Financial Derivatives & Risk Management
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Lecture 2
Forwards: Introduction & Pricing

In the last class I concluded the discussion of the various types of fundamental derivatives. I talked about forward contracts, futures, options and swaps, what basically they mean. Now, I will take up in detail the various aspects of forward contracts.

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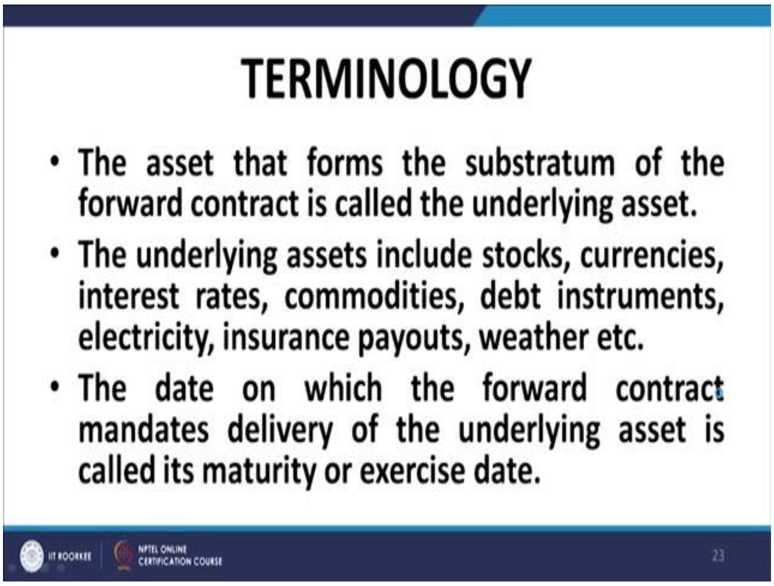

Terminology

The basic terminology first- The asset that is to be delivered i.e. that is going to change hands under the forward contract is called the underlying asset. For example, if you are having a forward contract in US dollars, that US dollars forms the underlying asset.

The maturity is the date on which the forward contract is going to be finally settled. In other words, when the US dollars shall be delivered to you by the bank and when you will make the payment against the forward contract. That is called the maturity of the contract.

The price at which the exchange is going to take place which is negotiated and finalized today is called the exercise price or the strike price. Exercise price or the strike price is also called the forward price.

Long & short positions

You are long in an asset if you own the asset. If you are owning an asset you said to be long in the asset. And if you are borrowing the asset you are said to be short in the asset. So if you hold an asset, you are owner of an asset, you have possession of the asset you said to be long in the asset and if you have borrowed the asset from somebody then said to be short in that asset.

In the context of a forward contract, a long forward position is a position where you are going to receive the asset under the forward contract. The short party is going to deliver the asset under the forward contract. So the party who is going to receive the asset is the long party, the party is going to deliver the asset is the short party.

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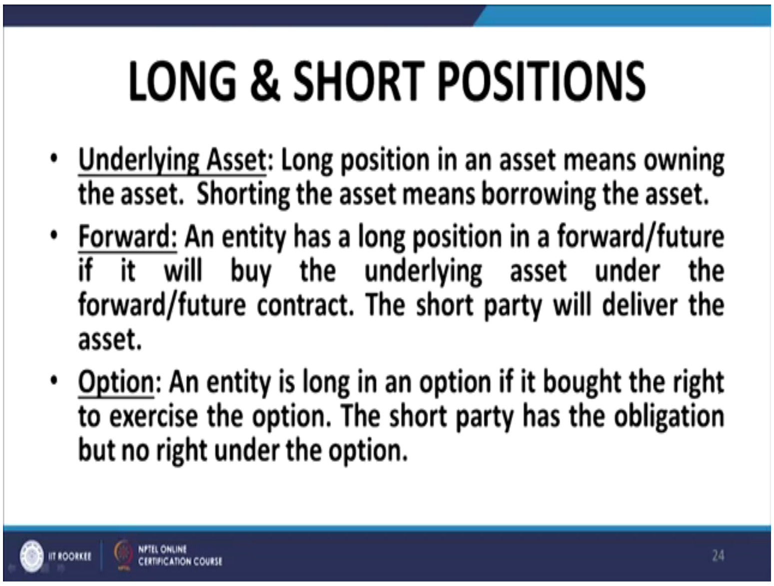

In the context of options, the party who owns the option or who has bought the option or who is the holder of the option, who gets the right under the option is said to be long in the option and the party who has written the option i.e. the party that is going to honor the option, which has no discretion, is said to be short in the option.

