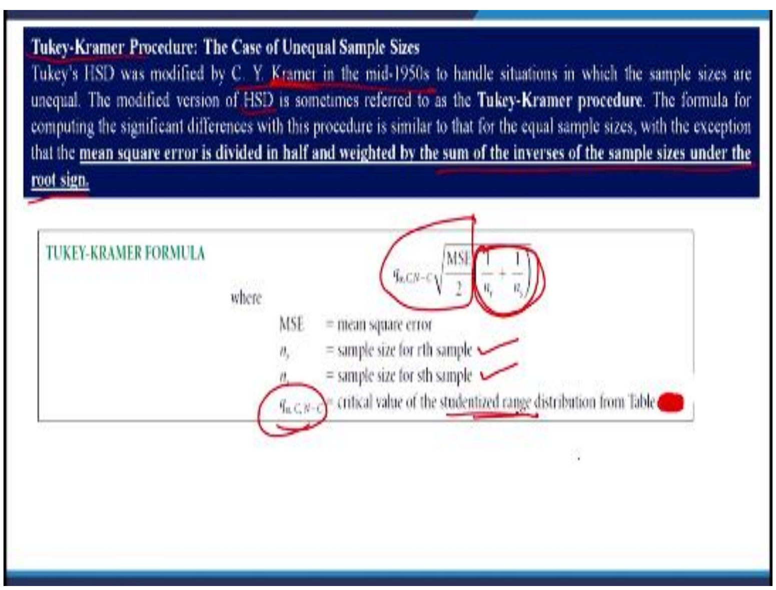
Business Statistics
Prof. M. K. Barua
Department of Management Studies
Indian Institute of Technology - Roorkee

Lecture – 49
Tukey Kramer Test

Hello friends, I welcome you all in the session as you are aware in previous session, we were discussing about one way ANOVA and we have seen that in one way ANOVA, we have got a dependent variable and an independent variable, in independent variable we have seen there were different levels for example, in one of the examples there were the sales of mall and the malls were in different geographical locations let us say in cities, in semi urban area and in villages area, right.

So, all these cities, village and semi urban all these are treatment levels, right under independent variable geography and that the dependent variable was let us say the sales of a particular product. We have seen Tukey test wherein we compared means of different groups, when the sample size was same.

(Refer Slide Time: 01:41)



Let us look at the extension of Tukey HSD test, it is called Tuley Kramer test now, this test is applicable when we have got unequal sample sizes, so this is basically extension of Tukey's test, it was; the extension was done by C. Y. Kramer in 50's so, the formula for these 2 tests are almost same with minor differences so, if you look at Tukey's test then this was the formula for HSD.

Now, these this particular value has been added to this formula, so what we have done, so there is nothing no difference except that mean square error is divided in half just see 1/2 and weighted by some of inverses of sample sizes under root sign, right, so this is what is the sum of inverse, right, so mean square error is a which we obtained in HSD test as well, sample size of rth sample, sample size for sth sample, right.

And these are different values which you will get in table, right, so at these particular combination, right, critical value of studentised range distribution from table, right.

(Refer Slide Time: 03:23)

Valve Openings by Operator

6.25±.10

1	2	3	4
6.33	6.26	6.44	6.29
6.26	6.36	6.38	6.23
6.31	6.23	6.58	6.19
6.29	6.27	6.54	6.21
6.40	6.19	6.56	
	6.50	6.34	
	6.19	6.58	
	6.22		

TUKEY-KRAMER FORMULA

$$q_{\alpha, C, N-C}\sqrt{\frac{MSE}{2}\left(\frac{1}{n_r}+\frac{1}{n_s}\right)}$$

where

MSE = mean square error

n_r = sample size for rth sample

n_s = sample size for sth sample

$q_{\alpha, C, N-C}$ = critical value of the studentized range distribution from Table

So, let us look at an example, this is an example wherein though we have seen this example, this is an example where let us say there are 4 operators and they are making valves and these are different valve opening diameters are put it in simpler words, there are 4 operators and they are drilling on an aluminium plate and let us say diameter of the hole they want to have is 6.25 ± .10 mm or centimetre whatever it is, right.

