Financial Derivatives & Risk Management
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Lec 20: Other Yield Measures

Ok, so I explained the relationship between the YTM and the bond price. If the coupon rate is higher than the YTM, it will increase the demand for the bond which will translate to an increased price and once the price increases the yield on the bond comes down towards the market rate. These are the underlying market dynamics.

Implied assumptions of YTM

The computation of YTM rests on two fundamental assumptions:

(i) The reinvestment rate equals the YTM;

(ii) The holding period of the bond coincides with its maturity.

So, what is reinvestment rate? Reinvestment rate is the rate at which the intermediate cashflows are reinvested or deemed to be reinvested. When an investor buys a bond, he receives interest payments (coupons) at periodic intervals. Usually, the principal is repaid on maturity. However, even the principal may be repaid as per a staggered schedule in multiple instalments. In any case, the holding of a bond, usually, results in periodic cashflows (whatever the source of such flows may be e.g. interest or principal or both). These intermediate cashflows may be reinvested by the investor until the end of his holding period when he wants to liquidate the entire investment. The interest rate at which he reinvests these intermediate cashflows is the reinvestment rate.

The first assumption above states that when we compute YTM of a bond, we IMPLIEDLY assume that all the intermediate cashflows are being reinvested at the YTM itself. The YTM formula is:

$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$ which can be written as $P_0 (1+y)^T = \sum_{t=1}^{T} C_t (1+y)^{T-t}$.

It is clearly seen from the second expression that the very formula used for calculating YTM implies that all the intermediate cashflows C_t are being implied to be reinvested at the YTM rate.

To reiterate, we do not explicitly make this assumption. It follows from the YTM computation formula itself that all intermediate cash flows are reinvested at the YTM itself. Whatever YTM is worked out, the formula implies that all intermediate cash flows are reinvested at the YTM itself. The investor does not make that assumption, there is nothing exogenous about it. The very computational formula implies this particular assumption (that all intermediate cash flows are reinvested at the YTM rate).

Since this is a very important matter, I illustrate with a simple example. For this purpose, I consider two bonds A & B. A is a 2-year zero coupon bond while B is a one-year bond. Both bonds are quoting at the same price of 916.77 with a YTM of 12%. The redemption values of A works out to 1,150.00 and that of B is 1026.80. Now, if both the bonds are to provide the same YTM over the 2 year period, then the proceeds received against redemption of bond B (1026.80) at the end of the first year must, on reinvestment, yield the redemption value (1,150) of the bond A at the end of the second year i.e. we must have $1026.80(1+r_{12})=1150$ which gives $r_{12}=12\%$ which is the YTM of the bonds. Thus, the reinvestment rate must equal the YTM.

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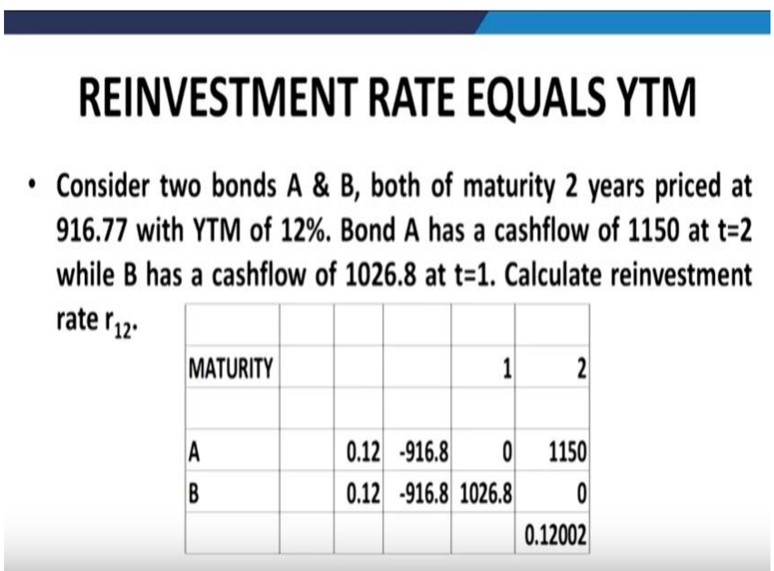

Let us study the implications of this assumption. They are very important and revealing. Consider the following example:

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ISSUES WITH YTM CONTD…

- **Which is better if the reinvestment rate is 9% pa?**

	Bond A	Bond B
Coupon	10%	3%
Face Value	100	100
Price	138.90	70.22
Maturity	15 years	15 years
Frequency of payment	Annual	Annual
Yield to maturity	6%	6.1%

We have two annual coupon bonds A & B both with maturity of 15 years. A pays a 10% annual coupon while B pays a 3% annual coupon. The current market price of A is 138.90 while B is quoting at 70.22. The face value is 100 for both the bonds. The yield to maturity of the bond A works out to 6% and the bond B is 6.1%. We need to study the impact of reinvestment rate of coupon payments on the return obtained by investing in the bonds.

