Financial Derivatives and Risk Management
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Lecture 26: T-Bill Futures: Arbitrage; Eurodollar Futures

Arbitrage with T-bill futures

We shall now discuss the condition for no-arbitrage in the context of the T-bill futures market. Let us construct two arbitrage portfolios A & B, as is the standard practice.

Portfolio A

(i) One N+90-day T-bill of face value one unit. On the maturity of this bill i.e. at the end of N+90 days (t=N+90) from today (t=0), the T-bill will yield a cash flow of one unit. Let the price of this N+90-day T-bill be D(0,N+90) today where D(0,N+90) is the present value of one unit of money worked out as per applicable day-count and other market conventions.

Portfolio B

(i) A long futures position a 90-day T-Bill future also maturing for delivery at the end of N^{th} day at a futures price of $D_F(0,N,N+90)$. Thus, the long position in this futures contract, on maturity of the futures i.e. at the end of N^{th} day, will deliver a 90-day T-bill on payment of the futures price of $D_F(0,N,N+90)$ on the same day. This 90-day T-bill will mature for payment, 90 days thereafter i.e. at t=N+90 day and will realize the face value of T-bill of one money unit. Since $D_F(0,N,N+90)$ is the futures price, it is determined at t=0 although actual payment is made at t=N.

(i) $D_F(0,N,N+90)$ N day T-bills of face value one unit each. On the maturity of each bill i.e. at the end of N days (t=N) from today (t=0), the T-bill will yield a cash flow of one unit. Let the price of one N day T-bill be D(0,N) today where D(0,N) is the present value of one unit of money worked out as per applicable day-count and other market conventions. Since one T-bill will yield one unit of money at t=N and the obligation against the long futures position above requires a payment of $D_F(0,N,N+90)$, the number of N-day T-bills longed (invested) is $D_F(0,N,N+90)$. Thus, the amount invested at t=0 for buying these $D_F(0,N,N+90)$ N-day T-bills is $D_F(0,N,N+90)*D(0,N)$.

Let us analyse the cash flows of the two portfolios:

NO ARBITRAGE	0	N	N+90
PORTFOLIO A			
(N+90) day T bill	-D(0,N+90)		1
PORTFOLIO B			

N day T bill	**-D(0,N)*D_F(0,N,N+90)**	**D_F(0,N,N+90)**	
90-day T bill future with delivery after N days		**-D_F(0,N,N+90)**	**1**
D_F(0,N,N+90)=D(0,N+90)/D(0,N)			

The cash flows at t=N+90 of both portfolios is one money unit. Further, in portfolio A there is no intermediate cash flow while in B, the intermediate cash flows annul each other so that there is no aggregate cash flow on the portfolio. Therefore, no arbitrage postulates that the cash flows of A & B at t=0 i.e. at inception must also be the same. This gives us: $D_F\left(0,N,N+90\right)=\frac{D\left(0,N+90\right)}{D\left(0,N\right)}$.

This is no-arbitrage condition as applied to the T bills futures. Unlike interest rates, this equation represents the no-arbitrage condition in terms of discount factors. D(0,N) is the present value of one money unit received at N. Similarly, D(0,N+90) is the present value of one money unit that is received at N+90, D_F(0,N,N+90) is the value at t=N of a cash flow of one money unit received at N+90.

This is a more extended depiction of the portfolio flows, where we have a transactional approach for arriving at the same condition.

NO ARBITRAGE	t=0		t=N	
	RECEIVE	GIVE	RECEIVE	GIVE
SHORT N+90 T-BILL A	D(0,N+90)	T BILL A		
BUY N T-BILL B	D(0,N+90)/D (0,N) T BILL B	D(0,N+90)	D(0,N+90) /D(0,N)	D(0,N+90)/D(0,N) T BILL B
LONG T BILL FUTURE DELIVERY AT t=N			T BILL A	F(0,T,N)
NET CASH			D(0,N+90) /D(0,N)	F(0,T,N)

Cash & carry arbitrage

Consider the case D(0,N+90) > D(0,N)D_F(0,N,N+90). These are discount factors. Expressed in terms of compounding factors, we will have S(0,N+90) < S(0,N)F (0,N,N+90). The compounding factors are inversely related to discount factors and therefore, the relationship gets reversed.

