Financial Derivatives and Risk Management

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Lecture 28: US T-Bond Futures: Salient Features, Pricing

Concept of conversion factor for T-bond futures

In the case of US T-bond futures, the short party can fulfil his delivery obligations by tendering one of a spectrum of bonds trading in the market at the point of delivery. In fact, any government bond that has more than 15 years to maturity on the first day of the delivery month and is not callable within 15 years from that date can be delivered by the short party for meeting the delivery obligation.

The choice of the delivery bond vests with the short party not the long party. The short party can select the bond of his choice from the number of possible delivery options available at delivery.

Now, the bonds trading in the market that meet the eligibility criteria would naturally differ in terms of maturity, coupon rates & indeed YTMs. Therefore, some mechanism, some methodology needs to be put in place by the exchanges for assessing relative worths of these various delivery options available to the short party. It is here that the role of conversion factor comes into play. The conversion factor provides the equivalence between a particular bond which is deliverable against a short position in a futures contract vis-à-vis a certain standard bond.

That standard bond is calculated on the basis of a certain algorithm; but fundamentally, in the context of CME T-bond contracts, it mandates a yield of 6% p.a. compounded semi-annually. The 6% is standardized in context of the US T-bond contracts traded on the CME. Of course, different exchanges may specify different yield rates and, indeed, different algorithms for working out the conversion factor.

So, the conversion factor provides a mechanism of assessing the relative worths of various bonds which are available for delivery against a futures contract. When a particular bond is delivered, its conversion factor determines the price received for the bond by the party with the short position. The applicable quoted price is the product of the conversion factor and the most recent settlement price for the futures contract. Taking accrued interest into account, the cash received for each USD 100 face value of the bond delivered is: Most recent settlement price x Conversion factor + Accrued interest.

The algorithm for working out the conversion factor shall be taken up in the sequel.

Importantly, the settlement prices of T-bond futures are quoted in terms of the standard bond. Therefore, it is necessary to multiply the settlement price with the conversion factor of the actual bond tendered for delivery to get the settlement price of that actual delivered bond. Added to this is the accrued interest for the period from the last coupon date to the date of delivery (the first day of the delivery month) and we get the total cash payable by the long party against its obligation under the T-bond futures on final settlement.

Hedging with T-bond futures

We use the following notation:

$$P_{0,s} = spot\ (cash)\ price\ of\ hedged\ asset\ per\ unit$$
$$V_{0,s} = total\ spot\ (cash)\ value\ of\ \exp osure\ of\ hedger$$
$$P_f^* = futures\ price\ of\ T-bond\ futures\ at\ hedging\ (t=0)$$
$$T = Maturity\ of\ T-bond\ futures$$
$$Q_s = no\ of\ units\ of\ hedged\ asset$$
$$N_f = No\ of\ T-bond\ futures\ shorted\ to\ hedge$$

We, then have:

$$The\ change\ in\ value\ of\ the\ hedged\ portfolio\ at\ T\ on\ clo\sin g\ out\ the\ hedge\ at\ t=T\ is$$
$$\Delta V = Q_s\Delta S - N_f\Delta F = -Q_s P_{s,0} D_s dr_s + N_f P_f^* D_f dr_f = -V_{s,0} D_s dr_s + N_f P_f^* D_f dr_f$$
$$If\ \ parallel\ shifts\ in\ \text{int}erest\ rates\ is\ assumed\ then: dr_s = dr_f = dr_h = dr\ and$$
$$\Delta V = -V_{s,0} D_h dr\ \ so\ that\ D_h = D_s - \frac{N_f P_f^*}{V_{s,0}} D_f;$$
$$For\ perfect\ hedging,\ D_h = 0\ \ or\ 0 = D_s - \frac{N_f P_f^*}{V_{s,0}} D_f\ \ or\ \ N_f = \frac{V_{s,0} D_s}{P_f^* D_f}$$

where P_f^* is the futures price of one T-bond futures at the time the hedge is instituted i.e. t=0 and N_f is the number of T-bonds shorted in the futures market at t=0 for the hedging exercise.

