Financial Derivatives & Risk Management
Professor J.P Singh
Department of Management Studies
Indian Institute of Technology, Roorkee
Lecture 15: Systematic & Unsystematic Risk

Recall

(i) Standalone risk is measured by standard deviation. However, it is not an appropriate measure of market risk. Market does not price standalone risk.
(ii) Beta is a measure of systematic risk, which is priced by the market.
(iii) The total risk of the portfolio can be segregated into two orthogonal components, the systematic risk, which is the beta risk and the unsystematic risk, which is the random risk. Market does not price total risk. Market prices systematic risk.
(iv) Market does not reward investors for taking unsystematic risk.

Systematic risk & unsystematic risk

The systematic risk which is the Beta risk is the component of total risk that is captured by the sensitivity of the security's returns to the market's returns. The unsystematic risk emerges from the random component of return *e*. Market prices only systematic risk but not unsystematic risk. Why? Because unsystematic risk can be eliminated through appropriate and adequate diversification.

From the CAPM equation: $E(R_S)=R_F+\beta_S[E(R_m)-R_F]$ it is obvious that the expected return on a security is related to its β. It is not related to its standard deviation σ. Market does not price total risk, market prices only systematic risk i.e. β risk.

Another important fallout is that the unsystematic risk must necessarily be random. Why? If there was some pattern embedded in the unsystematic risk, the market would decipher that pattern and if the market deciphers the embedded pattern, it would build it into the price. The price would reflect that pattern. That is the salient characteristic of efficient markets.

Thus, if this term *e* had an embedded pattern, it would be captured by the price. Therefore, *e* would no longer remain unsystematic because market would have priced this component of risk. Hence, by its very definition *e* has to be purely random in the CAPM model.

Implications of splitting of risk into systematic & unsystematic risk

Consider an example. We have a portfolio P having σ_P= 6% and β=1.00. Let σ_m=4%. Let R_F=3%, R_m=12%. This gives, R_P=12%, $\sigma_{P,sys}$= 4%, $\sigma_{P,unsys}$= 4.47%.

Obviously, the market portfolio has no unsystematic risk. Hence, the market total and systematic risk, both are 4%, β=1 for the market, unsystematic risk is 0, and expected return is 12 percent.

For portfolio P, expected return is 12%. Of course, every observation of return will not yield 12%. The systematic risk of P is 4%. Unsystematic risk is 4.47%. There will be a strong random component that contributes to the std dev of 4.47%. Since, std dev is equally influenced by upward deviations and downward deviations, this 4.47% could also be largely due to upward fluctuations. Nevertheless, if there was a pattern in these fluctuations, market would quickly decipher it and incorporate it in the price process.

Now, compare this portfolio P with another portfolio Q that has σ_Q= 6% and β=1.50. This gives, R_Q=16.5%, $\sigma_{Q,sys}$= 6%, $\sigma_{Q,unsys}$=0%. Thus, for the same total risk, we have a portfolio with higher expected return. Over a sustained period, Q is expected to provide a higher return.

Consider another portfolio S having σ_P= 4% and β=1.00. Let σ_M=4%. Let R_f=3%, R_m=12%. This gives, R_S=12%, $\sigma_{S,sys}$= 4%, $\sigma_{S,unsys}$= 0%. Thus, providing the same expected return as P but with a far smaller level of total risk. What does this mean? It means that S will provide the same average return as P. but with lesser fluctuations (variations) and hence, greater certainty.

Let us tabulate the findings:

PORTFOLIO	TOTAL RISK	SYS RISK	UNSYS RISK	EXPECTED RETURN
M	**4**	**4**	**0**	**12**
P	**6**	**4**	**4.47**	**12**
Q	**6**	**6**	**0**	**16.5**
S	**4**	**4**	**0**	**12**

Portfolio Q has a total risk of 6%, a systematic risk of 6% and therefore no unsystematic risk and expected return of 16.5%. Now, the interesting feature is, despite having the total risk as portfolio P, portfolio Q is able to generate a higher expected return. Why? Because the unsystematic risk has in Q been completely eliminated and shifted to systematic risk which adds value to the portfolio by enhancing its expected return. P has both systematic and unsystematic risk. Q has only systematic risk. The unsystematic risk component of portfolio P is not valued by the market; it is worthless from the market's perspective. However, in Q this worthless risk is converted to systematic which is accorded a positive value in the sense that market is willing to provide a higher expected return.

In other words, an investor can add worth to his portfolio by eliminating unsystematic risk i.e. by suitably diversifying. A portfolio with no unsystematic risk is likely to perform better than a portfolio which has a certain component of unsystematic risk.

A similar situation is presented in portfolio S, where for the same expected return (12%), we have a portfolio that has lesser total risk (standalone risk) compared to the portfolio P. Again, we have eliminated the unsystematic risk in the portfolio. Thereby we have been able to maintain the same expected return but with a lower level of total risk. S has the same systematic risk as P and hence, the same expected return but it has no unsystematic risk and hence is an efficient portfolio. P has a total risk of 6%, S has 4%. Why? Because eliminating unsystematic risk from the total risk is not reducing any value (expected return) from the portfolio.

