the OBLIGATION to sell/buy the asset by/on the said date for the said price, if the holder decides to exercise the option. He has no DISCRETION.***

One party to the contract is the holder of the option, also called long in the option or having a long position in the option and the other party is the writer of the option, also said to be short in the option or having a short position in the option.

Now, the party that is long in the option has a right, has a discretion, has the choice to buy or sell an asset at a price which is predetermined and that option can be exercised at a predetermined point in time or within a predetermined time period. The option holder has a right to buy/sell an asset at a point in time in the future at a price which is determined as of today.

The other party (option writer) has no right. The option writer has the obligation that if the option holder exercises the right, then he, the option writer, is duty bound to satisfy his leg of the contract. He must sell to/buy from the option holder the contracted asset at the contracted price and time. The option writer has no discretion, no choice to get out of the contract or let the contract lapse. This prerogative vests entirely with the holder.

So that is how the option contract operates, one party has the right, the other party has the obligation. This is unlike in the case of forwards. In the case of forwards, we never talk about a differential between two parties, both the parties had their respective obligations, both the parties were required to meet their respective mandates under the contract.

However, in the case of an option contract, as the name signifies, there is an option, there is a discretion, the discretion is with one party, he may exercise the option, he may just let the option lapse i.e. he may simply do nothing. But if he decides to exercise the option, then the other party's role comes into play and the other party's role is to honour his obligation under the contract.

The question, naturally arises as to why this differential rights? The answer is simple, that the option holder pays a certain price for acquiring that right under the contract. This price is called the option premium. By paying this premium to the option writer, the option holder acquires the right from him that if it exercises the option, the writer has the obligation to honour his commitment.

## European & American options

An option may be exercised at a given point in time (called a European option) or it may be exercised up to a given point in time (called an American option). European options can be exercised on a given day; American options can be exercised up to the date of maturity, up to the given day.

## Call & put options

An option which gives the holder of the option the right to buy the contracted asset is called a call option and an option that gives the holder of the option a right to sell the contracted asset is called a put option.

## Underlying asset

The asset that is contracted to be bought or sold is called the underlying asset. It can be commodities, currencies, stocks, indices, interest rates, T-bills even electricity.

## Strike price

The predetermined price at which the underlying asset can be bought or sold under the option contract is called the strike price.

**Maturity**

In the case of European options, the date on which the option becomes exercisable is called the maturity or the expiry date of the option. Similarly, in the case of American options, it is the date up to which the option can be exercised.