In other words, money is cheaper for the (0,N+90) holding relative to the (0,N) holding coupled with a forward investment (N,N+90). Thus, it would be logical to borrow at S(0,N+90) and invest at S(0,N) with a forward investment F(0,N,N+90) to earn arbitrage profits. The steps involved would be:

(i) Borrow a (N+90) day T-bill and short sell. The amount received would be D(0,N+90).

(ii) At the same time, invest this amount in N-day T-bills. Each T-bill would cost D(0,N). Hence, the number of N-day T-bills that can be purchased is D(0,N+90)/D(0,N). The amount that would be received on maturity of these N-day T-bills i.e. at t=N would be D(0,N+90)/D(0,N) (assuming face value of each T-bill to be one unit).

(iii) Take a long position in $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)]$ 90-day T-bill futures at $D_F(0,N,N+90)$ that mature for delivery at $t=N^{th}$ day. Accordingly, at $t=N^{th}$, $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)]$ number of 90 day T-bills would be received against the payment of D(0,N+90)/D(0,N) money units. On maturity of these $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)]$ T-bills at t=N+90, the amount received would be $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)]$. Out of this amount repay an amount of one money unit to the party from whom the (N+90) day T-bill was borrowed for short selling under step (i). This one unit represents the maturity value of the borrowed T-bill. Hence, the profit on this strategy is $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)]-1$. Alternatively, one T-bill out of $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)]$ T-bills received against the long futures can be replenished to the lender of the N+90-day T-bill shorted in step (i) when it is received on the N^{th} day. Both are equivalent.

But $D(0,N+90)/[D(0,N)*D_F(0,N,N+90)] >1$ since $D(0,N+90)>[D(0,N)*D_F(0,N,N+90)]$. Thus, this strategy yields a positive arbitrage profit.

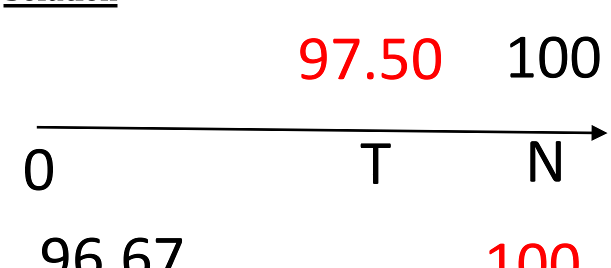
Alternatively:

t=0	t=N
Short sell N+90-day T bill. Receive D(0,N+90).	**Replenish T bill to the lender of the bill received against long futures.**
Buy D(0,N+90)/D(0,N) units of N day T-bill. Pay D(0,N+90)	**Receive cash D(0,N+90)/D(0,N)**
Long T-bill future with maturity N at price $D_F(0,N,N+90)$	**Receive 90-day T-bill Pay $D_F(0,N,N+90)$**
Net Cash =0	**$D(0,N+90)/D(0,N)-D_F(0,N,N+90)>0$**

<u>Example</u>

The quoted IMM index on a US 90-day T-bill futures is $Q_f = 90.00$. The futures contract matures in 30 days. A 120-day T-bill is also available with a discount yield $d_s = 10\%$ (day count is 'actual/360'). What is the implied 30-day spot rate? What arbitrage opportunities exist if the actual 30-day spot rate is r = 11%?

<u>Solution</u>



LET FACE VALUE OF T BILL	100
FOR THE 120 DAY BILL	
TENURE	0.333333
SPOT DISCOUNT YIELD	0.1
CURRENT PRICE	96.66667
FOR THE 30 DAY FUTURES	
IMM QUOTE	90
FUTURES YIELD	0.1
TRADED PRICE	97.5
IN BOTH CASES THERE WILL BE AN INFLOW OF	100
AT THE END OF 120 DAYS	
IN THE FIRST CASE, THERE IS A SPOT OUTFLOW OF	96.66667
IN THE 2ND CASE THERE IS OUTLOW AT 30 DAYS OF	97.5
THUS, FV OF CASE 1 AT 30 DAYS MUST EQUAL 2	
FOR NO ARBITRAGE	
HENCE REQUIRED 30 DAY SPOT RATE	0.103448

Let us see how we work it out. We can start with this spot instrument. This spot instrument has the life of 120 days and a discount yield of 10%. Therefore, its price as of today (t=0) is 96.67.