So, is there any significant difference amongst a means of these four groups or can we say that the mean of these groups same when the; these holes or the valves are being made by four different operators, right, so this is Tukey test, the formula for this we already seen all these values.

(Refer Slide Time: 04:32)

Valve Openings by Operator

1	2	3	4
6.33	6.26	6.44	6.29
6.26	6.36	6.38	6.23
6.31	6.23	6.58	6.19
6.29	6.27	6.54	6.21
6.40	6.19	6.56	
	6.50	6.34	
	6.19	6.58	
	6.22		

Means and Sample Sizes for the Valves Produced by Four Operators

Operator	Sample Size	Mean
1	5	6.3180
2	8	6.2775
3	7	6.4886
4	4	6.2300

For operators 1 and 2,

$$3.96\sqrt{\frac{.007746}{2}\left(\frac{1}{5}+\frac{1}{8}\right)} = .1405$$

The difference between the means of operator 1 and operator 2 is

$$6.3180 - 6.2775 = .0405.$$

Because this result is less than the critical difference of .1405, there is no significant difference between the average valve openings of valves produced by machine operators 1 and 2.

So, let us look at these solution to this particular question, we will test it at 0.05 significance level, so we will calculate sum of the square, columns sum of the square, error sum of a square, total sum of a square at appropriate degrees of freedom, so this is F value which we will calculated, right, so this is the type of distribution, this is your rejection region, so this is F value somewhere and will have critical value from table, right.

(Refer Slide Time: 05:12)

Valve Openings by Operator

1	2	3	4
6.33	6.26	6.44	6.29
6.26	6.36	6.38	6.23
6.31	6.23	6.58	6.19
6.29	6.27	6.54	6.21
6.40	6.19	6.56	
	6.50	6.34	
	6.19	6.58	
	6.22		

Means and Sample Sizes for the Valves Produced by Four Operators

Operator	Sample Size	Mean
1	5	6.3180
2	8	6.2775
3	7	6.4886
4	4	6.2300

For operators 1 and 2,

$$TK \rightarrow 3.96\sqrt{\frac{.007746}{2}\left(\frac{1}{5}+\frac{1}{8}\right)} = .1405$$

.1405

1, 3

The difference between the means of operator 1 and operator 2 is

$$6.3180 - 6.2775 = .0405.$$

Because this result is less than the critical difference of .1405, there is no significant difference between the average valve openings of valves produced by machine operators 1 and 2.

So, just keep in mind this particular values, so in this formula if you look at carefully, you have got when you take first 2 samples, so first sample has got sample size 5, right, so this is 5, second sample has got 8, so 8, so the value of Tukey is; Tukey test is.1405, is not it, so we know this that there are 5 units in first sample 8 in second, 7 in third, 4 in fourth sample, and these are different sample means.

So, if you look at the F value carefully, this is somewhere here right, in fact this is equal to 1 or is not near 1, so we will say that we are saying we will reject null hypothesis, right, we are rejecting null hypothesis, we are rejecting in the null hypothesis, it means that these means are different from each other, so this is what ANOVA does but it does not tell you which group of means are different from each other.

So that is why we are using something called TK test, right, okay, so when we calculate Tukey value, this is nothing but TK; Tukey Kramer value right, is 0.1405, now you have to compare this value with the differences of means, so let us take first and second group, so the mean difference is .04, this the difference first and second, so the difference is .0405, now compare this value with calculated TK value.

Now, this value is smaller than this, so we will say that there is no significant difference between group 1 and group 2, there is no significant difference, so it means they are same, so mean of group 1 and mean of group 2 are same, right, there is no statistical difference but what about between group 1 and 3, is not it, you need to calculate, then what about 1 and 4, 2324 and 34, so you need to calculate all those values, right.

