Financial Derivatives and Risk Management
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Lecture 23
Interest Rate Futures: Salient Features

Interest rate futures

The underlying asset of an interest rate future is an interest bearing instrument. It can be a Treasury bill (T-bill), Eurodollar deposit, Treasury bond (T-bond) or any similar interest bearing instrument. So interest rate futures are an example of interest rate derivatives. Common interest rate derivatives traded across the world are:

(i) Treasury bill (T-bill) futures

(ii) Eurodollar futures

(iii) Treasury bond (T-bond) futures

Of course there are innumerable interest rate futures trading across the world, but the ones that are representative and are going to discussed in the sequel are listed above.

Treasury bill futures

These have short term T-bills as the underlying instruments. The salient features of the T-bill futures contracts traded on the NSE in India are briefly enumerated below:

Underlying	91-day Government of India (GOI) Treasury Bill
Trading Unit (Lot Size)	2000 units (Face Value Rs 2 lacs)
Tick Size	INR 0.0025
Contract Trading Cycle	Three Serial monthly contracts followed by one quarterly contract of the cycle Mar/June/Sept/Dec
Price Quotation	$100-d_f$; d_f= futures discount yield
Contract Value	Rs 2000 x ($100 - 0.25d_f$)

Settlement

Daily Settlement Price	**Rs (100-0.25y_w) where y_w is weighted average futures yield of trades during the time limit as prescribed by NSE Clearing.**
Daily Contract Settlement Value	**Rs 2000 (100 - 0.25y_W).**
Final Contract Settlement Value	**Rs 2000 (100 - 0.25y_f)**
Mode of Settlement	**Settled in cash in Indian Rupees**

Daily settlement price

y_W (futures yield) shall be volume weighted average futures yield of traded futures contracts in the last 30 minutes of trading subject to there being at least 5 trades. Failing which, trades during the last 60 minutes shall be used for the calculation, subject to at least 5 trades. Failing which, trades during the last 120 minutes shall be used for the calculation, subject to at least 5 trades.

Final settlement price

y_f is weighted average discount yield obtained from weekly auction of 91-day T-Bill conducted by RBI on the day of expiry of the futures.
The tick size is the minimum fluctuation that is allowed by the exchange in terms of which quotations can be placed. It is INR 0.0025 per unit of contract or INR 5 per contract since each contract consists of 2,000 units.
Three serial monthly contracts are notified for trading by the exchange followed by one quarterly contract of the cycle Mar/June/Sept/Dec.

Price quotation for contracts

The method of quoting of the price of T-bill futures is similar to what we have for the treasury bills, which are the underlying instruments. Prices are quoted in terms of futures discount yields. The quotations are made as:

Q_f=100-d_f

where Q_f is the quotation and d_f is the futures discount yield expressed in percent, not the discount yield on the underlying T-bill. The quoted value of the contract is given by 2,000Q_f, because each contract has a lot size of 2000.

Traded value of contract

While the quoted value of a contract is given by 2,000Q_f, the actual traded value or price at which the contract is traded is given by:

P_f= 2000 x (100 - 0.25d_f)

where d_f is the futures discount yield in percent and is related to the quoted price Q_f as d_f=100-Q_f. The quoted price is the price at which market players quote their orders. However, settlement of trades takes place at the traded price.

The factor 0.25 comes up from the fact that the underlying instrument it a 91-day T-bill approximated by 90 days with the year being taken at 360 days which is the usual money market convention. Of course, each contract consists of 2,000 units.

The daily mark to market settlement is calculated on the basis of the daily settlement price 100-0.25y_w, where y_w is the weighted average discount yield worked out on the basis of the trades occurring in the last half hour subject to a minimum of 5 trades. If 5 trades do not take place in the last half hour, we take the average of the trades in the last one hour, provided at least 5 trades take place in that period. If that also does not happen, we take the average of trades over the last two hours of trading provided that at least 5 trades have taken place for working out the average. The average is volume weighted.

For the daily contract settlement price, you multiply (100-0.25y_w) by 2000.

The final contract settlement price is worked out in terms of y_f, y_f is the yield determined on the basis of the weekly auction by RBI of treasury bills which is the underlying instrument. In other words, at final settlement, we are again equating the futures yield to the spot yield on the date of the maturity of the contract. Thee spot yield is determined by the average yield on the weekly auction of treasury bills by RBI. Precisely, y_f is weighted average discount yield obtained from weekly auction of 91-day T-Bill conducted by RBI on the day of expiry of the futures. The T-bill futures are settled in cash.