Therefore, the outcome of this comparative evaluation is that the expected return of the portfolio does not get disturbed by eliminating or reducing unsystematic risk because market does not bother about unsystematic risk. It does not reward an investor for taking any level of unsystematic risk. The market believes that the investors are good enough or competent enough to do adequate diversification to eliminate all unsystematic risk in their portfolios and therefore, only the systematic risk component is captured by expected returns provided by the market.

<u>Quantitative features of systematic & unsystematic risk</u>

$$\text{The variance of an } N \text{ security portfolio is}: \sigma_P^2 = \sum_{j=1}^{N}\left(X_j^2\sigma_j^2\right) + \sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}\left(X_j X_k \sigma_{jk}\right)$$

If the portfolio is equally weighted in all securities, then

$$\sigma_P^2 = \sum_{j=1}^{N}\left(\frac{1}{N}\right)^2\sigma_j^2 + \sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}\left(\frac{1}{N}\right)^2\sigma_{jk} = \left(\frac{1}{N}\right)\sum_{j=1}^{N}\left[\frac{\sigma_j^2}{N}\right] + \left(\frac{N-1}{N}\right)\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}\left[\frac{\sigma_{jk}}{N(N-1)}\right]$$

$$\sigma_P^2 = \frac{1}{N}\bar{\sigma}_j^2 + \left(\frac{N-1}{N}\right)\bar{\sigma}_{jk} \Rightarrow Lim_{N\to\infty}\sigma_P^2 = \bar{\sigma}_{jk}$$

We have considered a portfolio of N securities equally weighted so that the weight of each security is 1/N. Now, the term $\sum_{j=1}^{N}\left[\frac{\sigma_j^2}{N}\right]$ is the aggregate of all the variances of the securities divided by the number of securities. So in some sense it is the average variance of the securities. Similarly, the term $\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}\left[\frac{\sigma_{jk}}{N(N-1)}\right]$ is the aggregate of all the covariances between the various pairs of securities divided by the number of such covariances. Hence, it represents the average covariance in some sense. To repeat, the 1st term is 1/N the average variance across the N securities, and the 2nd term N(N-1)/N average covariance, among all the pairs of securities.

Now in the limit that $N\to\infty$, the 1st term gives 0 and the 2nd term will give $\bar{\sigma}_{jk}$ and so the net result s $\bar{\sigma}_{jk}$. This analysis establishes that:

(i) There is some part of the total risk of a portfolio that tends to 0 as the number of securities in the portfolio increases i.e. as you diversify more and more, the total portfolio risk tends to decrease;

(ii) The component of risk that tends to get eliminated with diversification is dependent upon the intrinsic variances of the securities, rather than the inter se relationship among securities;

(iii) However, the total risk of a portfolio cannot be completely eliminated even with indefinite diversification;

(iv) The part which cannot be eliminated completely notwithstanding maximum possible diversification, that remains non-zero finite arises out of the covariances between pairs of securities.

Of course, our portfolio makes the assumptions of equal investments in the securities, but the message is loud and clear, diversification reduces risk albeit upto a certain finite non-zero threshold.

The component of risk that can be diversified away (unsystematic risk) arises due to the intrinsic variances of the securities and that which cannot be so diversified (systematic risk) is due to the inter-relationships between securities.

It is usual to derive the CAPM equation (SML) for a security by fitting a straight line in the given data on market and security returns using OLS regression. As a by-product of this model, the mutual non-correlation of the systematic variable R_m and the error term is ensured i.e. Cov (R_m, e)=0. A consequence of this Cov (R_m, e)=0 is that the risk can be partitioned into two orthogonal components. This reiterates that that we have two components of risk, one component of risk which is related to the regressor and the other component of risk, which is random. We have:

$$\sigma_S^2 = E\left[R_S - E\left(R_S\right)\right]^2 = E\left\{\left[R_F + \beta_S\left(R_m - R_F\right) + e\right] - E\left[R_F + \beta_S\left(R_m - R_F\right) + e\right]\right\}^2$$

$$= E\left[\beta_S\left(R_m - R_F\right) + e - \beta_S E\left(R_m - R_F\right)\right]^2$$

$$= E\left\{\beta_S\left[R_m - E\left(R_m\right)\right] + e\right\}^2 = E\left\{\beta_S\left[R_m - E\left(R_m\right)\right]\right\}^2 + E\left(e\right)^2 + 2E\left\{\beta_S\left[R_m - E\left(R_m\right)\right]e\right\}$$

$$= \beta_S^2 E\left[R_m - E\left(R_m\right)\right]^2 + E\left(e\right)^2 = \underbrace{\beta_S^2\sigma_m^2}_{SYS\ \ RISK} + \underbrace{\sigma_e^2}_{UNSYS\ \ RISK}$$

The first component $\beta_S^2\sigma_m^2$ is the systematic risk part of the total risk and is related to the sensitivity of the security return to the market return. The second component σ_e^2 is the unsystematic or random risk which can be eliminated by diversification.

To reiterate, the unsystematic return component must necessarily be random because if there were any pattern in it, that pattern would be deciphered by the market and built into the price and then it does not continue to be unsystematic return.