Financial derivatives and risk management
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Lecture 25
T-Bill Futures: Hedging

Before I proceed further, a relook at the following problem:

Hedging of a future investment: Prefixed redemption value

$$Let\ the\ amount\ to\ be\ invested\ V_{sH}\ at\ time\ H$$

$$Then,\ we\ have\ V_{sH} = F_s\left(1 - d_{s0}\frac{T-H}{360}\right)$$

$$Let\ the\ actual\ yield\ at\ time\ be\ d_{sH}$$

$$Actual\ investment\ V_{sH}^{*} = F_s\left(1 - d_{sH}\frac{T-H}{360}\right)$$

$$Change\ in\ investment\ V_{sH}^{*} - V_{sH} = -F_s\left(d_{sH} - d_{s0}\right)\left(\frac{T-H}{360}\right) = -F_s\Delta d_s\left(\frac{T-H}{360}\right)$$

$$Hence, change\ in\ value\ of\ exposure\ due\ to\ yield\ change$$

$$dS = -F_s\Delta d_s\left(\frac{T-H}{360}\right)$$

$$For\ each\ futures, P_{0,f} = F_f^{*}\left(1 - 0.25d_{f0}\right);\ P_{H,f} = F_f^{*}\left[1 - 0.25d_{fH}\right]$$

$$Change\ in\ value\ of\ N_f\ futures\ dF = -0.25N_fF_f^{*}\Delta d_f$$

$$N_f = \frac{\left(T-H\right)F_s}{360 * 0.25 \times F_f^{*}}\beta_{\Delta d_s,\Delta d_f}$$

Here we are considering the hedging by an investor of a future investment to be made in the future (say at t=H). He is worried about the change in the investment amount due to a change in interest rates between t=0 and t=H. However, he wants the redemption value to be constant say F_s. The interest rates are given in terms of yield rates.

At today's yield rates, the investment amount V_{sH} at t=H for a redemption amount of F_s on maturity at t=T is given by $V_{sH}= F_s\{1-d_{s0}[(T-H)/360]\}$.

Suppose the yield rates change at t=H to d_{sH}, then the actual investment to be made for the redemption proceeds of the face value of F_s at t=T will be $V^{*}_{sH} = F_s\{1-d_{sH}[(T-H)/360]\}$. Thus, the change in investment amount due to yield shift is $dS=- F_s\Delta d_s[(T-H)/360]$.

Using the notation as earlier the change in value of a hedge consisting of N_f futures is: $dF=-0.25N_fF^{*}_f\ \Delta d_f$.

We can set up the expression for the variance of the hedged portfolio. Thereafter on minimizing the variance, we get the number of contracts as:

$$N_f = \frac{(T - H)F_s}{360*0.25\times F_f^*}\beta_{\Delta d_s,\Delta d_f}$$

This is the number of contracts. F^*_f is the face value of underlying per futures contract.

EXAMPLE

It is t=0 and the treasurer of company X expects to make an investment of $1m redemption value in 6-month T-bills at t=3 months until t=9 months. The treasurer fears a fall in interest rates, which would imply that his investment beginning in t=3 months will earn less interest. The treasurer buys (goes long) 3-month T-bill futures today (t=0) with maturity in t=4 months. The IMM index quotes at t=0 and t=3 of the T-Bill futures turn out to be 89.2 and 90.3 respectively. Spot discount yield rates at t=0 and t=3 months are 11% and 9.6% respectively. Work out the extent to which the hedge has operated successfully. Assume spot yield shifts to be parallel.

