Business Statistics
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Lecture - 44
Examples of Hypothesis Testing

Hello friends. I welcome you all in this session. As you are aware in previous session we were discussing hypothesis testing of proportion for 2 sample and we have worked out couple of examples using Minitab. So I am going to give you 2 questions here for your let us say home assignment and I would request all of you to solve these 2 questions. And the question number one is like this.

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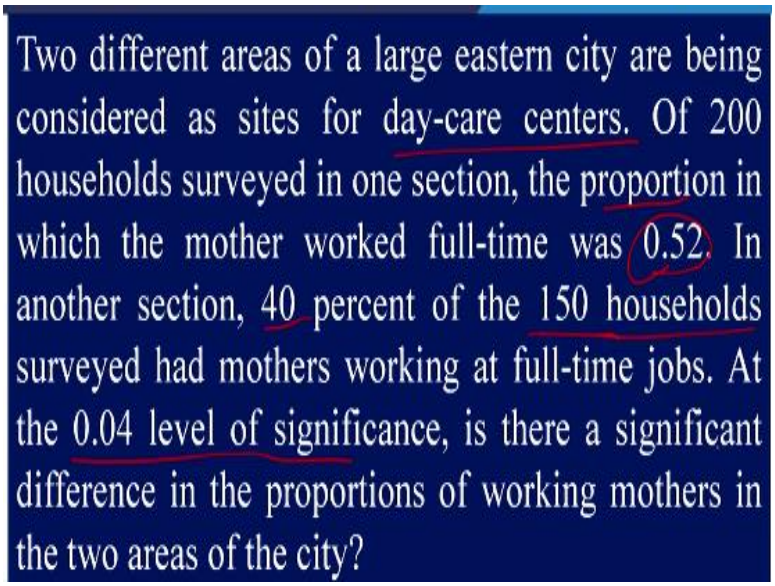

Two different areas of a large eastern city are being considered as sites for day-care centers of 200 household surveyed in one section. The proportion in which the mother worked full-time was 0.52 so proportion was 0.52. In another section 40% of the 150 household surveyed had mother working at full-time jobs. So 40% out of 150 households it means 60 women out of 150 households right.

At the 0.04 significance level is there a significant difference in the proportion of working mothers in 2 areas of the city. So you just frame null hypothesis and alternative hypothesis you can solve it using critical value approach or P value approach or confidence interval approach. So this is the solution, but I am not going to explain this because we have worked out several examples of this type. Similarly let us look at second example.

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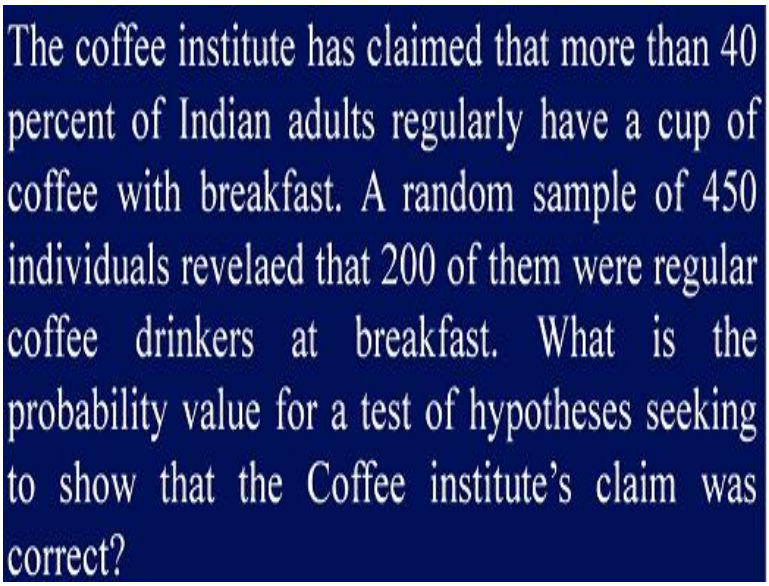

The coffee institute has claim that more than 40% of Indian adults regularly have a cup of coffee with breakfast. A random sample of 450 individuals revealed that 200 of them were regular coffee drinkers at breakfast. What is the probability value for a test of hypothesis seeking to show that the coffee institute claims was correct. So what is the claim of coffee institute more than 40% of Indian adults have coffee at the time of breakfast.