Prima facie, if one goes by the YTM as a measure of return, then bond B happens to be superior. It promises a higher YTM of 6.1% compared to A which has a YTM of 6%. However, let us analyse the issue further. Let us assume that the reinvestment is done at 6.66%. We then have:

$$For\ \ Bond\ \ A: Coupon\ rate: 10\%;\ Face\ value: 100$$
$$Annual\ \ Coupon\ \ Payment: 10;\ \mathrm{Re}investment\ \ rate: 6.66\%$$
$$\mathrm{Pr}oceeds\ \ from\ reinvested\ \ coupons\ \ for\ \ A = \sum_{t=1}^{15} cF\left(1+r\right)^{15-t}$$
$$= 10\times 1.0666^{14} + ... + 10\times 1.0666 + 10 = 243.6823$$
$$\mathrm{Re}demption\ \ at\ \ face\ value = 100$$
$$Total\ proceeds\ on\ \ maturity\ \ at\ \ t = 15\ \ years = 343.68$$
$$Current\ \ price: 138.90$$
$$Annualized\ \ return:\ r_A = \left(\frac{343.68}{138.90}\right)^{1/15} - 1 = 6.23\%$$

Similar calculations for bond B yield $r_B = 6.21\%$

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	A	B
REINVESTMENT RATE	6.66%	6.66%
REINVESTED COUPON	243.6823	73.1047
RED VALUE	100.12223	100.321
TOTAL	343.80453	173.426
PRICE	138.9	70.22
RETURN	1.062284	1.06213

Let us look at the figures carefully. At a reinvestment rate of 6.66% the coupons of A, each of 10, is being reinvested at 6.66%. The same happens for B as well, but the coupon rate of B is only 3% thus, the amount of each coupon that is reinvested for B is only 3. Clearly, higher the coupon rate, higher is the contribution of the reinvestment income to the yield. Higher also is the impact of the reinvestment rate.

Now, when we worked out the YTM of A we got 6%. It follows that we assumed that the coupons (of 10 each) were reinvested at 6%. Now, we are working out the yield at a reinvestment rate of 6.66%. Because of the relatively large coupon size, and hence larger reinvestments at this higher rate of 6.66%, more reinvestment income is generated and the overall yield jumps significantly from 6% to 6.23%. In the case of B, because coupon is small (3) and it was already being assumed to be reinvested at 6.1%, the increase in reinvestment rate to 6.66% is not having at large an impact as A.

The returns on A & B with different reinvestment rates are tabulated below:

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PRICE	A	138.9	B	70.22		
INTT RATE	TOTAL CASH FLOW		TCF/PRICE		RETURN	
	A	B	A	B	A	B
0.06	332.88	170.15	2.39654	2.4231	0.06	0.0608
0.061	334.66	170.68	2.40936	2.43065	0.0604	0.061
0.065	341.94	172.87	2.46177	2.46183	0.0619	0.0619
0.066	343.805	173.4261	2.47519	2.46975	0.0622	0.06212
0.07	351.41	175.71	2.52995	2.50228	0.0638	0.0631

From the above table, at 6%, 6.1%, 6.5% bond B continues to provide slightly higher returns than A but at 6.66% onwards, the situation reverses. So what is the bottom-line, the bottom line is that YTM is not unambiguous.

There is another issue with the YTM. We know that return of a portfolio of assets is the weighted average return of its constituents, beta of a portfolio is the weighted average beta of the portfolio constituents. Unfortunately, the YTM of a portfolio of assets is not the weighted average YTM of the constituent securities. I illustrate this by an example:

A portfolio is made up of equal proportions of securities A and B each of face value 100. The current price of A is 100 and its respective cash flows over the next three years are 15, 15, and 115. The price of B is also 100 and it gives cashflows of 6,106 and 0 at the end of years 1, 2 and 3. We then have,

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PORTFOLIO		0	1	2	3
A	0.15	-100	15	15	115
B	0.06	-100	6	106	0
PORTFOLIO	0.11289	-200	21	121	115

YTM (A)=15%; YTM (B)=6% while YTM(P)=11.29%.