Redemption (Face) Value of the portfolio (F_S): 1,000,000

Maturity of T-bills from now (T): 9-months

Timing of investment (Hedge Period) (H): 3-months

Face Value of T-bills underlying one futures (F^*_f): 1,000,000

$$No\ of\ \ futures: N_f = \frac{(T - H)F_s}{360\times 0.25\times F_f^*}\beta_{\Delta d_s,\Delta d_f} = \frac{(270 - 90)\times 1,000,000}{360\times 0.25\times 1,000,000}\times 1 = 2$$

PERFORMANCE OF HEDGE	0	3
FUTURES QUOTES	89.2	90.3
DISCOUNT YIELD	0.108	0.097
FUTURES PRICE	973000	975750
PROFIT PER CONTRACT		2750
PROFIT ON FUTURES POSITION		5500
CASH POSITION		
DISCOUNT YIELD RATE	0.11	0.096
MATURITY	0.5	0.5
INTEREST INCOME	55000	48000
LOSS DUE TO CHANGE IN YIELD RATE		-7000
NET LOSS		-1500

There are two important things:

(i) Spot as well as futures interest rates are expressed in terms of discount yields that relate to the redemption values than initial investments

(ii) The loss of 1,500 on the hedged investment is explained by the fact that the change in spot yields was from 11% to 9.6% i.e. by 1.4% whereas the change in futures yields was from 10.8% to 9.7% i.e. 1.1%. Thus, the spot yields had changed by 0.3% more than the futures yields but we had used a beta factor of 1.00 while calculating the number of contracts. By using beta of 1.00, we had implicitly assumed that the magnitude of changes in spot and futures yields will be equal which has not actually been the case. This is why the hedge has underperformed by 1,500 i.e. 1,000,000 x 0.3% x 0.5 year.

Let us, now, assume that the spot rates are given in terms of interest rates and not in terms of discount yields. The remaining data remains unchanged i.e. the spot interest rates change from 11% at t=0 to 9.6% t=9.6%. Then, we have, the original and revised investment amounts at t=3 months for getting a redemption of 1,000,000 at t=9 months are:

$$V_{sH} = F_s \Big/ \left(1 + r_{s0}\frac{T - H}{360}\right) = 1,000,000 \Big/ \left(1 + 0.11 \times \frac{180}{360}\right) = 947,867$$

$$Actual\ investment\ V_{sH}^{*} = F_s \Big/ \left(1 + r_{sH}\frac{T - H}{360}\right) = 1,000,000 \Big/ \left(1 + 0.096 \times \frac{180}{360}\right) = 951,498$$

$$Change\ in\ investment\ V_{sH}^{*} - V_{sH} = 6,331$$

showing that the net loss on the hedged portfolio as 831. Again, we must keep in mind that we have assume beta as one. However, while in the previous case beta was the regression of spot discount yields on futures yields, in the present case it is the regression of the factor 1/(1+r) with futures yields.

PERFORMANCE OF HEDGE	0	3
FUTURES QUOTES	89.2	90.3
DISCOUNT YIELD	0.108	0.097
FUTURES PRICE	973000	975750
PROFIT PER CONTRACT		2750
PROFIT ON FUTURES POSITION		5500
CASH POSITION		
INTEREST RATE	0.11	0.096
MATURITY	0.5	0.5
INTEREST INCOME	52132.7	45801.52672
LOSS DUE TO CHANGE IN INTT RATE		-6331.174704
NET LOSS		-831.1747042

Let us, now, consider another variant where the initial investment to be made at t=3 months is fixed and known say at 1,000,000 and the entity desires to hedge against fluctuations in the redemption value of the investment arising out of changes in interest rates/yields.

<u>Hedging of a future investment: Prefixed investment value</u>

The amount to be invested at a future date say t=H is prefixed and the impact of changes in interest rates is reflected in the changes in the redemption value of the investment at t=T. Let the changes in interest rates be represented in terms of the corresponding discount yields. We have,

Let the amount to be invested be V_{sH} *at time* H

Then, we have redemption value on the basis of discount yields at $t = 0$:

$$F_s = \frac{V_{sH}}{\left(1 - d_{s0}\frac{T-H}{360}\right)}$$

Let the actual discount yields at the date of investment i.e. at time $t = H$ *be* d_{sH}

$$\text{Actual redemption value } F_s^* = \frac{V_{sH}}{\left(1 - d_{sH}\frac{T-H}{360}\right)}$$

$$\text{Change in redemption proceeds } dF = F_{sH}^* - F_{sH}$$

$$= V_{sH}\left(d_{sH} - d_{s0}\right)\left(\frac{T-H}{360}\right) = V_{sH}\Delta d_s \frac{T-H}{360}$$