At this point, it is pertinent to explicitly point out certain features of these futures:

(i) There are two prices in relation to a traded T-bill futures viz

(a) the quoted price, it terms of which quotations by the exchange and the market participants are made Q_f=100-d_f where d_f is the futures discount yield at the time of the quotation.

(b) the actual traded price with respect to which trades are settled i.e. with respect to which profit and loss and cashflows to the margin account are computed. This is given by: $P_f=zF(1-0.25*d_f/100)$ where F is the face value of the underlying T-bill, z is the lot size i.e. the number of T-bills covered by one contract and d_f is the discount yield in percent $d_f=100-Q_f$.

(ii) If an investor enters a long position at a market quote of Qf,0 and exits at a quote of Qf,1, his profit from the holding works out to:

$$\Delta P_f = -zF(0.25\Delta d_f/100) = zF(0.25\Delta Q_f/100)$$

(iii) It is pertinent to mention here that the number "0.25" is not sacrosanct by any means. The traded price can very well be computed by reference to any other multiplier. To understand the role of this multiplier, let us assume that a futures contract is initially notified for trading by the exchange at a price F^*_0. On maturity, the contract is necessarily marked to the spot price on the date of maturity i.e. S_T. Thus, the price change over the life of the futures (S_T-F_0) is determined completely by the behaviour of S_T (since F_0 is known upfront) and is clearly independent of the futures prices at the various daily settlements. In other words, whatever value we may impute to the daily settlement prices or howsoever these daily settlement prices are computed, the overall price change S_T-F_0 remains unaffected. This implies that instead of using a certain formula for the daily settlement price e.g. the last traded price of the day, we use a different formula e.g. twice the last traded price, the overall change in price S_T-F_0 remains unaffected. What will happen in such a case is that the intermediate daily mark to market flows (positive as well as negative) will get doubled. Thus, the volatility of the transfers to margins on account of daily mark to market settlements will get doubled without influencing the overall character of the futures.

US T-bill futures

The US Treasury bills are short-term debt obligations backed by the U.S. government with a maturity of less than one year. T-bills are sold in denominations of $ 1,000 up to a maximum purchase of $ 5,000,000 and commonly have maturities of one month (4 weeks), three months (13 weeks), six months (26 weeks) and 1 year.

Salient features of US T-bill contracts are presented below:

Underlying Asset	**Usually 3 month T-Bill at the time of issue sold at a discount and quoted at Actual/360 day basis**
Unit of trading	**USD 1.00 million**

Delivery Months	**Mar/June/Sept/Dec**
Mode of Quote	**IMM Index (100-y_f)**
Tick Size	**0.50 BP = USD 12.50**
Settlement	**90,91,92 day T-Bill depending on the months in the contract, physical delivery.**

In US T-bills the underlying instrument that is tendered for delivery is a T-bill with 3 months to maturity and USD 1.00 million face value. Three months can mean 90, 91, or 92 days, depending on the month in which the contract was issued.

These bills are auctioned at regular intervals, and the delivery dates of the futures contracts are set so that on a delivery date, there are three possible underlying T-bills: the new 3-month T-bill, the "seasoned" 6-month T-bill, and the "seasoned" 1-year T-bill.

The seasoned 6-month bill is that which was issued three months ago and has three months left to maturity. The seasoned one-year bill was issued nine months ago and also has three months left.

Settlement occurs only in March, June, September, and December.

These dates set up a cycle of traded contracts. For example, in March, three T-bill futures contracts that are trading are those for delivery in June (of a currently 6-month T-bill), September (of a T-bill which currently has nine months to maturity), and December (of the T-bill with one year to maturity).

With each of these contracts, the underlying bond is also traded. However, you can trade futures with delivery dates even up to two years in the future. For the longer maturity dates, the underlying bond does not trade while the futures contract does.

On a given delivery date, three T bills are available for settlement the current 3-month bill, the 6-month bill issued 3 months ago, the 12-month bill issued 9 months ago.

On a given trade date (e.g. March), three futures are available for trading June delivery of the bill maturing in Sept, Sept delivery of December maturing bill and Dec delivery of March maturing bill.

The contract specifications are very similar to what we have for the Indian T-bill futures, except that the contract size here is USD 1.00 million. For Indian T-bill futures, the contract size was 0.20 million. The method of quotation is again similar i.e. 100-d_f.

The tick size here is given in terms of discount yield as is 0.50 bps i.e. 0.005% which translates to USD 12.50 as shown below:

ΔP_f =1,000,000x0.25x0.005/100 = USD 12.50.