Now, obviously a party who is long under a forward contract will benefit from a price rise. Why is that? For example, if you have undertaken a forward contract with the bank to buy US dollars i.e. you have taken a long position in US dollars at a price of say 75 rupees and the dollar price on the date of maturity of the forward contract turns out to be 80 rupees you can obviously take these dollars as 75 rupees from the bank and sell it in the open market at 80 rupees.

So higher the price on the date of maturity of the underlying asset greater is the profit to you and conversely if the price falls you are going to lose if you are long in the asset or long in the forward contract. The converse holds for the short position and short position benefits from a price fall and loses when the price rises.

Forward Price

Now, if I enter into a forward contract today the price at which I am going to enter into the forward contract is said to be the forward price of the asset. For example, if am going to get US dollars after three months at rupees 75 per dollar the 75 rupees constitutes the forward price of the US dollars under the forward contract for three months i.e. for delivery at the end of three months.

Now, net value of the forward contract as on the date of inception is zero. Why is that? Suppose I approach bank ABC and bank XYZ for getting a forward contract on 100 US dollars' maturity after three months. The forward contract that I envisage is the same in the context of both the banks and the credit worthiness of both the banks is also the same. And their assessment of my credit worthiness is also the same. The question is can they charge different prices? The answer is a very obvious 'no'. How can they do that? Because if they do that, arbitrage will take place and as equilibrium approaches the two prices will converge and that one price (the converged price) constitutes the forward price. So as on the date of the inception of the contract, negotiation of the contract, the net value of a forward contact is invariably zero. And the forward price is that price which makes this net value zero which is dictated by the market.

Why are forwards "derivatives"?

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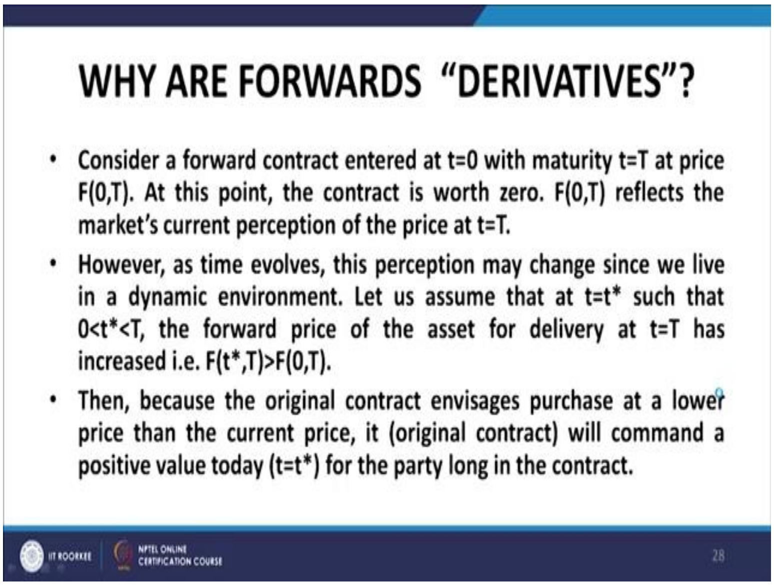

Another very interesting question is - Why are forward contracts termed as derivatives? At first sight it seems that the price at the date of maturity of the forward contract is fixed. So, if the price at the date of maturity of the forward contract is fixed, how does it become a derivative?

The answer to this is that - Let us say I start at t=0, that is today, I enter into a forward contract in US dollars at rupees 75 per dollar, maturity after 3 months. Let today be 1st January, let us assume I get to delivery on 1st of April, one US dollar @ 75 rupees as contracted on 1st of January.

Now we live in a dynamic world, dynamic environment, so as time passes the perception of the market about the rate of the US dollar, as on 1st April i.e. the rate for delivery on 1st April is going to change.

