# Financial Derivatives and Risk Management

**Professor J. P. Singh**
**Department of Management Studies**
**Indian Institute of Technology Roorkee**

**Lecture 12: Futures Hedging: Examples**

## Example 1

Suppose that the standard deviation of quarterly changes in the prices of a commodity is $\sigma_{\Delta S}=0.65$, the standard deviation of quarterly changes in a futures price on the commodity is $\sigma_{\Delta F}=0.81$, and the coefficient of correlation between the two changes is $\rho=0.8$. What is the optimal hedge ratio for a 3-month contract? What does it mean?

## Solution

Hedge Ratio (h) $=- \rho\sigma_{\Delta S}/\sigma_{\Delta F}=-0.80*0.65/0.81=-0.642=Q_f/Q_s$.

The hedge ratio of 0.642 implies that for hedging one unit of the cash asset, we are using 0.642 units of underlying units in the futures markets. The negative sign indicates that the position in the futures market (hedge) is opposite to the cash (hedged) position.

## Example 2

On March 1 the spot price of a commodity is 20 and the July futures price is 19. On June 1 the spot price is 24 and the July futures price is 23.50. A company entered into a futures contracts on March 1 to hedge the purchase of the commodity on June 1. It closed out its position on June 1. What is the effective price paid by the company for the commodity?

## Solution

Spot Price paid for buying the item in the spot market on hedge lifting (June 1) =- 24
Futures price on hedge inception (March 1) =19
Futures Price on hedge lifting (June 1) = 23.50
Profit on the futures during hedge period: 4.50
Net price paid on hedged purchase (June 1): -24+4.50=-19.50

## Example 3

The standard deviation of monthly changes in the spot price of wheat is 1.2. The standard deviation of monthly changes in the futures price of wheat for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A flour producer is committed to purchasing 200,000 pounds of wheat on November 15. The producer wants to use the December wheat futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of wheat. What strategy should the producer follow?

## Solution

Hedge Ratio (h) = $-\rho\sigma_s/\sigma_f$=-0.70*1.20/1.40=-0.6=$Q_f/Q_s$
$Q_f$=-0.6*(-)200,000=+120,000. No of contracts =$Q_f$/Lot size=120,000/40,000=3 (long)

$Q_s$ is taken as negative as it is a short position since the hedger wants to hedge its purchase.

**Impact of hedge ratio on hedge effectiveness**

| | | | | | | |
|---|---|---|---|---|---|---|
| SPOT PRICE t=0 | 60 | 60 | 60 | 60 | 60 | 60 |
| QUANTITY | 100 | 100 | 100 | 100 | 100 | 100 |
| SPOT PRICE t=T | 50 | 60 | 70 | 50 | 60 | 70 |
| HEDGE RATIO | 0.8 | 0.8 | 0.8 | 1.4 | 1.4 | 1.4 |
| FUTURES PRICE t=0 | 62 | 62 | 62 | 62 | 62 | 62 |
| FUTURES PRICE t=T | 52 | 62 | 72 | 52 | 62 | 72 |
| FUTURES QUANTITY | 80 | 80 | 80 | 140 | 140 | 140 |
| CHANGE IN SPOT VALUE | -1000 | 0 | 1000 | -1000 | 0 | 1000 |
| CHANGE IN FUTURES VALUE | -800 | 0 | 800 | -1400 | 0 | 1400 |
| NET CHANGE | -200 | 0 | 200 | 400 | 0 | -400 |

Now, the above chart gives an analysis of the impact on the hedge ratio on the hedged portfolio. Three different scenarios for the evolution of the spot and futures prices at the hedge maturity are considered. If an investor uses a hedge ratio, in the given set of data, of 0.80, he ends up with a net change of (-) 200 in the hedged portfolio in scenario A and a change of +200 in scenario C. On the other hand, the use of a hedge ratio of 1.40 yields a change in the hedge portfolio of +400 in scenario A & (-) 400 in scenario C. Thus, not only the magnitude of the value of the hedged portfolio changed but even its sign under the different scenarios changed. Again, the origin of this peculiarity can be traced to the linear payoff profile of the futures.