(Refer Slide Time: 08:00)

Means and Sample Sizes for the Valves Produced by Four Operators

Operator	Sample Size	Mean
1	5	6.3180
2	8	6.2775
3	7	6.4886
4	4	6.2300

Pair	Critical Difference	Actual Difference
1 and 2	.1405	.0405
1 and 3	.1443	.1706*
1 and 4	.1653	.0880
2 and 3	.1275	.2111*
2 and 4	.1509	.0475
3 and 4	.1545	.2586*

*Significant at $\alpha = .05$.

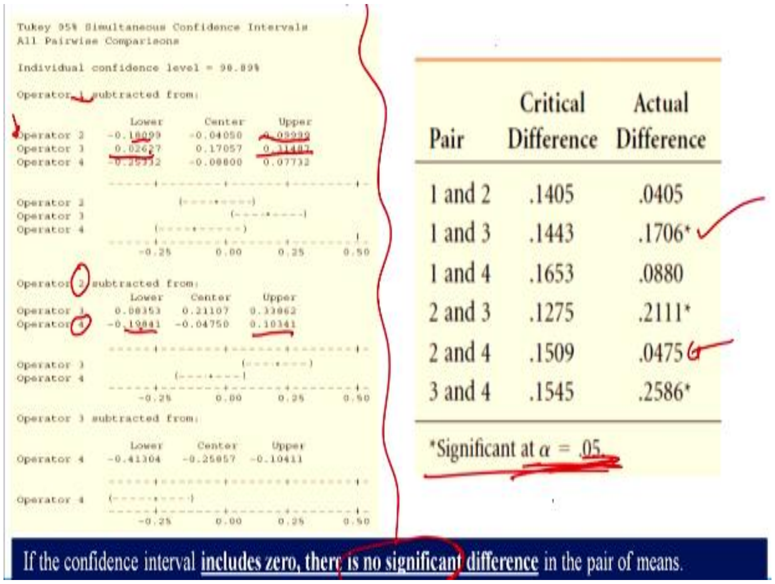
So, the actual differences are this, right between let us say group 1 and 2, this is the actual difference between first and third, this is the actual difference and so on, right, third and fourth; this is the actual difference and the critical value is .1444 for first and second group, sorry, first and third group, so if you take first and third group over here, so first is 5 and third is 7, so the moment you write 7 over here, this value becomes .1443, right.

When you take first and fourth group, it means 5 and 4, this becomes 4, so the critical value becomes .1653, right now you compare each pair with the actual difference, compare critical difference of each pair with the actual difference, so if the difference is more, if the actual difference is more than this, then there is a significant difference, if it is not then there is no significant difference.

So, let us look at this, this actual difference is less than critical difference, so we will say there is no significant difference, this difference is more than critical, so we will say there is a significant difference between first group mean and third group mean and so on right, so there is a significant difference between three different groups, right; first and third, second and third and third and fourth, right.

Otherwise, there is no significant difference between first, second, first fourth and second and fourth, so this is how you can solve a question like this.

(Refer Slide Time: 10:04)