The US economy may flourish or the Indian economy may flourish or whatever may happen. Certain extraneous circumstances can always happen, whatever happens there is the distinct possibility and in fact a distinct probability that the rates as on 1st April for delivery of US dollars will be different as we move along, move forward from 1st of January.

Let us say on 15th of January, the US dollar rates have increased, the US dollar rates have gone up to 80 Rupees for delivery on 1st of April, in other words if I enter into a fresh forward contract on 15th of January for delivery on 1st of April I have to pay a price of rupees 80 per US dollar. It can happen.

Now the earlier contract which was entered into on 1st Jan for delivery on the same date, that is 1st of April, was at 75 rupees per US dollar. As on 15th January a new contract envisaging the delivery of the same asset on the same conditions is costing you 80 rupees.

So naturally the earlier contract will be worth a positive value for the person who is holding the contract or who is going to buy dollars against the contract. Why? Because he is going to get dollars at 75 rupees. You are going to get dollars at 80 rupees. In other words, that forward contract is now commanding a positive value for the party who is long in the contract and a negative value for a party who is short in the contract. It is a zero-sum game.

So with the change in the perception of the price prevailing on the maturity date of the forward contract, the forward contract acquires a value and that value gradually changes in relation to that change in perception. And that is why it is termed as a derivative. Because there is a functional relationship between the price of the dollar and the value of the contract, this qualifies as a financial derivative.

Profit & Payoff Diagrams

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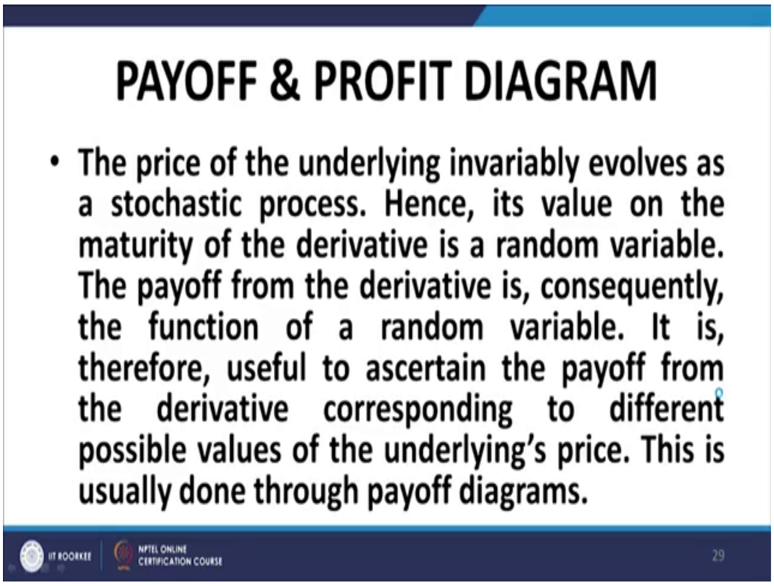

Spot prices of assets at a future date are random variables They cannot be precisely predicted. Now, derivatives are contracts that are settled at a future date. Hence, it becomes useful to analyse the payoffs from the derivatives corresponding to various projected values of the price of the underlying on the date of maturity of the derivative. The way this is done is through payoff and profit diagrams.

The price of the underlying on the maturity of the forward contract, for example, will have a spectrum of values, possible prices. In context of those possible prices what are the possible payoffs from the derivatives becomes a very relevant question because that is what is ultimately going to decide my investment strategy in these derivative instruments. And one very simple way of analysing them is the payoff diagram.

I reiterate, the underlying's price process at a future date is a random variable and the derivative is a function of that random variable so what possible values the derivative's payoff can take in context of various values of that random variable (price of underlying at a given future date) is a very interesting problem and that can be analysed through payoff and profit diagrams.

So what is the difference between payoff and profit diagrams? Payoff gives you the payoff on maturity, the net cash flow on the date of maturity. The profit diagram gives you the cash flow less the cost of inception of the strategy.