So null hypothesis and alternative hypothesis you are supposed to frame and then solve this question. So this is the solution I am not going to explain this solution just work out using Minitab and let us move on to exercise on hypothesis testing. So there are several statements and you have to tell me tell whether these statements are true or false.

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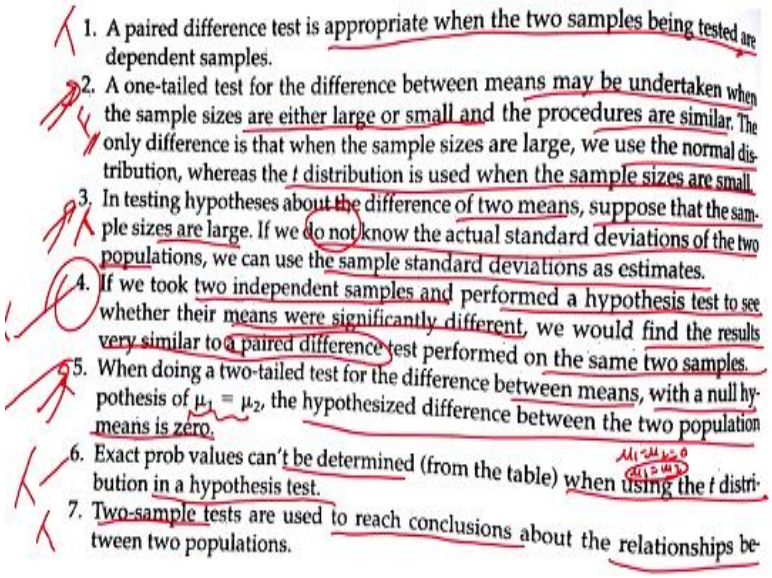
1. A paired difference test is appropriate when the two samples being tested are dependent samples.
2. A one-tailed test for the difference between means may be undertaken when the sample sizes are either large or small and the procedures are similar. The only difference is that when the sample sizes are large, we use the normal distribution, whereas the *t* distribution is used when the sample sizes are small.
3. In testing hypotheses about the difference of two means, suppose that the sample sizes are large. If we do not know the actual standard deviations of the two populations, we can use the sample standard deviations as estimates.
4. If we took two independent samples and performed a hypothesis test to see whether their means were significantly different, we would find the results very similar to a paired difference test performed on the same two samples.
5. When doing a two-tailed test for the difference between means, with a null hypothesis of $\mu_1 = \mu_2$, the hypothesized difference between the two population means is zero.
6. Exact prob values can't be determined (from the table) when using the *t* distribution in a hypothesis test.
7. Two-sample tests are used to reach conclusions about the relationships between two populations.

So let us look at first question. A paired difference test is appropriate when 2 samples being tested or dependent samples true or false. This is true statement right because in the very first class we have talked about what is dependent sample and independent sample right. Let us look at second question a one tailed test for the different between means may be undertaken when the samples sizes are either large or small and procedures are similar.

The only difference is that when sample sizes are large we use normal distribution whereas as t distribution is used when the samples sizes are small. Just read this question once again and try to answer, is this true or false. This is completely false statement whether we use normal distribution or t distribution it not only depends on sample size, but other parameter as well right. So that is why this statement is false.

Let us look at third one in testing hypothesis about the difference of 2 means. Suppose that the sample sizes are large we do not know the actual standard deviation of the 2 population we do not know right. So we do not know the actual standard deviation of the population. We can use the sample standard deviation as estimates. Is it true or false? If we do not know the standard deviation of the population then we use sample standard deviation to estimate population standard deviation this statement is true.

Let us look at fourth one. If we took 2 independent samples and performed a hypothesis test to see whether their means were significantly different right. We would find the results very similar to a paired difference test performed on the same 2 samples what do you think. So there are 2 samples 2 independent samples. You have performed hypothesis testing of means and you found some result.