Now, let us look at the T bill future. The IMM quote is 90. That means the discount yield is 10%. The life is 90 days. Therefore, the futures trade price works out to 97.50. The price at which the 90-day T-bill will be delivered on maturity of the futures (t=30 days) is 97.50. This is a 90-day T-bill. Hence, it will be redeemed at t=30+90=120 days with a maturity payment of 100. Thus, the investment of 96.67 at t=0 gives a cash flow of 100 at t=120. Also an investment of 97.50 at t=30 gives the same cash flow of 100 at the same time (t=120). It follows that 96.67 at t=0 must be worth 97.50 at t=30. This gives a rate of 10.34% p.a.

Eurodollar futures

Eurodollars

A Eurodollar is a dollar deposited in a U.S. or foreign bank outside the United States. By being located outside the United States, Eurodollars escape regulation by the Federal Reserve Board, including reserve requirements.

The Eurodollar interest rate is the rate of interest earned on Eurodollars deposited by one bank with another bank. It is essentially the same as the London Interbank Offered Rate (LIBOR).

Contract specifications for Eurodollar futures

Underlying Asset:	90 day Eurodollar deposits at the time of issue quoted at Actual/360 day basis.
Unit of trading:	USD 1.00 million
Delivery Months:	Mar/June/Sept/Dec
Mode of Quotation:	IMM Index: $(100\text{-}y_f)$
Tick Size:	0.5 BP = USD 12.50
Settlement:	Cash settled by setting quote at 100-R where R is the maturity date spot rate.

Eurodollar futures: underlying

The underlying instrument in Eurodollar futures is a Eurodollar time deposit, having a principal value of $1 million with a three-month maturity. Thus, a three-month Eurodollar futures contract is a futures contract on the interest that will be paid on $1 million for a future three-month period.

Contract maturities

Eurodollar futures contracts have maturities in March, June, September, and December for up to 10 years into the future. This means that in 2019 a trader can use Eurodollar futures to take a position on what interest rates will be as far into the future as 2029.

Eurodollar futures at CME Group are based on the three month LIBOR underlying rate and listed under the March quarterly cycle for 40 consecutive quarters, plus four serial contracts at the front end of the curve.

Eurodollar futures quotes

The traded price is given by: 1,000,000 {1-0.25[(100-Q)/100]}. Thus, a settlement price of 99.3100 corresponds to a traded (contract) price of 998,275.

Tick size

The minimum allowable price fluctuation, or tick size, is generally established at 0.50 basis point, or 0.005%. Based on a million-dollar face value 90-day instrument, this equates to $12.50.

However, in the nearby expiring contract month, the minimum price fluctuation is set at 0.25 basis point, or 0.0025%, equating to $6.25 per contract.

Change in value of contract=Contract Face Value * Change in Yield* 0.25 =$1,000,000*0.005%*0.25 =$12.50

Settlement

The contract is settled daily in the usual way until maturity date. On maturity the settlement price is set equal to 100-R, where R is the actual three-month Eurodollar interest rate on that day, expressed with quarterly compounding and an actual/360 day count convention. Once a final settlement has taken place, all contracts are declared closed.

Example

XYZ Co is expecting to borrow $34,000,000 in five months' time. It expects to make a full repayment of the borrowed amount in 11 months' time. Assume it is June 1, 2018 today. XYZ Co can borrow funds at LIBOR+70 bp. LIBOR is currently 3.6%, but XYZ expects that interest rates might increase by as much as 80bp in five months' time. The following information and quotes from an appropriate exchange are provided on LIBOR based $ futures.

3 months EURODOLLAR December futures are currently quoted at 95.84. The contract size is $1,000,000. The tick size is 0.01% and the tick value is $25.

Initial assumptions: It can be assumed that settlement for futures contracts is at the end of the month, that basis diminishes to zero at a constant rate until the contract mature and time intervals can be counted in months that margin requirements may be ignored. Assess hedge performance.