Payoff & profit on stock positions

For example, let us look at the payoff of a stock position. Let us say you own this stock. Because you own the stock you are long in the stock. Suppose you have bought the stock at t=0 at price S_0. Let us assume that you want to remain invested upto t=T. Now, the stock price at t=T can take a spectrum of possible values, say S_T. The various possible prices S_T are depicted along the X axis which gives you the price of the stock on maturity. The price that you will realize on selling the stock at t=T is S_T. This is your payoff at t=T. However, your profit is S_T-S_0. We have:

Payoff of long stock= Cash-flow of sale of stock at T = $\Pi_{\text{long stock}}=S_T$

Profit of long stock=$\pi_{\text{long stock}}=S_T-S_0$

Both the above diagram are +45 degree straight lines with respective intercepts of 0 and $-S_0$ on the Y-axis.

In case you borrow the stock at t=0 and sell it forthwith at S_0 i.e. have a short position, you will replenish it to the owner at t=T by buying it from the market at that time (t=T) at S_T. In this case, your payoff at t=T is $-S_T$ as you have to pay for the stock and your profit is $-S_T+S_0$.

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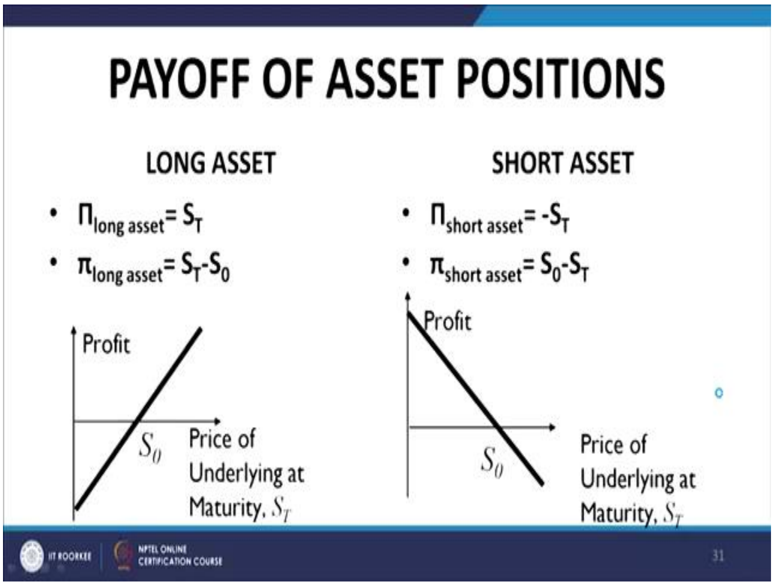

Payoff of short stock= Cash-flow of sale of stock at T = $\Pi_{short\ stock}=-S_T$

Profit of short selling stock=$\pi_{short\ stock}=-S_T+S_0$

Payoff & profit on forward positions

A very similar situation is in the case of a forward position. You are guaranteed a certain price of acquisition on along forward position. The forward price, K, is agreed today but settled at T. So you get the stock against the long forward position at price K at time t=T. At this time, the market price of stock is S_T. Hence, the payoff (and profit) of a long forward position is S_T-K.

Payoff of long forward on stock= Net Cash-flow at T = $\Pi_{long\ forward}=S_T-K$

Profit of long forward on stock=$\pi_{long\ forward}=S_T-K$

Long possessions and short possessions are mirror images of each other about the X axis since they represent a zero-sum game.

Payoff of short forward on stock= = $\Pi_{short\ forward}$=-S_T+K

Profit of short forward on stock=$\pi_{short\ forward}$=-S_T+K

Pricing of forwards

Now, we come to a very important issue. I have mentioned little while earlier that pricing of any asset including derivatives is determined by demand and supply. Nevertheless, we have certain benchmark prices as per certain assumptions of the behaviour of the financial markets. The most common assumption that we make while pricing set financial assets, is the concept of no-arbitrage.

This no-arbitrage price gives us a certain theoretical price of the asset being priced. And at this price, the philosophy is that the market trading or the market will accept this without any arbitrage opportunities. But it must be emphasised that trading does take place. There can be situations where the actual market price does not necessarily coincide with this no-arbitrage price at all points in time.

There can be various types of asymmetries in the market which may result in the actual market price being slightly different from the no-arbitrage prices and as a result of it trading may take place. My perception of the price of an asset or the value of an asset as determined by me maybe say rupees hundred, that same asset maybe trading in the market at rupees hundred and ten.