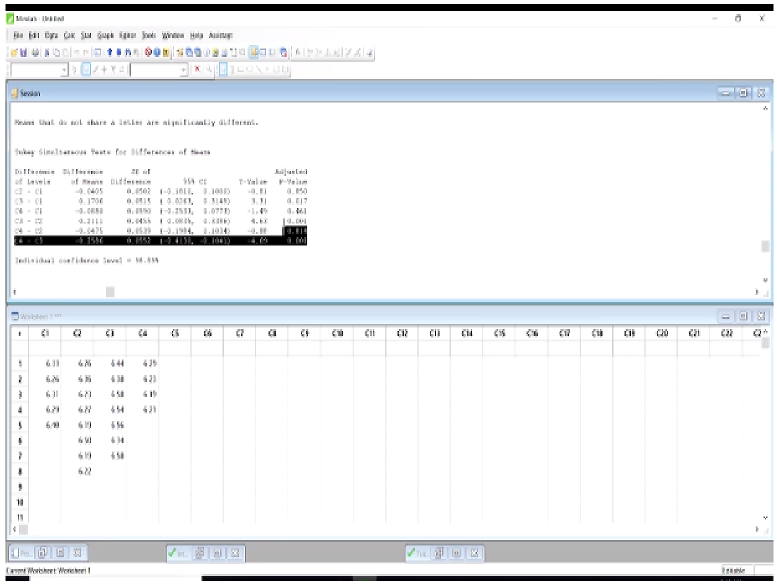


And this is the computer output, you can use any when I say computer, it means you can use any software be it, so SAS, be it Minitab, or be it SPSS right, so if you look at this output then let us compare group first and second, right are operator 1 and operator 2, so if you look at the upper limit and lower limit, so this is the lower limit minus this and this is upper limit, right, so if the confidence interval includes 0, there is no significant difference between those two groups, right.

So, if you look at group 1 and 2, is there any 0 between upper and lower limit; yes, there is 0, so we will say there is no significant difference, they are same. Let us look at 1 and 3, this .31, .02, so there is no 0 in this range, so we will say there is a significant difference between 1 and 3 which can be seen here as well, is not it, let us look at any other group, for example group 2 and 4, okay so, of course there is a zero value between lower and upper limit.

So, there is no significant difference between 2 and 4, just see, there is no significant difference, star means there is a significant difference, right this value of alpha, .05, okay.

(Refer Slide Time: 11:51)



So, we will solve this question using Minitab and this is the output we would be getting, so let us first go for data entry, so is the question and let us look at the software, we will go to software and we will go for data entry, right, so this is how you can do data entry for this question, so let us look at this values, 6.33, 6.26 so, this is how you should be doing data entry for each of these groups, keep in mind that we are solving this question using TK test.

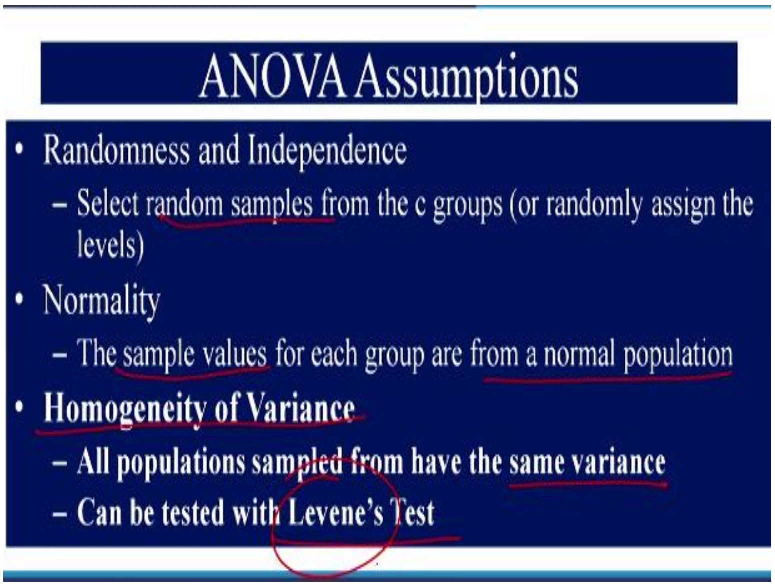
Because the sample sizes are not equal, so in first group there were only 5 samples and second group we have got 8 samples, in third group and fourth group again will make data entry, so 6.22 is the last value for 8th observation in second sample, 6.58, 6.54, 6.56, 6.34, 6.58 so, there are only seven observations in third group, so far as fourth group is concerned the first value 6.29, 6.23 and the last one is 6.21.

So, we will go to stat, will go to ANOVA, will go one way ANOVA, okay, so this is yeah, so we can select so all these values, C1 to C4, select, so all these values will come over here, will go to options, we will check the significance level, let us check once again significance level for this question and the significance level is .05, right it is written over here, so will go to here, it is .05, so this is let tailed test or low tailed test, it is not a; so this lower bound, we will click okay.