Now would you find the same results if you take those 2 samples as dependent samples or related sample. Will you find the same very same result? No this is false you will have 2 different results right. Let us look at the next one when doing a 2 tailed test for the difference between means 2 tailed test for difference between means with a null hypothesis as this the hypothesized difference between these 2 population means is 0 is it. Do we hypothesized that the difference between these 2 population means is 0 yes, we do. This is true.

Because we always say mu1-mu2-0 right. So either you say mu1= mu2 are the same can we written as this is not it mu1= mu2 right. So statement number 5 is true. Exact probabilities

values cannot be determined when using the t distribution in a hypothesis test, is it true or false. Exact probability values cannot be determined when using t distribution in hypothesis test. Yes, this is true right.

Let us look at next one. Two sample tests are used to reach conclusions 2 samples are used to reach conclusion about the relationship between 2 populations, is it correct. 2 sample test are used to reach conclusion about relationship between 2 populations. Yes, this is absolutely true okay.

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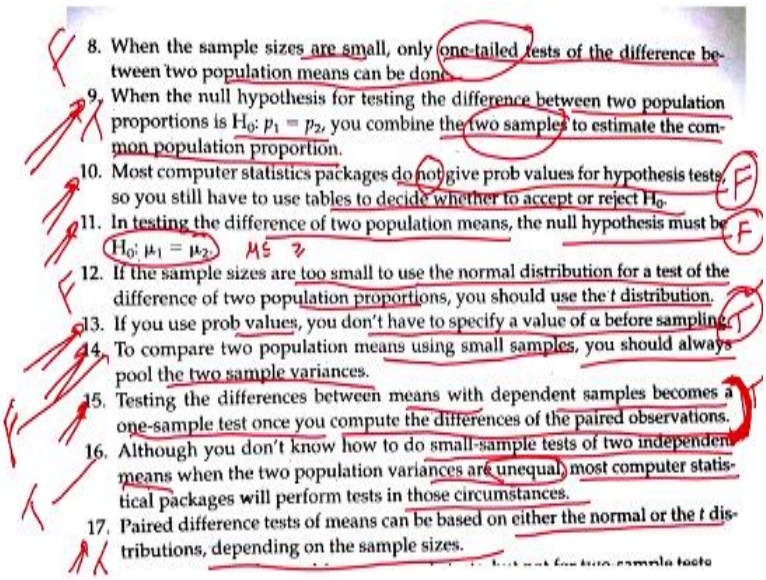
8. When the sample sizes are small, only one-tailed tests of the difference between two population means can be done.
9. When the null hypothesis for testing the difference between two population proportions is H_0: $p_1 = p_2$, you combine the two samples to estimate the common population proportion.
10. Most computer statistics packages do not give prob values for hypothesis tests, so you still have to use tables to decide whether to accept or reject H_0.
11. In testing the difference of two population means, the null hypothesis must be H_0: $\mu_1 = \mu_2$.
12. If the sample sizes are too small to use the normal distribution for a test of the difference of two population proportions, you should use the *t* distribution.
13. If you use prob values, you don't have to specify a value of α before sampling.
14. To compare two population means using small samples, you should always pool the two sample variances.
15. Testing the differences between means with dependent samples becomes a one-sample test once you compute the differences of the paired observations.
16. Although you don't know how to do small-sample tests of two independent means when the two population variances are unequal, most computer statistical packages will perform tests in those circumstances.
17. Paired difference tests of means can be based on either the normal or the *t* distributions, depending on the sample sizes.

Let us look at the next one. When the sample sizes are small only one tailed test of the difference between 2 populations means can be used, is it. When sample size is small do we use only one tailed test. No in fact one tail or 2 tail test has nothing to do with sample size is not it. So this statement is false number 8 is false. Let us look at 9 when the null hypothesis for testing the difference between 2 populations proportion is this when null hypothesis is this you combine the 2 samples to estimate the common population proportion.

Do we really combine? You combine the 2 samples to estimate the common population proportion yes we do combine. So ninth one is true. Let us look at tenth one. Most computer statistics packages do not give prob values for hypothesis tests. So you still have to use table to decide whether to accept or reject null hypothesis. Is it correct what do you think? Most computer statistics packages do not give prob values in fact the other way round is correct.