What is my immediate reaction? I may feel that market is over pricing that asset. That asset is worth hundred according to me. Market is paying 110 for that so let me sell that asset. On the other hand, if my perception of the value of the asset as ascertained by me is hundred and forty rupees and the market price is hundred and twenty. What will I do? I will say the market is under-pricing that asset and therefore, I will buy that asset because I feel that I am getting something worth 140 rupees at a cost of 120 rupees. And that is how trading operates. How I work out that perception is a separate issue I will be talking about it gradually.

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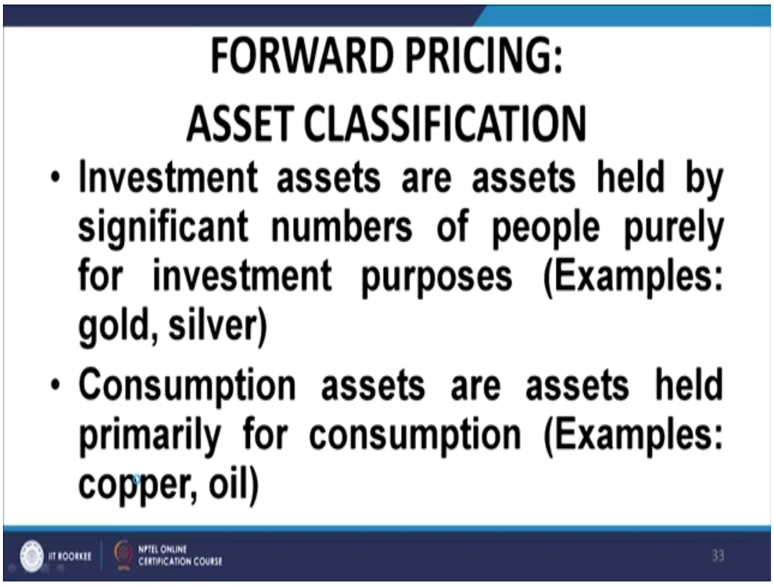

But when we talk about forward pricing as I mentioned, the most important principle that we invariably follow is the principle of no-arbitrage. However, in the context of forward pricing we need to segregate assets into two types of assets. Why we do that will be clear in due course, when we talk more about arbitrage procedures.

The classification is (a) investment assets i.e. assets that are not held directly for consumption e.g. gold, silver, shares, stocks, bonds etc. (b) consumption assets which are held for actual consumption e.g. coal, wheat, copper and so on.

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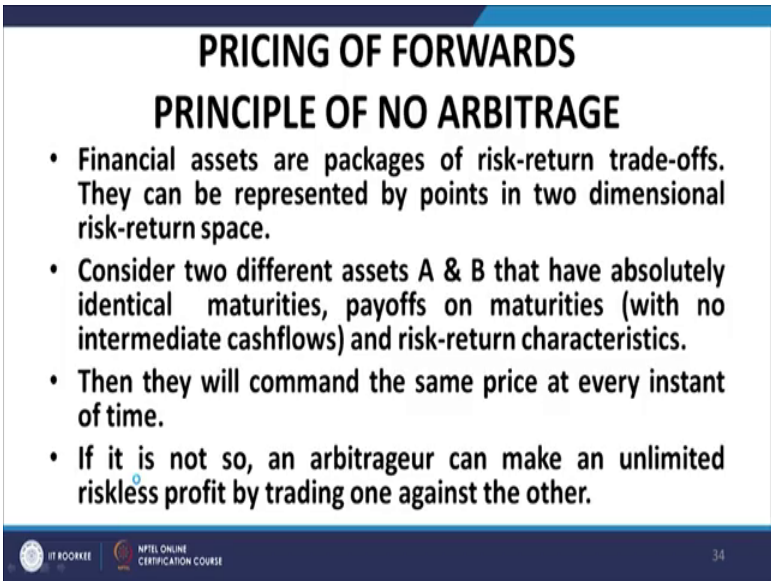

There is a significant difference in the pricing strategies of these two types of assets. We first talk about investment assets.