We will go to comparisons, will do Tukey, right and test, right, click okay, once again there is of course there is no need of all those gross, we will click okay, so this is our we can minimise these values, so just minimise this, this as well so, look at this particular table values, right. Now, if you look at carefully this P value is .85 which is >.05, so there is no significant difference between first and second group.

What about this .01; it is < alpha, so we will say that there is a significant difference between C1 C3 similarly, this C2 and C3 there is a significant difference and C3 and C4, this what the same; we are getting the same result which you were getting in Minitab, let us look at once again, so there is a significant difference between 1 3, 2 3 and 3 4, right, so 1 3, 2 3 and 3 4, right so we are getting same answer using Minitab output, right.

(Refer Slide Time: 16:20)

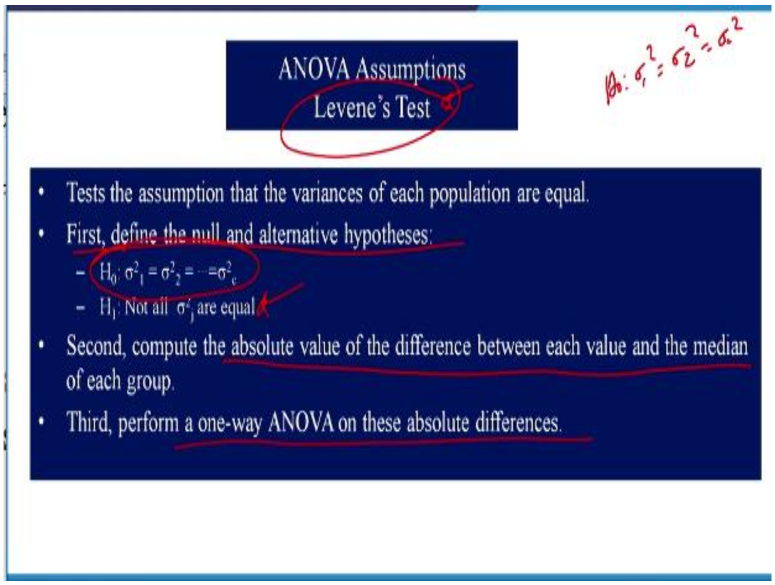


So, this how you can solve a question using Minitab on ANOVA having different sample sizes, now, let us look at some assumptions about ANOVA, we did work out couple of examples using one way ANOVA, in other words, we have worked questions on completely randomised design, right, what is completely randomised design; wherein there is only one independent variable having different treatment levels, right.

So, before going for ANOVA, you need to fulfil certain assumptions for example, the sample should be randomly selected from the different groups, we should ensure that the samples from where we are taking the populations are normal and the most important assumption is homogeneity of variance, the variance of the populations from where we are selecting samples should be same.

So, there should not be any difference amongst variances of different populations from where we are selecting samples, so all this; all the populations will have same variance and it can be tested using a test called Levene's test, so let us see what is Levene's test.

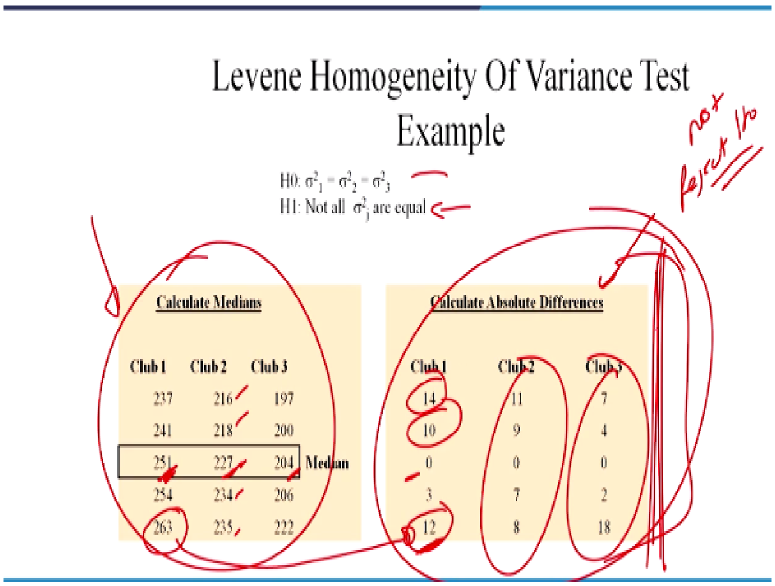
(Refer Slide Time: 17:45)