They do give prob values right so statement is false right tenth is false. Let us look at

eleventh one. In testing the difference of 2 population means in testing the difference of 2 population means the null hypothesis must be mu1=mu2. Is it necessary that null hypothesis has to be mu1=mu2? No there are in fact we have seen examples where you can have null hypothesis like this or you can have null hypothesis like this is not it.

So you can have all kinds of symbols in null hypothesis so this is false 11 number is false okay. Let us look at the 12th one. If the sample sizes are too small to use normal distribution for test of the difference of 2 population proportion you should use the t distribution is it. If the sample sizes are too small to use normal distribution for the test of the differences of 2 population proportion you should use t distribution. No this is false.

Again as I said whether to use normal distribution or t distribution you look at sample size as well population standard deviation. Let us look at 13th number question. If you use prob you do not have to specify value of alpha before sampling. If you use prob values, you do not have to specify yes this is true. There is no need to specify alpha before sampling right. So 13th is true okay.

Let us look at 14th one. To compare 2 population means using small samples you should always pool the 2 sample variances. Is it to compare 2 population means using small samples you should always pool the 2 sample variances. No this is false. We do not pool the 2 sample variances right. Let us look at 15th. Testing the differences between means with dependent samples become a one sample test once you compute the difference of the paired observation.

Testing the differences between means with dependent samples becomes one sample test once you compute the differences of paired observation yes this is correct. 15th is true. In fact, we have worked out one example wherein we had a situation where a gym instructor, the fitness training center instructor claim that the weight loss would be more than 70 pounds right and then we framed null and alternative hypothesis is just one sample rather than 2 sample right.

Let us look at 16th one. Although you do not know how to do small sample test for 2 independent means when the 2 population variances are unequal they are unequal. Most computer statistics packages will perform test in those circumstances, is it correct? Yes, this is correct statistical packages will perform test in those circumstances where variances are

unequal right.

Let us look at 17th paired difference test of means can be based on the normal or t distribution depending upon sample size. Paired difference test of means can be based on either normal or t distribution depending upon sample size. Yes, this is true right okay.

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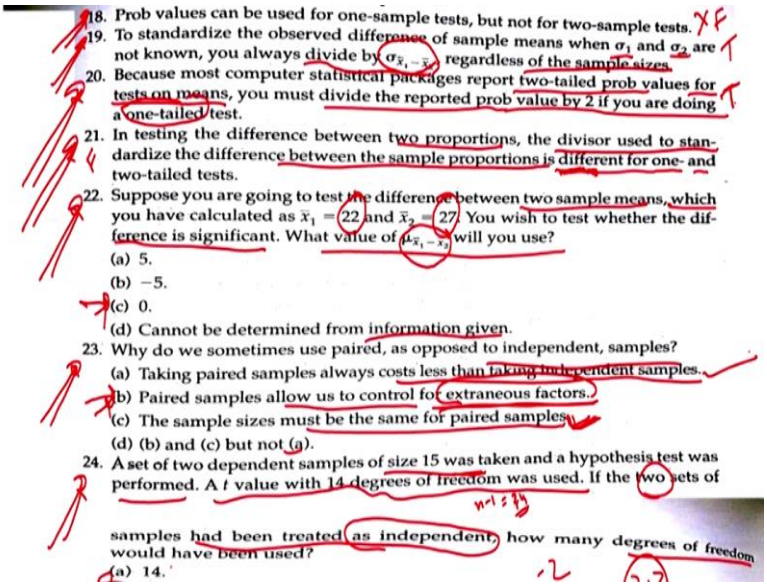
18. Prob values can be used for one-sample tests, but not for two-sample tests.
19. To standardize the observed difference of sample means when σ_1 and σ_2 are not known, you always divide by $\sigma_{\bar{x}_1 - \bar{x}_2}$ regardless of the sample sizes.
20. Because most computer statistical packages report two-tailed prob values for tests on means, you must divide the reported prob value by 2 if you are doing a one-tailed test.
21. In testing the difference between two proportions, the divisor used to standardize the difference between the sample proportions is different for one- and two-tailed tests.
22. Suppose you are going to test the difference between two sample means, which you have calculated as $\bar{x}_1 = 22$ and $\bar{x}_2 = 27$. You wish to test whether the difference is significant. What value of $\mu_{\bar{x}_1 - \bar{x}_2}$ will you use?
    (a) 5.
    (b) −5.
    (c) 0.
    (d) Cannot be determined from information given.
23. Why do we sometimes use paired, as opposed to independent, samples?
    (a) Taking paired samples always costs less than taking independent samples.
    (b) Paired samples allow us to control for extraneous factors.
    (c) The sample sizes must be the same for paired samples.
    (d) (b) and (c) but not (a).
24. A set of two dependent samples of size 15 was taken and a hypothesis test was performed. A t value with 14 degrees of freedom was used. If the two sets of samples had been treated as independent, how many degrees of freedom would have been used?
    (a) 14.