A's YTM is 15% and B's YTM is 6% because both of them are quoting a par. Therefore, coupon rates must equal the YTM. Using the aggregate cashflows from the combination of the two bonds, the YTM of the portfolio works out to 11.29%. However, the weighted average YTM of A & B is 10.5%. This is another shortcoming of the YTM measure.

<u>Bond Prices & YTM</u>

Bond prices and YTM are inversely related:

(i) The value of a bond would be equal to its face value only when the coupon interest rate and YTM are equal.

(ii) The value of a bond would be higher (lower) than its face value when the coupon interest rate is higher (lower) than YTM.

(iii) The value of a bond would decrease (increase) as it approaches maturity, if it is a premium (discount) bond

While (i) and (ii) have already been explained earlier, (iii) follows from the fact that bonds are usually redeemed at face value. Hence, on the maturity date the value of a bond must equal its face value. It, therefore, follows that if a bond is quoting at a price higher than face value (premium bond) at a particular date before maturity, the excess above face value (premium) must be amortized in the period leading upto maturity. At maturity, it must be valued at the face value because it will be redeemed on that date itself at face value. In other words, the value of the bond will gradually decline to face value by the time the bond reaches maturity.

In view of the shortcomings of the YTM concept, we, now, introduce the concept of “holding period” yield or “realized” yield.

Holding period yield or realized yield

In the context of YTM, we had made two fundamental assumptions viz. that of (i) the reinvestment rate equalling YTM and (ii) the holding period equalling maturity. We, now, relax these assumptions and introduce the concept of holding period yield.

We define holding period yield/ realized yield is the yield actually projected/earned by the investor on his investment, taking account of his investment horizon and the projected/ actual reinvestments. It is given by the eq;

$$
\begin{aligned}
&P_0\left(1+r_y\right)^H = TOTAL\ CASH\ FLOWS\ AT\ END\ OF\ HOLDING\ PERIOD\ H \\
&= PROCEEDS\ FROM\ REINVESTED\ COUPONS + SALE\ PRICE\ AT\ t=H
\end{aligned}
$$

It is pertinent to mention that *t=H* is the investor’s investment horizon which may or may not coincide with the maturity of the instrument *t=T*. Besides, the cash proceeds from reinvested coupons are worked out on the basis of projected/actual reinvestment rates rather than any implicit rates (like in the case of YTM).

We, now, address an issue relating to the second YTM assumptions viz. that YTM assumes that (i) reinvestment rate equals YTM and (ii) the bond is held unto maturity. We revert to the definition of holding period yield. We have, in the case when *r=y* and the holding period *H* coincides with the maturity *T*:

$$
\begin{aligned}
&P_0\left(1+r_y\right)^{H=T} = TOTAL\ CASH\ FLOWS\ AT\ END\ OF\ HOLDING\ PERIOD\ H=T \\
&= PROCEEDS\ FROM\ REINVESTED\ COUPONS + SALE\ PRICE\ AT\ t=H=T \\
&= \sum_{t=}^{T} cF\left(1+y\right)^{T-t} + F = \left(1+y\right)^T \left[\sum_{t=}^{T} cF\left(1+y\right)^{-t} + F\left(1+y\right)^{-T}\right] = P_0\left(1+y\right)^T \\
&so\ that\ r_y = y
\end{aligned}
$$

This shows that only under the two assumptions that we made while defining YTM, the YTM so obtained gives us the appropriate/correct measure of yield.

There is a second point. The cashflows at maturity of a bond are fixed by contract at issue of the bond and hence, are known upfront. Thus, the making of these assumptions means that the YTM is not susceptible to any randomness or price changes at the time of exiting the investment. If a bond is held for a period shorter than its maturity, then the investor has to exit the investment by selling the bond in the market because the issuing company will redeem it only on maturity. Since there is an inherent uncertainty about the price that may exist at the time of sale, it exposes the investor to a certain randomness/risk. Further the yield also cannot be ascertained/predicted with precision. The situation is obviously different if the bond is held to maturity. The redemption value is known at the time of issue. Therefore it has no randomness attached to it. Hence, it is simply the current market price that determines YTM. No market price at a future date is involved. This makes YTM have a 1-1 correspondence with the current market price.