Traded price vs index price

The "quoted" or "index" price Q_f = 100-d_f where d_f is the quoted discount yield on the futures contract.

The actual futures traded value at which the contract is traded per face value F of the underlying is $P_{trade} = F\left(1\text{-}0.25\frac{d_f}{100}\right)$.

If the contract multiple is z, then traded value per contract is $P_{trade} = zF\left(1\text{-}0.25\frac{d_f}{100}\right)$.

For the NSE T-bill futures the face value of one contract is zF=INR 200,000 while for the US T-bill contract it is USD 1,000,000.

Futures discount yield

We work the spot yield on T-bill as:

$$d_s(\%) = \frac{P_1 - P_0}{P_1} \times \frac{360}{N} \times 100$$

The process of calculating futures discount yield is absolutely similar. In the case of futures N=90 days, P_1 is the face value of the T-bill to be delivered and P_0 will be the price at which the futures is traded i.e. the price at which the futures will be settled against the delivery of the underlying T-bill. Thus, for futures yield, the above expression gets modified to:

$$d_f(\%) = \frac{F - P_{trade}}{F} \times \frac{360}{90} \times 100 \ or \ P_{trade} = F\left(1 - 0.25 \times \frac{d_f}{100}\right)$$

Discount yield vs interest rates

It is interesting to make a comparison on the relationship between discount yields and interest rates. \They are respectively defined by:

$$P_0 = P_1(1 - dN) \ and \ P_1 = P_0(1 + rN)$$

where d is a discount yield while r is an interest rate. It is clear from these formulae that the discount yield operates on the terminal value of an investment to yield its current value the interest rate operates on the initial vale of the investment to give the terminal value. To examine the relationship between d and r, we note that:

$$P_0 = P_1(1-dN) = \frac{P_1}{(1+rN)} = P_1(1+rN)^{-1} = P_1\left[1-rN+O(rN)^2+...\right]$$

so that d,r would yield approximately equivalent results so long as rN is small. A comparison of the computation of P_0 using d,r for different maturities is tabulated below:

COMPARISON OF YIELD RATE & INTEREST RATE					
MATURITY	0.25	0.25	0.25	0.25	0.25
RATE	5%	7%	10%	12%	15%
PRICE (YIELD)=F(1-yN)	98.75	98.25	97.5	97	96.25
PRICE (INTT)= F/(1+rN)	98.7654	98.280098	97.561	97.0874	96.3855

Hedging with interest rate futures

Let us take an example. Say at t=0, an investor takes a long position in a T-bill futures at a certain price, say, P_f. It means that on the date of maturity of the futures i.e. t=T, he will receive a 90-day T-bill i.e. a T-bill that will mature 90 days after that date of delivery (i.e. the date of maturity of the futures) for payment. The investor in the futures will pay the price P_f at t=T but will receive the maturity proceeds of the T-bill 90 days later i.e. at t=T+90. Obviously, P_f is determined at t=0.

Thus, the contract is negotiated at t=0, Price at which the futures will be settled i.e. at which the T-bill will be received by the long futures is agreed at t=0.

The contract is settled at t=T, the maturity date of the futures by the short delivering a 90-day T-bill to the long and the long paying the price P_f.

The T-bill now belongs to the party who was long in the futures. He will receive the proceeds (face value) of the T-bill when it matures at the end of 90 days from delivery i.e. at t=T+90.

The important thing is that the price that the investor paid at t=T to get the 90-day T-bill would normally be the market price of the bill at that date. However, because he has taken a long futures on the bill, this price if prefixed at t=0.

Secondly, what the long futures holder receives is a 90-day T-bill which will be redeemed by the issuer of the bill after 90 days. In other words, after 90 days the long futures party will receive the face value of the T-Bill.

Now, if during the period (0,T) there is a downward shift in interest rates, the price of the bill in the cash market will, obviously, appreciate since price and interest rates are inversely related. It is likely that the futures price of the bill will also rise.

But the long futures holder is already assured of getting this bill at a prefixed price. A price which is fixed beforehand, before this interest change occurred. The price has now increased. Therefore, the value of the long futures position will rise. because that position holder will be getting the bill at a lesser price compared to the updated futures price. Thus, the long futures holder makes a profit on his long futures in the event of an unanticipated decline in interest rates.

Thus, If the investor’s portfolio is such that a downward shift at interest rate will result in a loss, and he desires to protect his portfolio against such an eventuality, he can, by taking up long T-bill futures create a hedge. The hedge would so operate that if the interest rates move downwards, his long T-bill futures would generate profits that would compensate his losses on the portfolio. Of the converse will also hold.