So in Levene's test, the first thing is we define our null and alternative hypothesis, so hypothesis is this, so we will say that the all these variances of the populations from your where we are taking samples are same, so all these variances are same, alternative hypothesis is that they are not same, they are different from each other or at least one of the variances is different from others.

The second is; the second point in Levene's test is we compute the absolute value of difference between each value and the median of each group, so every group will have median, so that median value from that median value will subtract each of those values and then we will perform one way ANOVA on this absolute differences. So, let us test one of the questions for equality of variances, right.

(Refer Slide Time: 18:56)

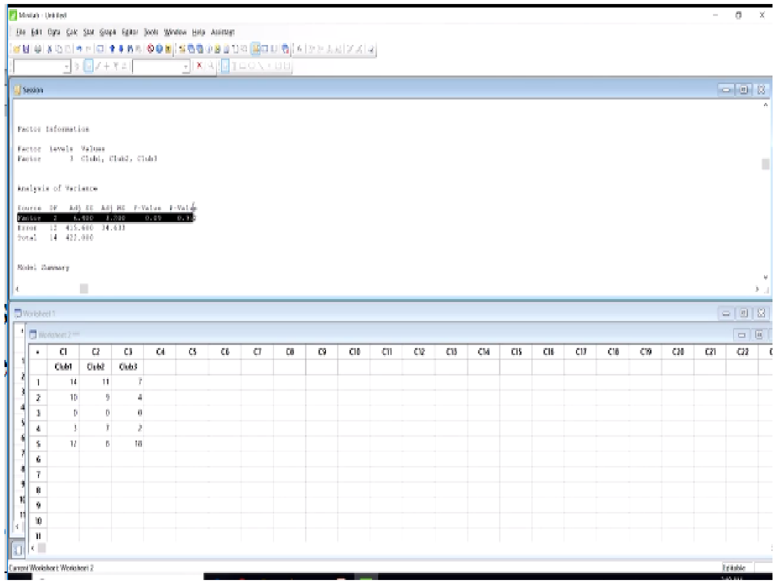


So, let us look at this question is though we have already worked out this question but what about variance; we have not checked equality of variance so far, so null hypothesis, alternative hypothesis, so what we have said in this Levene's test is that from each group we will select the median, so from first group, this is the median value, second group this is the median value, third group this is the median value, right.

So, it is 251- 237, we will get 14, right then 251- 241, we will get 10, so we have to write absolute differences over here, 251- 251, 0 right, 251-263, this is; right, this is actually 251-263 is -12 but we have to write absolute value, right, so this is 12 similarly, 227 – this - this - this – this – this, right, you will get all these values, similarly from group 3, so this is the; this table is about absolute differences.

And what is the next step; we have to perform one way ANOVA on these absolute differences, so when we perform this one way ANOVA on this, we should reject the null hypothesis, now we should not reject null hypothesis, is not it, so let us see whether we will rejecting null hypothesis or not on this value, so we will perform one way hypothesis testing on this data, right, let us look at this.