The following important assumptions are made in the above analysis:

(i) The above computations use interest rates and not discount yields.
(ii) Duration is based on continuous compounding of interest rates.

$$We\ have,\ with\ continuous\ compounding\ \ P = \sum_{t=1}^{T} C_t e^{-rt}$$
$$so\ that\ \frac{\partial P}{\partial r} = -\sum_{t=1}^{T} tC_t e^{-rt}\ \ or\ \partial P = -PD_s\partial r\ \ where\ \ D_s = \frac{\sum_{t=1}^{T} tC_t e^{-rt}}{P}$$

(iii) Duration is a first order approximation of the change in price, convexity and higher order derivatives are ignored.
(iv) The analysis includes change in value due to passage of time even if interest rates remain constant.
(v) Initial and MTM margin transfers are ignored.

We start with the usual expression, the change in value of the hedged portfolio comprising of the cash position (exposure) and the futures position is given by $\Delta V = Q_s \Delta S - N_f \Delta F$. Q_s is the number of units of hedged asset and ΔS is the change in unit price of the hedged asset between the time points the hedge is instituted and the time it is lifted. In terms of duration (with continuous compounding) with respect to spot rates D_s, we can write $\Delta S = -P_{s,0} D_s dr_s$ where dr_s is the change in spot interest rates between the above two time points.

Now, $Q_s * P_{s,0} = V_{s,0}$, the value of the exposure in monetary terms. Similarly, N_f is the number of T-bond futures shorted for hedging and ΔF is the change in futures price per T-bond futures between t=0(institution of hedge) and t=T (lifting of hedge). In analogy with the cash exposure, we can write $\Delta F = -P_f^* D_f dr_f$ where D_f is the duration of the T-bond futures with respect to the future interest rates r_f.

The two interest rates are the only random variables. Their evolution is stochastic. Now, we make the cardinal assumption in this model. We assume parallel shifts in interest rates that is the spot interest rates change (whenever they change) by the same amount for all maturities. Thus, the interest rate curve moves up or down only parallel to itself. If that is the case, then the futures rates also move by the same amount as the spot rates. Hence, we can write: $dr_s = dr_f = dr_h = dr$.

Under this assumption we obtain that the duration of the hedged position is $D_h = D_s - \frac{N_f P_f^*}{V_{s,0}} D_f$.

We know that duration is a measure of the sensitivity of a position to interest rates. Thus, if we want our exposure to completely hedged i.e. immune to interest rate changes, we must mandate that $D_h = 0$ whence $N_f = \frac{V_{s,0} D_s}{P_f^* D_f}$.

It may be noted that the value of the futures position at hedge inception i.e. $V_{f,0}=0$ since there is no initial investment in the futures to set up the hedge. We ignore margin requirements, so there is no initial investment in the futures contract.

Let us compare the expression for the number of contracts i.e. $N_f = \frac{V_{s,0} D_s}{P_f^* D_f}$ with the expression we obtained in the context of hedging using T-bill futures $N_{f,T-bill} = \frac{F_{s,0} D_s}{F_f^* D_f}$, both assuming parallel shifts. A perusal of the two formula brings out an interesting point. The number of contracts which we have arrived at in the case of T-bond futures and those in the case of T-bill futures differ in one fundamental respect viz in the former case we use the current value of the exposure and the current futures price for working out the optimum number of bonds/contracts while in the latter case we use the face values of the exposure and the T-bill futures i.e. the basic difference is the use of present values vis a vis the face values. If we explore further, the reason is not hard to find. In the case of T-bonds futures we work out the optimal futures position using interest rates whereas in the case of T-bill futures we use discount yields and not interest rates. Indeed, the durations that are worked out are on the basis of interest rates and discount yield respectively e.g.

$$\Delta V_s = -F_s \Delta d_s \left(\frac{N - T}{360} \right) = -D_s F_s \Delta d_s; \quad dF = -0.25 N_f F_f^* \Delta d_f = -D_f N_f F_f^* \Delta d_f$$

Now, we know that while interest rates operate on the current value of an investment (principal) to provide the future value, discount yields work the other way round i.e. they operate on the face (redemption) value to yiled the current value. This difference in the working of the rates manifests itself in the computation of the optimal futures position. Interest rates apply on current prices and give the future values. Yields apply on the future values and give the present values.