We know the maturity and the amount at which the bond is going to be redeemed. Therefore, to that extent, the YTM is deterministic. The YTM is clearly defined. The issue of the price variation at which the investment is to be liquidated is not involved in the YTM computation.

In the case of holding period yield, we have a holding period which may be less than the maturity of the bond. It is the period for which the investor remains invested in the bond. He wants to work out the return for that period. For this, he needs to work out the total cashflows that he will derive from the investment at the end of the holding period. These cash flows will be the future value of all reinvested coupons at this point plus the sale proceeds of the bond in the market (if the holding period is less than the maturity).

Properties of holding period yield

(i) The realized yield will always lie between the YTM and the reinvestment rate.

(ii) For bonds with longer term to maturity realized yield will be closer to the reinvestment rate. For bonds with shorter term to maturity, realized yield will be closer to the YTM.

If the bond has a longer maturity, a larger proportion of its total return is contributed by reinvestment income, so that the yield tilts towards the reinvestment rate rather than the YTM and vice versa.

$$\begin{aligned}
&We\ have\ (r = reinvestment\ rate)\\
&P_0\left(1+r_y\right)^T = cF\left(1+r\right)^{T-1} + ... + cF + F\\
&P_0\left(1+y\right)^T = cF\left(1+y\right)^{T-1} + ... + cF + F\\
&so\ that\\
&P_0\left[\left(1+r_y\right)^T - \left(1+y\right)^T\right] = cF\sum_{i=1}^{T-1}\left[\left(1+r\right)^i - \left(1+y\right)^i\right]\\
&Let\ r > y, then\ RHS\ is\ positive\ so\ that\ r_y > y\\
&P_0\frac{\left(1+r_y\right)^T}{\left(1+r\right)^T} = cF\sum_{i=1}^{T}\frac{1}{\left(1+r\right)^i} + \frac{F}{\left(1+r\right)^T}\\
&\text{Sin}\,ce\ it\ is\ assumed\ that\ r > y, the\ RHS\ is\ less\ than\ P_0\\
&Hence,\ r_y < r\ whence\ r > r_y > y\\
&The\ reverse\ inequality\ will\ hold\ if\ r < y
\end{aligned}$$

Treasury bill yield

Securities which have a very short maturity period of one year or less are called money market instruments e.g. treasury bills. These T-bills are very common across the world; they are issued by the central bank of the country to finance the government expenditures. They are auctioned by the central bank. They are usually issued at a discount with maturities ranging from 90 days to 364 days, carry no coupon payments and are redeemed at face value. They carry the guarantee of the government of the country for repayment of redemption proceeds. Then we have certificates of deposits, commercial papers (which are issued by corporates, usually privately placed).

Thus, treasury bills are zero-coupon instruments with a maturity of one year or less. The yield on T bills is quoted on a discount basis. It is defined by:

$$d = \frac{P_1 - P_0}{P_1} \times \frac{360}{N_{sm}}$$

Normally, we compute the yield as annualized change in price over the initial price. The T-bill yield has three significant differences over the conventional yield measures viz.

(i) The normal yield is computed as the annualized percentage change in price with reference to the initial price. However, the T-bill yield is calculated as the annualized percentage change in price with reference to the redemption proceeds i.e. the final price.

(ii) In normal yield measures, annualization is done with 365 days in the year. However, in T-bill yield, annualization is done with reference to 360-day year.

(iii) Usually, compounding is assumed while computing yield. However, T-bill yield is computed in a linear manner i.e. without compounding on a simple interest format.

<u>Relation between T-bill yield and effective yield</u>

By definition of T − bill yield

$$d = \frac{P_1 - P_0}{P_1} \times \frac{360}{N_{sm}} \quad so\ that\ \frac{P_1}{P_0} = \left(1 - \frac{dN_{sm}}{360}\right)^{-1}$$

Now, effective return

$$i_e = (1+r)^{365/N_{sm}} - 1 = \left(1 + \frac{P_1 - P_0}{P_0}\right)^{365/N_{sm}} - 1$$

$$= \left(\frac{P_1}{P_0}\right)^{365/N_{sm}} - 1 = \left[\left(1 - \frac{dN_{sm}}{360}\right)^{-1}\right]^{365/N_{sm}} - 1$$