So we will move on to next slide and next question is this 18 number. Prob values can be used for one sample test, but not for 2 sample test, is it? No it is false. Prob values can be used for both one as well as 2 samples very simple questions. Number 19 to standardize the observed difference of sample means when sigma 1 and sigma 2 are not known. To standardize the observed difference of samples when sigma 1 and sigma 2 are not known.

You always divide by this regardless of the samples size is it so. So 19 is true. Yes, we divided it by standard the difference of standard error right. So 19 is true. Let us look at 20th because most statistical packages report 2 tailed prob values for test on means. You must divide the reported prob value by 2 if you are doing one tailed test. Yes, we have to divide by 2 so this is true statement question number 20 is true.

Let us look at the next one. In testing the difference between 2 proportions the divisor used to standardize the difference between sample proportion is different for one and 2 tailed test. In testing the difference between 2 proportion the divisor used to standardize the difference between sample proportion is different. Is it? No I do not think it is different. It is same right so this statement is false. So it is same for one and 2 tailed test right it is not different.

Let us look at 22. Suppose you are going to test the difference between 2 samples means which you have calculated as 22 and 27 right. You wished to test where the difference is significant what value of this will you use. It is 5-5 0 are cannot be determined from information given below. So value of this right will be 0 in this case okay. Let me move on question number 23. Why do we sometimes use paired as opposed to independent samples.

Why do we sometimes use paired test as opposed to an independent sample what is the reason. Taking paired sample always cost less than taking independent samples is this the reason. Paired samples allow us to control for extraneous factors. Extraneous factors are those factor which affect relationship between dependent and independent variables and there are several extraneous factors.

The sample sizes must be same for paired sample of course it has to be there for pair sample b and c, but not a. So the answer is b. We always use paired sample so that we can remove the effect of extraneous factors right. So it allows us to control for extraneous factors. Let us look at 24th. A set of 2 dependent samples of size 15 a set of 2 dependent samples of size 15 was taken and a hypothesis test was performed.

A t values with 14 degrees of freedom was used because n-1 right. So that is why 14 degrees are used. If 2 sets of sample had been treated as independent sample how many degrees of freedom would have been used. So it was n1+n2-2 (()) (21:50) 15+15-2 is 28. So b is the correct answer for this. B is correct answer okay.