Summary

$$\Delta V = Q_s \Delta S + z N_f \Delta F \Rightarrow N_f = -\frac{Q_s}{z} \beta_{\Delta S, \Delta F} (\text{General})$$

$$\Delta V = Q_s S_0 \frac{\Delta S}{S_0} + z N_f F_0 \frac{\Delta F}{F_0} = V_s \frac{\Delta S}{S_0} + N_f V_F^* \frac{\Delta F}{F_0}$$

$$\Rightarrow N_f = -\frac{V_s}{V_F^*} \beta_{\frac{\Delta S}{S_0}, \frac{\Delta F}{F_0}} = -\frac{V_s}{V_F^*} \beta_{R_P, R_M} = -\frac{V_s}{V_F^*} \beta_P (\text{Stock Index})$$

$$\Delta V = Q_s P_{0,s} \frac{dP_s}{P_{0,s}} + z N_f P_{0,f} \frac{dP_f}{P_{0,f}} = Q_s P_{0,s} D_s dy_s + z N_f P_{0,f} D_f dy_f$$

$$\Rightarrow N_f = -\frac{P_{0,s} Q_s D_s}{z P_{0,f}^* D_f} \beta_{\Delta y_s, \Delta y_f} = -\frac{V_s D_s}{V_F^* D_f} \beta_{\Delta y_s, \Delta y_f} (\text{Duration, c.c.})$$

$$\Delta V = Q_s P_{0,s} D_s (1 + y_s)^{-1} dy_s + z N_f P_{0,f} D_f (1 + y_f)^{-1} dy_f$$

$$N_f = -\frac{P_{0,s} Q_s D_s (1 + y_f)}{z P_{0,f}^* D_f (1 + y_s)} \beta_{\Delta y_s, \Delta y_f} = -\frac{V_s D_s (1 + y_f)}{V_F^* D_f (1 + y_s)} \beta_{\Delta y_s, \Delta y_f} (\text{Duration, annual. c.})$$

Arbitrage and source of cash flows

t=0	t=0	t=T		
		COUPON	PRINCIPAL	TOTAL
A	98	10	100	110
B	96	20	90	110

Let us look at an example. We have 2 bonds A & B. Both the bonds are maturing at t=T. Only one final payment of principal + interest remains on the both the bonds. Bond A will pay a principal of 100 and a coupon of 10 at t=T as its final payment while Bond B will make a final payment on the same date of principal of 90 and a coupon of 20. Thus, the aggregate cash flows at t=T of both bonds are the same. Both have the same level of risk i.e. same creditworthiness. So, no other effects are there and the total cash flow in both cases at maturity are 110.

Let us assume that Bond A is priced at 98, while Bond B is priced at 96 today (t=0). The question is: Would this situation allow for an arbitrage opportunity? The answer is obviously yes. How? By simply shorting the Bond A (98) and buying Bond A (96). In other words, by constructing a portfolio comprising of short A & long B. The maturity payoffs of both bonds will be 110 so that the receipts against the long B can be used for exactly paying off the obligations against the short B. So, it is all squared. But at t=0, the portfolio yileds a cash inflow of 2. In effect, the arbitrageur has bought 110 at t=T for 96 at t=0 and sold 110 at t=T for 98 at t=0.

So, the message that is conveyed by this illustration is that the arbitrage process does not depend on the bifurcation of cash flows between principal and interest or coupons. It simply depends on what total cash flows are there at various points of time, irrespective of what is their relationship with the instrument, so long as a relationship does exist. It does not depend on whether those cash flow arise due to a coupon payment or principal or partly one and the other, provided the risk is unaffected.

Arbitrage will relate itself only to total cash flows because at the end of the day, arbitrage converts everything to money and money does not depend on what source you are generating it from. So, if the coupon is higher or the principal, so long as the total cash flow is the same arbitrage will take place if there is a price differential and there is no change in risk.

If the risk is unchanged and the prices are different and the total cash flows are the same on maturity then arbitrage will take place. In other words, arbitrage will take place in relation to total cash flows.

Determining the price of a T-bond futures

The futures price, F_0, is related to the spot price, S_0, by $F_0=(S_0-I)\exp(rT)$ where T is the maturity of the futures contract and I is the present value of the coupons during the life of the futures contract. ***Most important, both F_0 and S_0 are cash prices and NOT clean or quoted prices***.