$$Hence, change\ in\ value\ of\ \exp osure\ due\ to\ yield\ change$$

$$dF = V_{sH}\Delta d_s\left(\frac{T - H}{360}\right)$$

$$For\ each\ futures, P_{0,f} = F_f^*\left(1 - 0.25d_{f0}\right);\ P_{H,f} = F_f^*\left[1 - 0.25d_{fH}\right]$$

$$Change\ in\ value\ of\ N_f\ futures\ dF = -0.25N_f F_f^*\Delta d_f$$

$$N_f = \frac{(T - H)V_{sH}}{360 * 0.25\times F_f^*}\beta_{\Delta d_s,\Delta d_f}$$

Let us continue with the above example. We, now, assume that the amount to be invested at t=3 months as 1,000,000. Let the spot discount yields change from 11% initially to 9.6% when the investment is actually made and the futures quote be respectively 89.2 and 90.3 and the initiating and lifting of the hedge. The number of contracts assuming a beta of 1 is:

$$No\ of\ futures : N_f = \frac{(T - H)V_s}{360\times 0.25\times F_f^*}\beta_{\Delta d_s,\Delta d_f} = \frac{(270 - 90)\times 1,000,000}{360\times 0.25\times 1,000,000}\times 1 = 2$$

Thus, the profit from the hedge is calculated as earlier and works out to 5,500. This is received at t=3 months when the hedge is lifted. Let us work out the change in the redemption value of the investment. It is

$$F_s = \frac{V_{sH}}{\left(1 - d_{s0}\frac{T - H}{360}\right)} = \frac{1,000,000}{\left(1 - 0.11\times\frac{180}{360}\right)} = 1,058,201$$

$$F_s^* = \frac{V_{sH}}{\left(1 - d_{sH}\frac{T - H}{360}\right)} = \frac{1,000,000}{\left(1 - 0.096\times\frac{180}{360}\right)} = 1,050,420$$

$$dF = 7781$$

so that the loss on the unhedged portfolio is 7781 but it arises at the maturity of the investment i.e. at t= 9 months. Because of this timing mismatch between the profit on the futures (t=3 months) and the loss on the investment (t=9 months), the two are not per se comparable. We need to eliminate this timing difference either by taking the present value of the loss on the investment or the future value of the profit on the hedge. Let us do the former using the current discount yield of 9.6%. The present value of the loss of 7781 works out to: 7781(1-0.096 x 0.50) =7407 so that the net loss on the hedged investment worked out at t=3 months is 1907.

Finally, if the spot rates are given as interest rates of 11% and 9.6% instead of discount yields, the change in the redemption value and the loss on the hedged investment works out as:

$$F_s = V_{sH}\left(1 + r_{s0}\frac{T - H}{360}\right) = 1,000,000\left(1 + 0.11\times\frac{180}{360}\right) = 1,055,000$$

$$F_s^* = V_{sH}\left(1 + r_{sH}\frac{T - H}{360}\right) = 1,000,000\left(1 + 0.096\times\frac{180}{360}\right) = 1,048,000$$

$$dS = 7000;\ PV\ of\ dS = 6,679;\ Net\ Loss : 1,179$$

PERFORMANCE OF HEDGE	0	3
FUTURES QUOTES	89.2	90.3
DISCOUNT YIELD	0.108	0.097
FUTURES PRICE	973000	975750
PROFIT PER CONTRACT		2750
PROFIT ON FUTURES POSITION		5500
CASH POSITION		
INTEREST RATE	0.11	0.096
MATURITY	0.5	0.5
INTEREST INCOME	55000	48000
LOSS DUE TO CHANGE IN INTT RATE		-7000
PV OF THE LOSS		-6679.389313
NET LOSS		-1179.389313

Thus, while hedging a future investment certain important issues need to be taken care of:

(i) Whether the interest rates on the investment are being quoted I terms of interest rates or discount yields. The beta equals unity is a reasonable assumption only when the method of quoting of spot interest rates and futures interest rates is the same, otherwise a small correction for the difference should be made for precise outcomes.