(Refer Slide Time: 20:50)

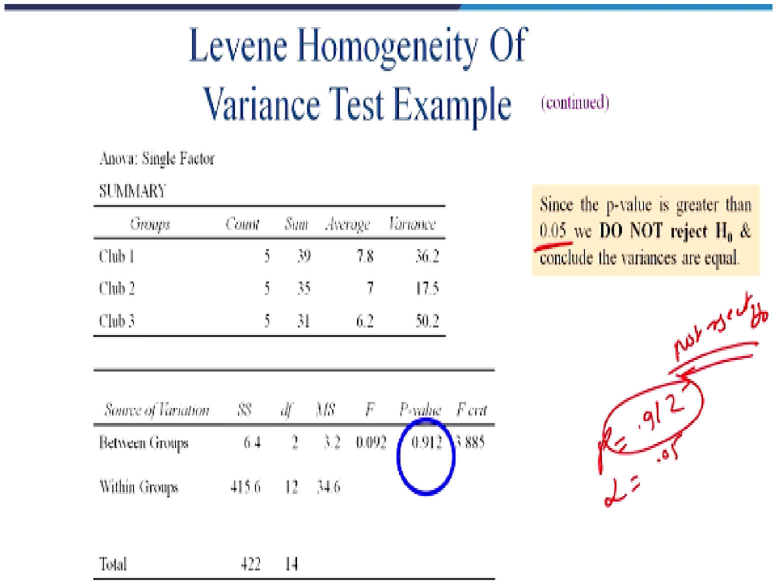


So, we will go to okay, we will have a new file, so this is let us say club 1, club 2 and club 3 and for club 3, now we will enter the absolute differences, right, we will not write the actual values, right, so this is for club 2; 11, 9, 0, 7 and 8, for club 3; 7, 4, 0, 2, 18, we will go to stat, will perform one way ANOVA, right, okay so we will select all these three values, we will go to options in fact, we should go to options.

And of course, we are testing it at 95% significance level let us check whether it is 95 or not, yes it is 95% significance level, so let us look at this, so this lower bound will go to okay, then a comparison, okay there is no need to click any other option, so we will go to okay, click okay, so let us look at ANOVA table, so just minimise these 2, right, so let us look at ANOVA table, now if you look at the differences over here will go downward.

Let us look at what is F value, can we have some F value somewhere here, just go up, yeah here it is right, so look at P value first, first we will decide whether we are rejecting null hypothesis or alternative hypothesis, right, so this P value.

(Refer Slide Time: 23:14)



Groups	Count	Sum	Average	Variance
Club 1	5	39	7.8	36.2
Club 2	5	35	7	17.5
Club 3	5	31	6.2	50.2

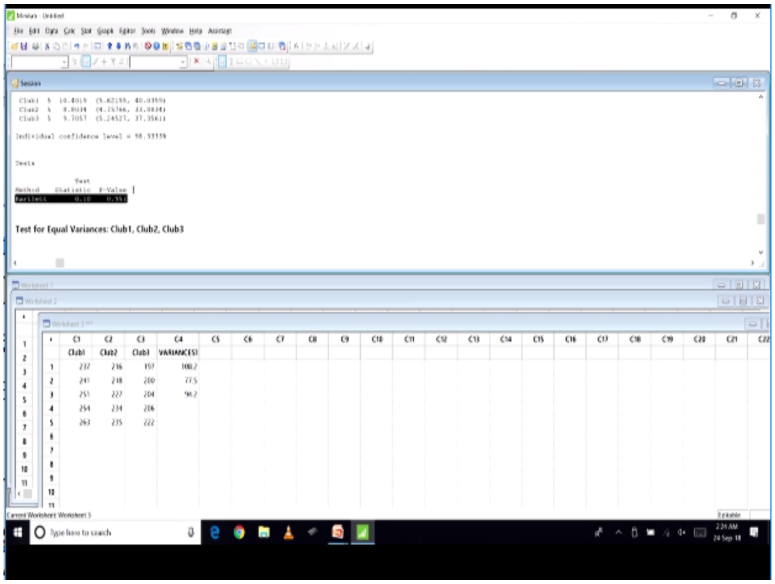
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	6.4	2	3.2	0.092	0.912	3.885
Within Groups	415.6	12	34.6			
Total	422	14				

So, P value is just look at .91; .912 and alpha is .05, is P value < alpha; no, so we will not reject null hypothesis, this is the P value, right, so what conclusion we are trying, we will not reject null hypothesis, what is our null hypothesis; null hypothesis is that all these variances are same, so we are not rejecting, it means we are saying that all these variances are same and that is one of the conditions for performing ANOVA, right.