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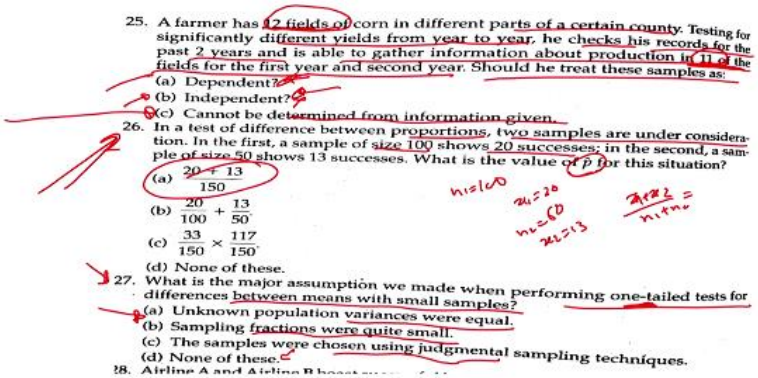
25. A farmer has 12 fields of corn in different parts of a certain county. Testing for significantly different yields from year to year, he checks his records for the past 2 years and is able to gather information about production in 11 of the fields for the first year and second year. Should he treat these samples as:
(a) Dependent?
(b) Independent?
(c) Cannot be determined from information given.
26. In a test of difference between proportions, two samples are under consideration. In the first, a sample of size 100 shows 20 successes; in the second, a sample of size 50 shows 13 successes. What is the value of $\bar{p}$ for this situation?
(a) $\frac{20 + 13}{150}$
(b) $\frac{20}{100} + \frac{13}{50}$
(c) $\frac{33}{150} \times \frac{117}{150}$
(d) None of these.
27. What is the major assumption we made when performing one-tailed tests for differences between means with small samples?
(a) Unknown population variances were equal.
(b) Sampling fractions were quite small.
(c) The samples were chosen using judgmental sampling techniques.
(d) None of these.
28. Airline A and Airline B boast

Let us look at 25 a farmer has 12 fields of corn in different parts of certain country. Testing for significantly different yields from year-to-year. He checks his records for the past 2 years

and is able to gather information about production in 11 of the fields for the first year and second year. Should he treat these samples as dependent independent cannot be determined from information.

So for having dependent sample you should have data collected before and after right. So if you had let us say data of all 11 fields then it would have become dependent. In case of independent again initially in this question it is written as farmer is 12 fields, but information is available only on above 11 so it cannot be independent right so the answer is c cannot be determined from information given right. Let us look at 26th one.

In a test of difference between proportions 2 samples are under consideration. In first sample the size is 100. So n1 is this shows 20 successes so x1 is 20. In the second sample this was 50 and x2 13 right. So the value of (()) (24:01) will be it was x1+ x2/n1 +n2 is not it. So this is 33/50 right. So A is the correct answer for this. Let us look at 27th number what is the major assumption we made when performing one tailed test for different between means with same samples major assumption.

What is the major assumption we made when performing one tailed test for differences between means with small samples. Unknown population variances were equal. So unknown population variances were equal. Sampling fractions were quite small. The sample were chosen using judgmental sampling technique and none of this. So the answer to 27 is of course A is what we have assumed. Let the population variances were unknown and equal right

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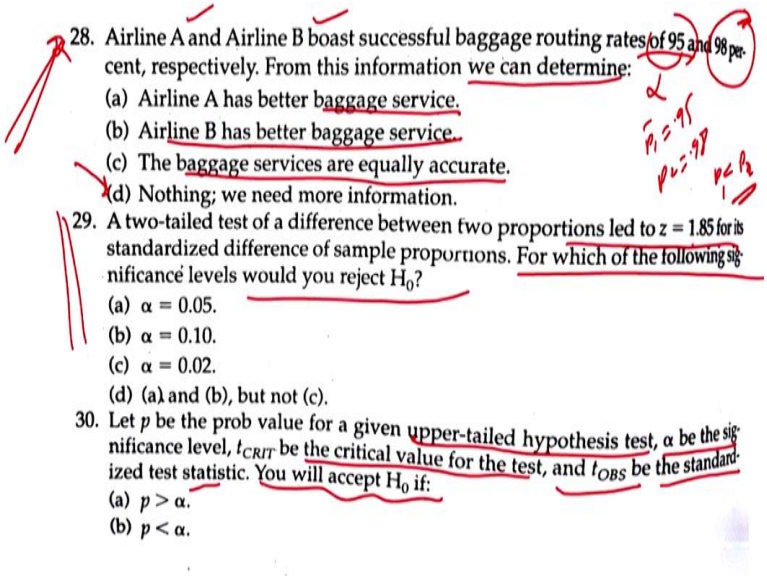