Recall that the relationship $F_0=(S_0-I)\exp(rT)$ was arrived at by invoking no-arbitrage between spot and futures markets. Also recall the explanation above that arbitrage relates to the total cash values of the instruments and does not discriminate between the coupons and principal payments. It, therefore, immediately follows that both S_0 and F_0 are cash prices i.e. inclusive of all accrued interest etc. For example, S_0 would represent the total cash that would be received on selling one unit of the underlying bond in the spot market at that point in time (t=0). "Total cash" is reiterated. It is not and does not relate to the market quotes of the underlying bond. Same holds for the futures price F_0. To emphasize again, the spot price and the futures price are both cash prices. They are not the quoted prices. This equilibrium formula arises from no-arbitrage requirements and because it arises from no-arbitrage, it relates only cash prices.

Example of futures pricing

In a T bond futures contract, the cheapest to-deliver bond will be a 12% coupon bond with a conversion factor of 1.6000. Suppose that delivery will take place in 270 days (t=270). Coupons are payable semiannually on the bond. The last coupon date was 60 days ago (t=-60), the next coupon date is in 122 days (t=122), and the coupon date thereafter is in 305 days from t=0. Rate of interest (cont compd) is 10% p.a. The underlying bond is currently quoted at S115. Calculate the quoted futures price.

Solution

Step 1

Calculate the cash spot price of the bond S_0 from the quoted price. The quoted price of a bond does not include the accrued interest since the last coupon payment date. However, when acoupon bond is sold in the market between two coupon dates, the buyer must compensate the seller for the amount of interest that has accumulated (accrued) since the last coupon date to the transaction date. This is because the next coupon (which will relate to a period, for a part of which the bond was owned by the seller and for the other part the bond was owned by the buyer), will be received entirely by the buyer. Thus, the total cash at which the bond is transacted is the quoted price plus the accrued interest. In other words, the cash price of the bond is obtained by adding to this quoted price the proportion of the next coupon payment that accrues to the holder. The cash price is therefore: 115+{[(60/(60+122)] x 6}= 116.978

Step 2

Calculate the present value I of all coupons that would be received on the bond during the remaining life of the futures. A coupon of $6 will be received after 122 days (0.3342 years). The present value of this is 6 x exp(-0.1 x 0.3342)= = 5.803

Step 3

Calculate the future value of the net price i.e. (S_0-I) at the time of maturity of the futures. The futures contract lasts for 270 days (= 0.7397 years). The cash futures price, if the contract were written on the 12% bond, would therefore be (116.978 - 5.803)exp(0.1 x 0.7397) = 119.711

Step 4

We, now, have the futures cash price. To worth out the futures quote we need to eliminate the accrued interest component in this futures value. As mentioned earlier, the quotes are made on the basis clean price i.e. the price excluding the accrued interest. At delivery (t=270), there are 148 days of accrued interest on the underlying bond. The quoted futures price, if the contract were written on the 12% naked bond, is calculated by subtracting the accrued interest 119.711 - {[(148/(148+35)] x 6}= 114.859. This is futures quote of a futures contract on the given 12% bond.

Step 5

However, futures quotes are given in terms of the standard bond. Hence, we need to convert this futures quote on the given 12% bond to a quote on the standard bond. The conversion factor is given as 1.60 so that our 12% bond equals 1.60 standard bonds. The quoted futures price per standard bond is 114.859/1.6000 = 71.79.

TIMELINE		-60	0	122	270	305	487
EVENT		COUPON	VALUE	COUPON	DELIVERY	COUPON	
CASHFLOW		6		6		6	
QUOTED PRICE							
OF BOND			115				
AI			1.97802				
CASH PRICE OF BOND			116.978				
COUPON DURING LIFE OF FUTURE				6			
PV OF THIS COUPON			5.80277				
S(0)-D(0)=X(0)			111.175				
CASH PRICE OF FUTURE							
FOR DELIVERY AT 270			119.711				
HENCE SHORT WILL RECEIVE					119.71		
AND DELIVER ONE T-BOND							
BUT QUOTE FOR FUTURE							
AT T=0 IS IN TERMS OF PURE BOND PRICE							
THIS CASH FLOW PRICE			119.711				
THAT SHORT RECEIVES							
INCLUDES INTEREST SINCE LAST COUPON							
THAT IS SINCE T=122 TO T=270 I.E 148 DAYS					4.8525		
QUOTE PRICE OF FUTURE ON GIVEN BOND					114.86		
CONVERSION FACTOR					1.6		
QUOTE OF FUTURE OF STD BOND					71.787		
WITH SAME CONDITIONS AS GIVEN BOND							