So, this how you can use Levene's test, in fact before performing ANOVA, you should perform Levene's test, so if you let us say if you reject null hypothesis in Levene's test, you should not go for ANOVA, so this is in fact a precondition for ANOVA, okay. So, what we have done; we have performed one way ANOVA on absolute differences but there is an option in Minitab wherein you can directly compare all the variances.

So, let us look at the same question once again, this is the question we have got, this is the question so, for simplicity let me remove all these points so we will take the same question and we will find out variances, we will directly compare their variances not using this particular data so, let us look at the Minitab solution to this question, right and we will directly calculate whether the variances are same or not, right.

(Refer Slide Time: 25:26)



So, we will go for data entry, let us look at this, so in fact this can be maximised, okay, so club 1, club 2, club 3, so now we will write original values of the question not the absolute differences, so 263, then we have got 216, 218, 227, 234 and 235, 197, 200, 204, 206 and 222, now go to stat and then go to ANOVA, right. In ANOVA, there is an option over a test for equal variances, right, so you can directly go to there that value that point test for equal variances.

So, respondents that are not in one column but they are in separate columns right, so just select all these 3 values and we will go to options, so use based on normal distribution of course you can click this okay, let us look at what are other things, so we will click it okay first, then graphs, okay there is no need of having other graphs, let us look at storage, let us look at click at variances, right, okay, then okay.

Let us look at the P value, right, we will go there, right, so we have this test for equality of these 3 groups right, so significance level is this alpha will go downwards, P value is this, P value is .9 51, so P value is .951 which is more than alpha, so we will not reject null hypothesis, is not it, since P value is more than alpha, we will not reject null hypothesis and when we say we are not rejecting null hypothesis, it means all these variances are same, all variances are equal.

Hence we proved the same thing, so this is how you can in fact use either absolute differences for testing Levene's test or you can directly enter original values in Minitab and compare variances, so before moving on to the randomised block design, let me summarise what we have done in today's session. In today's session, we have looked at couple of examples on ANOVA, as all of you know that ANOVA is basically a tool for comparing means of 2 or more than 2 groups.

Now, in ANOVA you have dependent variable and independent variable, so in independent variable you have got different levels which we call them treatment levels, so if the number of simple sizes are same in different groups, then we should use something called Tukey HSD test, honestly significant difference test, we did workout couple of examples using a Tukey HSD test, in today's class, we discussed about Tukey Kramer test.

Now, this is extension of Tukey HSD test, there is little change in the formula otherwise, every thing is same as we have seen in HSD test, now apart from this, we have looked at couple of assumptions of ANOVA, so the assumptions were there were different assumptions, the first one was that the samples from where we are taking let us say the populations from where we are taking samples should be normally distributed.

The second assumption was that we should take samples in a random manner, random selection of the sample, the third assumption was Levene's test and these assumption was for equality of variances or homogeneity of variances, so this homogeneity of variances, we have tested using Levene's test, now Levene's test can be done in 2 ways; the 1st is directly you can compare variances of different groups, there is an option in Minitab, so you can directly enter your data.

But keep in mind that in Levene's test, you should always, you should not reject the null hypothesis, is not it, if you reject null hypothesis you cannot do ANOVA, so before performing ANOVA in any question you should do Levene's test, the other way of testing equality of variance is first you calculate the absolute differences from the given question and then enter those values in software and perform one way ANOVA.

So, this is what we have done in today's session, in next session we will talk about randomised design which is the second type of experiment design, remember that I have told you that there are three types of experimental designs, the first one was completely randomised design where there is only one independent variable and a dependent variable in randomised block design will have a second variable as well, right so we will discuss about randomised block design in next session till then good bye, thank you very much.