28. Airline A and Airline B boast successful baggage routing rates of 95 and 98 percent, respectively. From this information we can determine:
    (a) Airline A has better baggage service.
    (b) Airline B has better baggage service.
    (c) The baggage services are equally accurate.
    (d) Nothing; we need more information.
29. A two-tailed test of a difference between two proportions led to $z = 1.85$ for its standardized difference of sample proportions. For which of the following significance levels would you reject H_0?
    (a) $\alpha = 0.05$.
    (b) $\alpha = 0.10$.
    (c) $\alpha = 0.02$.
    (d) (a) and (b), but not (c).
30. Let p be the prob value for a given upper-tailed hypothesis test, α be the significance level, t_{CRIT} be the critical value for the test, and t_{OBS} be the standardized test statistic. You will accept H_0 if:
    (a) $p > \alpha$.
    (b) $p < \alpha$.

Let us look at next question 28. Airline A and Airline B boast successful baggage routing rates of 98% and 95% right. So 95% and 98% from this information we can determine Airline A has better baggage service, Airline B has better baggage service the baggage services are actually accurate nothing we need more information yes. So for this question we need more information right.

So if we had the itself let us say alpha then we would have checked whether the performance is more in Airline B or not right. So you would have a P1=0.95 P2=0.92 right and you could have framed let us say P1<P2 okay. A 2 tailed test of difference between 2 proportion lead to 1.85 for its standardized difference of sample proportions for which of the following significance level would reject.

So we would just look at z table and you can find out answer to this question I am skipping this question. Let P be the prob values for a given upper tailed test hypothesis. Alpha be the significance level tCRIT be the critical value of test and tOBS is the standardized test statistics you will accept null hypothesis if these 2 are correct so answer to this is B. So B and C but not A right.

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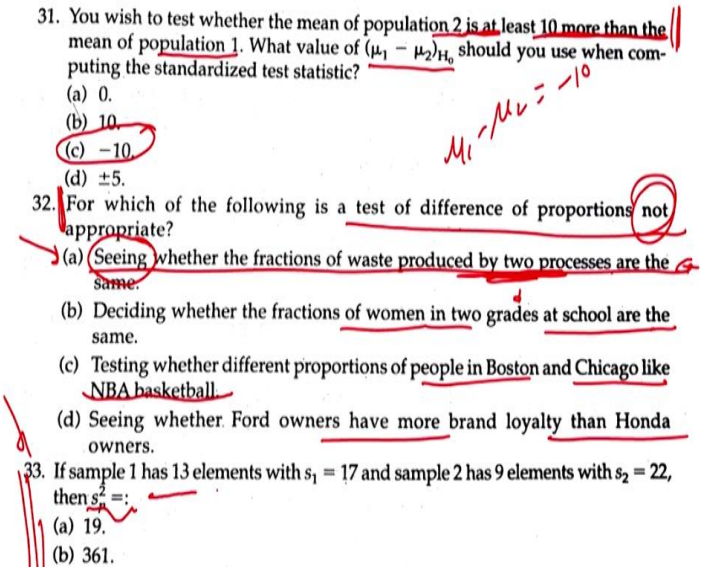
31. You wish to test whether the mean of population 2 is at least 10 more than the mean of population 1. What value of $(\mu_1 - \mu_2)_{H_0}$ should you use when computing the standardized test statistic?
(a) 0.
(b) 10.
(c) −10.
(d) ±5.

32. For which of the following is a test of difference of proportions not appropriate?
(a) Seeing whether the fractions of waste produced by two processes are the same.
(b) Deciding whether the fractions of women in two grades at school are the same.
(c) Testing whether different proportions of people in Boston and Chicago like NBA basketball.
(d) Seeing whether Ford owners have more brand loyalty than Honda owners.

33. If sample 1 has 13 elements with $s_1 = 17$ and sample 2 has 9 elements with $s_2 = 22$, then $s_p^2 =$:
(a) 19.
(b) 361.

Let us look at next question you wish to test whether the mean of population 2 is at least 10 more than the mean of population 1. So mean of population 2 is higher right. You wish to test whether the mean of population 2 is at least 10 more than mean of population one right. What value of mu1-mu2 you should use when computing standardized test statistics. So in this case you will be using this right. Mu-Mu2=-10 right.