(ii) Whether the objective of the hedge is to hedge the future (redemption) value of proposed investment (which is predetermined) or whether the redemption value of the investment is fixed and it is desired to hedge the investment value. In the former case, the number of contracts need to be calculated with reference to the proposed investment value while in the latter case they are obtained with reference to the redemption value. Furthermore, in the former case, it is particularly convenient to work out the hedge parameters and assess its effectiveness if spot rates are quoted as interest rates rather than discount yields while in the latter case use of discount yields (since they operate on face values) are particularly appropriate. However, beta in each case needs to be worked out in accordance with the convention used.

(iii) The possibility of timing mismatch between the point at which the hedge is lifted and hence the profit/loss on the hedge is generated and the point at which interest/discount is obtained on the hedged investment must be carefully considered. It is paramount to translate both the profits/losses on the hedged asset and the hedge at the same point in time by taking present/future values before the figures are compared and hence, hedge effectiveness is assessed. For example, when discount yields are used in spot rates, they operate on the face (future, redemption) value and return the spot value. Hence, in such a situation, the profit on the hedge and the loss of interest calculated using discount yields may be assumed to occur at the point in time and are therefore per se comparable. However, the same may not be the case when we use interest rates that operate on initial value of investment. In this case the profit on futures arises when the investment is initiated (which coincides with the timing of the investment) while the loss of interest occurs at the end of the investment. Hence, this loss needs to be discounted before it is compared with the profit on the hedge. So the present value correction was necessary because we were using different parameters for the measurement of return. Discount yields in the futures markets and interest rates on spot investments. This non-compatibility between the two mandated that we work out the present value.

Hedging a future borrowing

We take up an example:

EXAMPLE

Suppose that it is t=0 and a treasurer realizes that at t=5 months the company will have to issue $5 million of commercial paper with a maturity of 6 months. If the paper were issued today, the company would realize $4,820,000. (In other words, the company would receive $4,820,000 for its paper and have to redeem it at $5,000,000 in 6 months' time). The T- Bill futures for delivery at t=7 months from now are quoted at 92.00. How should the treasurer hedge the company's exposure? SHORT! Why???

Suppose at t=5 months from now, the T bill futures are quoting at 88.20 and the CP yields have risen by 1% over this period. Calculate the net profit/loss on the hedged position. You may assume parallel shifts in yields.

Now let us look at the above problem. Suppose that it is t=0 and a treasurer realises that at t=5 months, the company will have to issue USD 5,000,000 face value of commercial paper with a maturity of 6 months.

The amount that it will actually receive will be the present value of this face value. Commercial paper a discount instrument, so it will be redeemed at face value and it will be issued at discount to face value. The difference between the two will constitute a return for the investor.

So, USD 5,000,000 is the face value and 6 months is the tenure of the instrument. Then the problem says if the instrument was issued today, the company would realise 4,820,000. The treasurer is worried that if the interest rates increase then the present value (at t=5 months) of USD 5,000,000 which constitutes the amount realizable by issuing the CP, will decrease. So if the interest rates increase, the amount that he will receive as at t=5 months will fall. The treasurer wants to protect against a rise in interest rates as a result of which his collection from the issue proceeds of the CP do not fall down.

To hedge against the above rise in interest rates, he takes a short position in T bill futures contracts. If the interest rates rise, the T bill futures will decrease in value and he will make a profit on the hedge. Thus, if the interest rates in the T-bill market mirror the behaviour of interest rates in the CP market which is usually the case since both are short term fixed rate instruments, and the T-bill futures market correlates positively with the T-bill spot market, the T-bill futures hedge can be effective in smoothing down the CP price fluctuations.