So, that is how very significant the choice of the hedge ratio is, in context of a hedging assignment. The hedge outcome radically depends on an appropriate hedge ratio. Hedge ratio is very significant, and so it is very important that the hedger uses the correct hedge ratio.

Any error in the choice of hedge ratio will create hedge ineffectiveness. It means is that we are either over-hedging or under-hedging in the futures market. Because the futures payoff profile is linear, this error affects us both ways, when the spot price goes up as well as when it declines. This is well illustrated in the above table. Not only the magnitude of values of the hedged asset fluctuate more but even the directions of value changes also get reversed simply by change in the hedge ratio.

**Regression & number of contracts**

We can write the regression equation for the changes in spot prices when regressed against changes in futures contract as: $\Delta S=\alpha+\beta_{\Delta S,\Delta F}\Delta F$. Thus, $Q_S\Delta S=\alpha Q_S+\beta_{\Delta S,\Delta F}Q_S\Delta F=\alpha Q_S+\beta_{\Delta S,\Delta F}(Q_S/Q_F)Q_F\Delta F$

For perfect hedging, we would require that $Q_S\Delta S+Q_F\Delta F=0$ so that the changes in the spot and futures markets annul each other. Thus, $Q_S\Delta S[1+\beta_{\Delta S,\Delta F}(Q_S/Q_F)]-\alpha Q_S=0$
This will be independent of $\Delta S$ (the risk factor) only if $[1+\beta_{\Delta S,\Delta F}(Q_S/Q_F)]=0$ or $Q_F=-\beta_{\Delta S,\Delta F}Q_S$

Since we want a perfect hedge, the net value change of the hedged position should be zero so that $Q_S\Delta S+Q_F\Delta F=0$. The result is the same as we arrived at earlier viz with $|\rho|=1$, though through a slightly different approach.

**Cross hedging**

The asset that gives rise to the hedger's exposure is sometimes different from the asset underlying the futures contract that is used for hedging. This is known as cross hedging. It leads to an increase in basis risk. Define $S_N^*$ as the price of the asset underlying the futures contract at time t=N. As before, $S_N$ is the price of the asset being hedged at time t=N.

By hedging, a company ensures that the price that will be paid (or received) for the asset is $S_N + F_0^* - F_N^* = F_0^* + (S_N^* - F_N^*) + (S_N - S_N^*)$.

The terms $S_N^* - F_N^*$ and $S_N - S_N^*$ represent the two components of the basis. The $S_N^* - F_N^*$ term is the basis that would exist if the asset being hedged were the same as the asset underlying the futures contract. The $S_N - S_N^*$ term is the basis arising from the difference between the two assets.

Cross hedging arises when the futures that are being used for hedging have an underlying that is different from the hedged asset. In other words, the asset that constitutes our exposure may not have futures available on it for trading in the market to which the hedger has access. He may then need to take recourse to futures on some alternative underlying. This is a situation which is called cross hedging. In cross hedging another factor that contributes to the basis risk comes into play. It relates to the prices of the asset that is the underlying asset of the futures contract vis-à-vis the exposed asset.

Consider a hedger who wants to sell an asset A in the spot market at t=N and he wants to hedge that asset by taking a futures position. The futures that he uses for hedging have an underlying asset B which is different from the asset that the investor owns and wants to sell.

The price that he will receive on selling the asset A is $S_N$. But the futures that he used for the hedge have a different underlying B. The profit on the short futures position will be with respect to the actual futures position i.e. on B i.e. $F_0^* - F_N^*$.

Thus, the net price realised on the hedged sale is: $S_N + F_0^* - F_N^* = F_0^* + (S_N^* - F_N^*) + (S_N - S_N^*)$.