Solution

XYZ company is expecting to borrow USD dollars 34 million in five months (t=5 months) from now (t=0). It expects to make a full repayment of the borrowed amount in 11 months (t=11 months) from now. Thus, the borrowing period is 6 months from t=5 months to t=11 months. Today is June 1, the firm can borrow funds at LIBOR+70bp. LIBOR is currently 3.6%. So its actual borrowing rate is 4.3% if it borrows today. But it desires to borrow at t= 5 months and interest rates may increase by up to 80 basis points during this interim period t=0 to t=5 months. LIBOR may increase up to 4.4% and the firm's borrowing cost may increase to 5.1%. So it wants to hedge by LIBOR futures.

The 3-month LIBOR December futures are currently quoted at 95.84. The tick size is 0.01% which translates to a value USD 25.00 per contract. Given that the settlement for futures contract may be assumed at the end of each month and that basis diminishes to 0 at a constant rate from its current value.

The basis has to converge to zero on the date of maturity of the futures contract. That is mandated by no-arbitrage. The futures and spots must converge on the maturity of the futures. Thus, it is given that convergence will be approached at a uniform rate. Margin requirements may be ignored.

AMOUNT OF BORROWING			34000000	
PERIOD OF BORROWING			6	MONTHS
CURRENT LIBOR			3.6	PERCENT
SPREAD			0.7	PERCENT
CURRENT BORROWING COST			4.3	
PROJECTED INCREASE			0.8	PERCENT
PROJECT INCREASE IN BORRORING COST			136000	

The projected increase in borrowing cost=USD 34,000,000*0.008*6/12=USD 136,000.

Now let us see what happens to the hedge. As far as the number of contracts goes, the borrowing is for 6 months, the underlying has a maturity of 3 months, the amount borrowed is USD 34,000,000 and the face value per contract is USD 1,000,000. Thus, number of contracts N_f=34,000,000*6/1,000,000*3 =68.

The current futures quote is 95.84. The corresponding futures interest rate is 100-95.84=4.16%. The traded futures price is USD 1,000,000(1-0.25*0.0416)=USD 989,600.

The spot LIBOR rate at t=0 is 3.6%. Hence, the t=0 value of a USD 1,000,000 face value Eurodeposit is =USD 1,000,000(1-0.25*0.036)= USD 991,000. Thus, the current basis is USD 991,000-989,600=USD 1,400.

The maturity of the futures in in December. The assumptions given say that settlement may be assumed at the end of the month, so maturity may be assumed end-December. Today is given as June 1. Hence, the remaining life of the contract is 7 months. At the end of 7 months from now, the basis must converge to zero. Further, as per assumption the convergence is to be at a uniform rate. Hence, the basis would amortize at the rate of USD 200 every passing month. Now, the hedge is to be lifted at t=5 months when the borrowing is to take place. Thus, by that time USD 1,000 out of the current basis of USD 1,400 would have been amortized so that the basis at hedge lifting would be USD 400.

Now, the spot LIBOR rate at hedge lifting (t=5 months) is expected to be 4.4% whence the value of a face value 1,000,000 90-day Eurodeposit as on that date would be USD 1,000,000(1-0.25*0.044)=USD 989,000.

With the basis at 400, the December futures on a 1,000,000 USD Eurodeposit would be trading on that date (t=5 months) at USD 989,000-400= USD 988,600.

Hence, the profit per futures works out to USD 989,600-988,600=1000. The profit on 68 futures =USD 68,000.

Recall that the increase in borrowing cost was 136,000 so that the shortfall after hedging is USD 68,000.

HEDGE USING IRF				
SHORT IRFS				
CURRENT FUTURES QUOTE			95.84	
CORRESPONDING FUTURES INTT RATE			4.16	
CORRESPONDING FUTURES TRADED PRICE			989600	
NO OF CONTRACTS			68	68
SPOT INTEREST RATE			3.6	
SPOT VALUE OF EUD DEPOSITS			991000	
BASIS			1400	
UNEXPERIRED BASIS AT LIFTING OF HEDGE			400	
SPOT INTEREST RATE			4.4	
CORRESPONDING VALUE OF SPOT EUD DEPOSITS			989000	
TRADED VALUE OF FUTURES ON HEDGE LIFTING			988600	
GAIN PER FUTURES			1000	
TOTAL GAIN ON HEDGE			68000	

REVISED RATE OF BORROWING		5.1
COST OF BORROWING		867000
TOTAL REPAYMENT		34867000
ACTUAL AMT RECD		34068000
NET COST		1.023453094
ANNUALIZED		1.047456235
		0.047456235