The number of contracts constituting the hedge are worked out as follows:

NO OF CONTRACTS (SHORT)			
FACE VALUE OF BORROWINGS	5000000		
MATURITY OF BORROWINGS FROM NOW	330	DAYS	
COMMENCEMENT OF BORROWINGS	150	DAYS	
PERIOD OF BORROWINGS	180	B319	
FV OF EACH BILL FUTURE	1000000		
NO OF FUTURES	10	(T-H)*F(s)/360*0.25*F(f)	

Hence, the treasurer shorts 10 T-bill contracts each of the face value of USD 1,000,000. We, now, assess the effectiveness of the hedge:

At t=5 months from now, he liquidates the hedge at 88.20 and the yields on the CP have increased by 1% over their value at t=0. The word used as yields, please note this. Since the yields has increased by 1%, therefore this proceeds from the CP issue will decline when the actual issue is made. and we have to analyse whether the T bill futures hedge has been effective or not.

PERFORMANCE OF HEDGE	0	5
FUTURES QUOTES	92	88.2
DISCOUNT YIELD	0.08	0.118
FUTURES PRICE	980000	970500
PROFIT PER CONTRACT		9500
PROFIT ON FUTURES POSITION		95000
CASH POSITION	0	5
TENURE OF BORROWINGS	0.5	0.5
DISCOUNT YIELD	0.072	0.082
ISSUE PROCEEDS	4820000	4795000
LOSS DUE TO CHANGE IN INTT RATE		-25000
NET PROFIT ON HEDGED POSITION		70000

The futures prices corresponding to the quotes at hedge inception (92.00) and hedge lifting (88.20) work out respectively to 980,000 and 970,500 so that the profit on the hedge of 10 short futures is 95,000.

As far as the cash position goes, it is given that the CP would have realized 4,820,000 at the original yield rate, which, thus, works out to 7.2%. It is also given that the yield rate has increased by 1%. So the current yield is 8.2%. Hence, the realization at current yields will be 4,795,000 i.e. a fall in realization of 25,000. Thus, the entity makes a profit of 70,000 on the overall hedged position.

If we analyse this profit made on the hedged borrowings, we note the following:

(i) Let us assume that the discount yields in both the CP markets and the T-bill futures market had both changed by the same respective magnitudes as in the earlier but in the opposite direction i.e. that CP yields had decreased by 1% and the T-bill yields had decreased by 3.8% instead of increasing by these values. The changed situation would, then, have been as follows:

PERFORMANCE OF HEDGE	0	5
FUTURES QUOTES	92	95.8
DISCOUNT YIELD	0.08	0.042
FUTURES PRICE	980000	989500
PROFIT PER CONTRACT		-9500
PROFIT ON FUTURES POSITION		-95000
CASH POSITION	0	5
TENURE OF BORROWINGS	0.5	0.5
DISCOUNT YIELD	0.072	0.062
ISSUE PROCEEDS	4820000	4845000
LOSS DUE TO CHANGE IN INTT RATE		25000
NET LOSS		-70000

Thus, the hedged borrowing would have still carried a loss of 70,000. In fact, in this case, the borrowings would have generated a higher cash inflow of USD 25,000 at the point of borrowing (t=5 months) but the hedge would have eroded that profit by generating a loss of USD 90,000. This is a manifestation of the fact that the payoff from a futures is a linear function of the price of the underlying at maturity. The important thing is that both yields are stochastic processes, and thus, randomness is intrinsic. In the first case, when the yields increased we generated a profit on the hedged borrowings while in the latter case, when yields fell, we collected a loss. But either outcome and, in fact, a spectrum of outcomes, is possible and because these outcomes are random, they may be aligned with a probability distribution.

(ii) There is another intricate issue. And that is, why did the profit of USD 70,000 arise in the first place? Why did the hedge not result in neutralization of the spot market loss and no more? Obviously, this is because there was over-hedging. Let us see how. The change of yields in the spot markets was only 1%. However, the yields in the futures markets changed by as much as 3.8%. thus, there was massive mismatch between the change in yields in the two markets. But, when we worked out the number of contracts, what beta did we assume? We assumed a beta of one. That means that we worked out the number of futures contracts required for hedging on the premise that the changes in yields in the two markets would be close to each other approximately same. To justify this contention, let us examine the situation if we use a beta of 1/3.8 since beta is the ratio of changes in spot to changes in futures being the slope of the regression of changes in spot on changes in futures. We have:

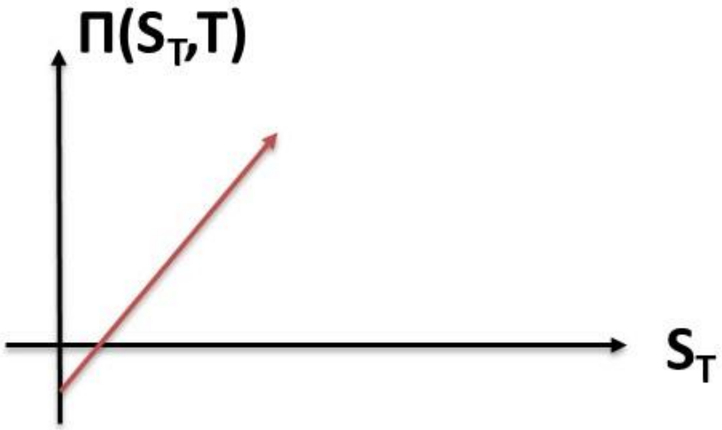
NO OF CONTRACTS (SHORT)		
$\beta_{(ys,yf)}$=1/3.8		
FACE VALUE OF BORROWINGS	5000000	
MATURITY OF BORROWINGS FROM NOW	330	days
COMMENCEMENT OF BORROWINGS	150	days
RESIDUAL LIFE	180	days
FV OF EACH BILL FUTURE	1000000	
NO OF FUTURES	2.631579	

PERFORMANCE OF HEDGE	0	5
FUTURES QUOTES	92	95.8
DISCOUNT YIELD	0.08	0.042
FUTURES PRICE	980000	989500
PROFIT PER CONTRACT		-9500
PROFIT ON FUTURES POSITION		-25000
CASH POSITION	0	5
TENURE OF BORROWINGS	0.5	0.5
DISCOUNT YIELD	0.072	0.062
ISSUE PROCEEDS	4820000	4845000
LOSS DUE TO CHANGE IN INTT RATE		25000
NET LOSS		0

We clearly end up with a perfect hedge with the entire loss being precisely compensated by the profit on the hedge. Thus, on the basis of our perception, which was that the spot and futures yields would move in equal magnitudes, we planned a suitable hedging strategy. And, indeed, if the future had evolved according to our perception, we were protected. However, in actual fact the future yields have far overshot the spot yields, the spot yield has increased by 1%, the futures yields have increased by 3.8% and that is the reason why the exposure has been over hedged. Now, the important thing is I took beta as 1/3.8 but wherefrom did I get this figure? I got them from actual data, this actual data would not be available at the time of planning of the hedge, one needs to make a forecast of it. The forecast that we made was 1which turned out to be wrong. So this a post facto analysis. This is not what one could have planned with absolute accuracy, because these prices and yields are random variables.

<u>Linearity of futures payoff</u>

Linearity of futures payoff $\Pi(S_T,T)=S_T-F_0$
For every unit increase & decrease in S_T, there is a unit increase/decrease in payoff.
Slope is 45˚.
Parallel shifts.



The futures payoff is linear. It means that as S_T increases, futures payoff increases in a straight line. In fact, the futures payoff is given by $S_T - F_0$, F_0 is the price at which the long futures position is taken and S_T is the spot price of the underlying on the maturity date of the futures. So, unit increase in S_T implies a unit increase in the payoff from futures contract.

Parallel shifts

The second thing is we assume parallel shifts while computing number of contracts. This manifests as beta equal to usually which is usually assumed. In other words, for all maturities, the yield curve shifts parallel to each other i.e. given any maturity say a maturity of 5 years and maturity of 3 years, the increase in yields is the same. This implies that the forward rates would also shift by the same amount.

Recap

$d_f=100-Q_f$

$d_f= (100-P_f)/(100*0.25)$

$P_f=100*(1-0.25d_f)$

Just to recap again, yields given by $d_f=100- Q_f$, Q_f is the quoted price. The above are very fundamental equations.