Then only this statement become correct right. For which of the following is a test of difference of proportion not appropriate. For which of the following is a test of difference of proportions not appropriate. Seeing whether the fractions of waste produced by 2 processes are the same seeing right. Deciding whether the fraction of women in 2 grades at school are the same. Deciding whether fractions of women in 2 grades at school are the same.

Testing whether different proportion of people in Boston and Chicago like NBA basketball. Seeing whether Ford owners have more brand loyalty than Honda owners. So answer to this is A. So fraction of waste produced by 2 process are the same. You cannot have this kind of situation with this statement is true. So for which of the following is a test of difference of proportion not appropriate.

So seeing whether the fraction of waste produced by 2 process okay. Let us look at next one of course you would be requiring table for this. In fact, you have the formula for this right (()) (30:15) variance. You just put these values in formula you will get the answer right. For which of the following is a 2 test 2 sample test not appropriate.

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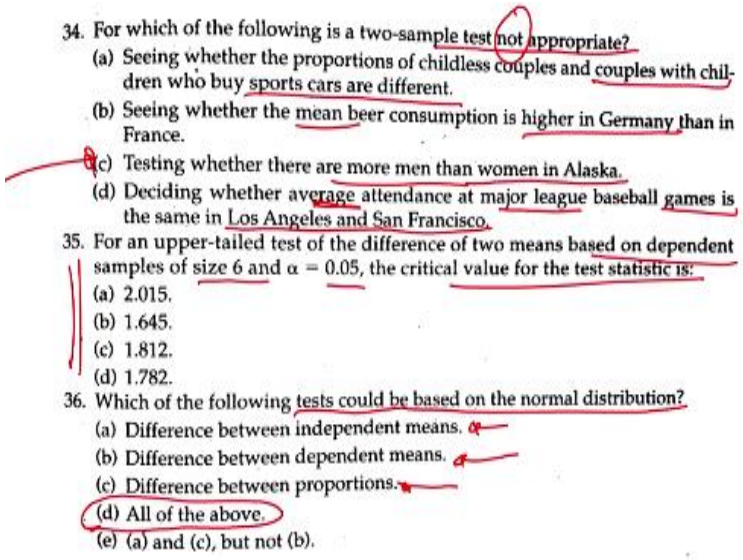

34. For which of the following is a two-sample test not appropriate?
    (a) Seeing whether the proportions of childless couples and couples with children who buy sports cars are different.
    (b) Seeing whether the mean beer consumption is higher in Germany than in France.
    (c) Testing whether there are more men than women in Alaska.
    (d) Deciding whether average attendance at major league baseball games is the same in Los Angeles and San Francisco.
35. For an upper-tailed test of the difference of two means based on dependent samples of size 6 and $\alpha = 0.05$, the critical value for the test statistic is:
    (a) 2.015.
    (b) 1.645.
    (c) 1.812.
    (d) 1.782.
36. Which of the following tests could be based on the normal distribution?
    (a) Difference between independent means.
    (b) Difference between dependent means.
    (c) Difference between proportions.
    (d) All of the above.
    (e) (a) and (c), but not (b).

For which of the following is 2 sample test not appropriate right not appropriate. Seeing whether the proportion of childless and couples with children who buys sports car are different. Here we are not saying whether it is a 2 sample (()) (30:48) test are we are not saying whether it is a means or proportion or variance. Which of the following is a 2 sample test not appropriate.

Seeing whether the proportion of childless couples and couples with children who buys sports car are different. Seeing whether the mean beer consumption is higher in Germany than France. Testing whether there are more men than women in Alaska. Deciding whether average attendance at major large baseball game is same in Los Angeles and San Francisco. So the correct answer for this is C.

So let us look at these 2 questions and then we will finish this session. For an upper tailed test of the difference of 2 means based on dependence sample of size this alpha this the critical value of test is statistics just look at t table you will get the answer. Which of the following tests could be used on the normal distribution difference between independent means, difference between proportion all of this are this.

So the answer to this question is all of this. So with this let me finish today's session. In this session, we have worked out couple of examples on hypothesis testing and we have refreshed the concept related to hypothesis testing. Thank you very much.