Thus, the price realized on the hedged position can be split into three parts:

(i) the actual futures prices at the time of institution of the hedge (t=0) on the actual underlying B i.e. $F_0^*$;
(ii) the basis at hedge maturity (t=N) of the actual futures on the actual underlying B i.e. $(S_N^* - F_N^*)$;
(iii) a correction term representing the excess of the actual spot price at maturity of hedge (t=N) of the asset A (that was sold spot by the hedger) and the asset B (that constituted the underlying of the futures) i.e. $(S_N - S_N^*)$.

## Example 4

It is 15 October and a treasurer has identified the need to convert Euros into dollars to pay a US supplier USD 12 million on 20 November. The treasurer has decided to use December Euro futures contracts to hedge his exposure. For this purpose, he opens a position on 15 October and closes it on 20 November. Relevant spot and futures prices and other details are as follows:

- Contract size: Euros 200.000;
- Date Spot Price (USD/Euro) Futures price (USD/Euro)
- 15 October 1.3300 1.3350
- 20 November 1.3190 1.3240

He rounds off the number of futures contracts to the nearest integer. Calculate the net price paid by him (in million Euros) for the entire USD 12 million after implementing the futures hedge on 20 November.

## Solution

| | | | | | |
|---|---|---|---|---|---|
| AMT TO BE PAID | | USD | 12000000.0000 | | |
| SHORT EURO FUTURES | SINCE USD FUTURES NOT AVAILABLE | | | | |
| CONTRACT SIZE | | | 200000.0000 | EUROS | |
| | USD/EUR | SPOT | FUTURES | | |
| OCTOBER | | 1.3300 | 1.3350 | | |
| NOVEMBER | | 1.3190 | 1.3240 | | |
| | | 0.0110 | 0.0110 | | |
| AMT TO BE PAID | EUR | 9022556.3910 | EUROS | 12000000/1.33 | |
| NO OF CONTRACTS | | 45.1128 | | 9022556.39/200000 | |
| ROUNDED OFF | | 45.0000 | | | |
| SPOT PAYMENT IN EUROS IN NOV | | 9097801.3647 | EUROS | 12000000/1.3190 | |
| PROFIT ON FUTURES | USD | 99000.0000 | | 45*200000*0.0110 | |
| | EUROS | 75056.8613 | | 99000/1.3190 | |
| NET PAYMENT IN EUROS | | 9022744.5034 | | 9097801-75056 | |

It is 15 October and the treasurer identified the need to convert Euros into USD to pay a US supplier. He has to pay the US supplier USD 12 million on 20 November. The home currency is Euros. The treasurer has decided to use December Euro futures.

For this purpose, he opens a futures position on 15$^{th}$ October and closes it on 20$^{th}$ November when he lifts the hedge. We need to keep track of the manner in which the prices are quoted. Prices are quoted in terms of USD/Euro.

Now, the amount that he has to pay is USD 12 million. Because he has to pay USD, he has to buy USD from the spot market. Therefore, to hedge he would have taken a long position in USD futures to protect against a rise in price of USD. Equivalently, however, he takes a short position in Euro futures.

The amount to be paid in Euros at the current (15th October) rate is Euros 12,000,000/1.33= 9,022,556.391. The contract size is 200,000 Euros. So, the number of contracts come out to be 45.1128 which is rounded off to 45 contracts. Thus, he will short 45 Euro futures to cover his exposure.

Now, on 20th November he makes the payment of USD 12,000,000 by buying in the spot market at the then spot rate. He pays Euros 12,000,000/1.319=9,097,801.3647.

He makes some profit on his short Euro futures. The change in price of the futures is from USD 1.3190 to USD 1.3240 and his position is short. The number of contracts is 45, lot size is 200,000. Hence the profit is USD 45*200,000*(1.3350-1.3240)=USD 99,000=Euros 99,000/1.319=75,056.86. Hence his net payment is 9,097,801.3637-75,056.86=9,022,744.50.