I have already explained the principle of no-arbitrage. No-arbitrage means the two assets having identical risk profiles, identical maturities, identical pay offs on maturity cannot be priced differently. Because arbitrage i.e. buy-sell process will ensure that at equilibrium the two have to coincide, converge.

Forward pricing without dividends & yields

Let us take a simple situation, we ignore transaction costs altogether as also market frictions of any type, commissions, broker commissions etc. We ignore bid ask spread, borrowing-lending spread to keep the exposition as simple as possible.

Secondly, we assume that holding the underlying asset during the life of the forward contract does not result in any cost nor does it give rise to any income. For example, there is no consideration of any dividend on the underlying asset during the life of the forward contract and there is no consideration of any cost involved for example insurance in holding the asset. Let us, keep the exposition as simple as possible.

We look at two alternative portfolio strategies:

Strategy A	**t=0**	**t=T**
Borrow at risk-free rate for time T	S_0	
Buy one unit of underlying	$-S_0$	
Sell this one unit of underlying		S_T
Repay the loan with interest		$-S_0\exp(rT)$
Net Cash-flow	**0**	$S_T - S_0\exp(rT)$

Strategy B		
Take a long forward position		
Take delivery and make payment under the forward		**$-F_0$**
Sell the stock acquired under the long forward position		**S_T**
Net Cash-flow	**0**	**S_T-F_0**

Both strategies (i) do not involve any net initial cash-flow (ii) both strategies do not involve any cash-flow in the interval (0,T) and (iii) we assume that the forward contract is default free, so that both strategies do not involve any risk-taking. Hence, by the principle of no arbitrage, the cash-flows at maturity must match so that $F_0=S_0\exp(rT)$.

So this is F_0, F_0 is the theoretical forward price or the no-arbitrage forward price as of today, for delivery of the stock whose current spot price is S_0, T years from today. r is the risk free rate.

<u>Cash & carry arbitrage & reverse arbitrage</u>

<u>Cash & carry arbitrage</u>

Cash & carry arbitrage takes place when $S_0\exp(rT)<F_0$. It involves the following steps:

At t=0

1. Borrow an amount S_0 at @r% p.a. continuously compounded for (0, T);
2. Buy one unit of the stock at current spot price S_0;
3. Take a short position in a forward contract on one unit of stock with maturity at t=T.

At t=T

1. Tender delivery against the short forward position and receive the forward price F_0;

2. Repay the borrowed amount with interest aggregating to $S_0exp(rT)$.

The net cash inflow is $F_0-S_0exp(rT)>0$. Hence, an arbitrage profit is achieved.

Reverse cash & carry arbitrage

Reverse cash & carry arbitrage takes place when $S_0exp(rT)>F_0$. It involves the following steps:

At t=0

1. Borrow one unit of stock for (0, T);

2. Sell the stock in spot market at current price S_0

3. Invest this amount of S0 at r% p.a. continuously compounded for (0, T);

5. Take a long position in a forward contract on one unit of stock with maturity at t=T.

At t=T

1. Liquidate the investment to get $S_0exp(rT)$;

2. Receive delivery of stock against long forward position and pay forward price F_0;

3. Replenish the stock to the owner.

The net cash inflow is $S_0exp(rT)-F_0>0$. Hence, an arbitrage profit is achieved.

Why risk-free rate?

So the next question is – why r is the risk free rate? Let us answer that question. Let us look at this arbitrage process again.

I borrow S_0. This borrowing is to be repaid from the proceeds of the forward contract on tendering delivery.

Now, because I have the stock in my possession, I will definitely honor my commitment under the forward contract i.e. I shall deliver the stock on the maturity date.

Since we assume the forward contract to be default free, it follows that the forward price will definitely be received by me. Since repayment of the borrowings is to be made out of these forward proceeds, it follows that I shall definitely make the repayment. Hence, from the perspective of the lender, the lending may be assumed riskfree and therefore the riskfree rate.

Forward pricing with dividends

Let us, now, assume that we have a stock as the underlying of a forward contract of maturity T. Further, it is expected that during the period (0, T), say at $t=\tau$, the stock will pay a dividend equal to D_τ. We ignore transaction costs altogether as also market frictions of any type, commissions, broker commissions etc. We ignore bid ask spread, borrowing-lending spread to keep the exposition as simple as possible.

We look at two alternative portfolio strategies:

Strategy A	**t=0**	**t= τ**	**t=T**
Borrow at risk-free rate for time T	S_0		
Buy one unit of underlying	$-S_0$		
Receive dividend		D_τ	
Repay the amount of loan equal to the present value of D_τ say D_0		$-D_\tau$	
Sell this one unit of underlying			S_T
Repay the rest of the loan			$-(S_0-D_0)\exp(rT)$
Net Cash-flow	**0**	**0**	$S_T-(S_0-D_0)\exp(rT)$
Strategy B			
Take a long forward position			

Take delivery under forward and make payment			**$-F_0$**
Sell the stock acquired under the long forward position			**S_T**
Net Cash-flow	**0**	**0**	**S_T-F_0**

Both strategies (i) do not involve any net initial cash-flow (ii) both strategies do not involve any net cash-flow in the interval (0, T) and (iii) we assume that the forward contract is default free, so that both strategies do not involve any risk-taking. Hence, by the principle of no arbitrage, the cash-flows at maturity must match so that $F_0=(S_0-D_0)\exp(rT)$.

Thus, when a dividend is expected, there is a correction equal to the present value of dividend. It reduces the effective spot price because it is something which you are getting by simply holding the asset. Merely by holding the asset, you are getting this amount of money. So obviously it reduces the cost of carrying the asset. Out of the borrowing of S_0 for buying the asset spot, you meet the repayment of the amount D_0 (with interest) when you receive the dividend D_τ at $t=\tau$. D_0 is the present value of D_τ. Thus, your repayment at maturity i.e. t=T is only for the amount S_0-D_0 (of course, with interest thereon). In other words, your effective borrowing for the asset goes down from S_0 to S_0-D_0 and therefore, with the no-arbitrage condition we have $(S_0-D_0)\exp(rT)$ is equal to F_0.

\Forward pricing with yields

Let us, now, assume that we have an underlying asset of a forward of maturity T, say an index or a currency e.g. USD, In this case, when we buy the asset in the spot leg and retain it for (0, T), it generates a percentage yield i.e. it grows in value at a rate proportional to its instantaneous value or we can invest it and earn a percentage yield or a return on it e.g. in USD deposits. Let q be the continuous compounded yield rate on p.a. basis. We ignore transaction costs altogether as also market frictions of any type, commissions, broker commissions etc. We ignore bid ask spread, borrowing-lending spread etc. We look at two alternative portfolio strategies:

Strategy A	**t=0**	**t=T**
Borrow at risk-free rate for time T	**S_0exp(-qT)**	
Buy exp(-qT) unit of underlying	**-S_0exp(-qT)**	
Sell this one unit of underlying		**S_T**
Repay the loan		**-S_0exp[(r-q)T]**
Net Cash-flow	**0**	**S_T – S_0exp[(r-q)T]**
Strategy B		
Take a long forward position on one unit of underlying		
Take delivery under forward and make payment		**-F_0**
Sell the stock acquired under the long forward position		**S_T**
Net Cash-flow	**0**	**S_T-F_0**

Both strategies (i) do not involve any net initial cash-flow (ii) both strategies do not involve any net cash-flow in the interval (0, T) and (iii) we assume that the forward contract is default free, so that both strategies do not involve any risk-taking. Hence, by the principle of no arbitrage, the cash-flows at maturity must match so that F_0= S_0exp[(r-q)T].

The important thing to note here is that we buy exp(-qT) units of the underlying in the spot leg and not one full unit. This is because the underlying when held during (0, T) yield a continuously compounded yield @ q% p.a. Hence, to get one unit of underlying at time t=T, we need only to buy exp(-qT) units in the spot market at t=0.

F_0 is the theoretical forward price or the no-arbitrage forward price as of today, for delivery of the stock whose current spot price is S_0, T years from today. r is the risk free rate.

We will continue from here on, we have talked about the pricing of forward contract in two contexts, pricing where we are having no income and pricing where we have in different types of income. Either income in terms of currency units or income in terms of percentage unit, we will move on from here in the next lecture. Thank you very much all of you.