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Lecture – 41
Hypothesis Testing: Two Samples – III

Hello friends, I welcome you all in this session as you are aware in previous session, we were solving a couple of examples on two sample based and we worked out a couple of examples using a probability value approach and critical value approach and we solved examples using critical value approach and the answer which we were getting. We verified those answers using mini tab software. Today we will look at some more questions related to two simple hypothesis testing. So let us look at this example.

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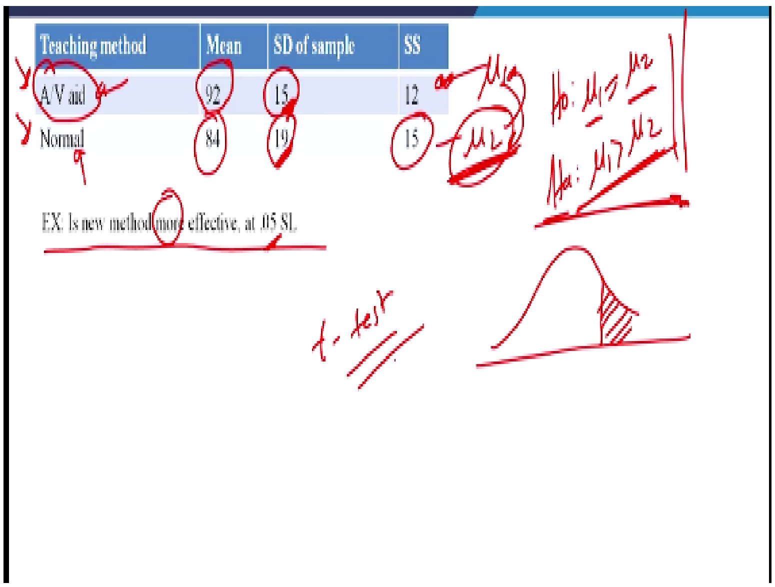

There is a faculty member who is in fact teaching students using audio visual aids and just normal lecturing method. So, there are two ways of teaching methods either just normal lecturing method or using audio video visual aids. So, he wants to know the effectiveness of these two methods we wants to know whether this audio visual method is over effective compared to the normal teaching method or teaching about it.

So, what he did he selected a sample of 15 students and he just took the mean of marks of those 15 students in a subject and the standard deviation of marks was 19. Now when this same teacher thought the students using audio visual aids the mean marks of the students was 92 and the same size he choose was 12, sample standard deviation 15 so the question is new method more effective to test this hypothesis at .05 significance level.

So, now you have to frame null hypothesis alternative hypothesis then you need to decide on which test you are suppose to apply whether it t test or Z test so the question is very clear. Is new method is more effective? And new method is audio visual aids, so how would you write null hypothesis? Null hypothesis is that μ1 and μ2 are same the mean marks obtained by normal method and mean marks obtained by audio visual method are same.

Right alternative hypothesis is what it would be mu 1 is better method than mu2 so here we are saying that μ1 is this and μ2 is in fact here old method right. Mean of the old method this is after how you are you know assigning new one and μ2 you have this as μ2 and this as μ1 only the thing is this change so this is your null and alternative hypothesis. This is now we have to decide on what type of test you are applying.

So, first is first thing is very clear this is 1 tail test example so it is right tail test this is the rejection region in right tail test and what about z test t test which test you would be using emphasis in both this cases < 30. You have been given simple standard deviation not the population standard deviation. We will use t test over here so let us look at the solution to this question.

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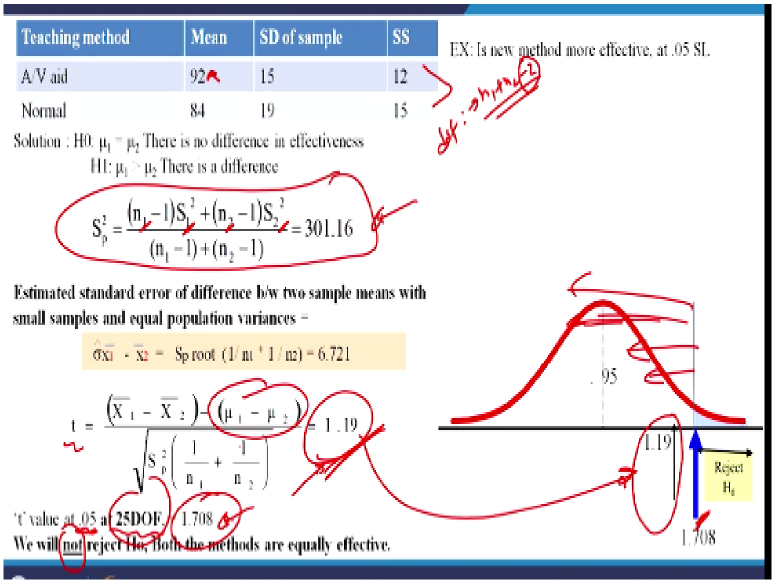

So, first of all we can calculate pooled variance SP square so this n1 – 1* S1 square so this is standard is variance of first sample and variance of second simple. first sample size and second sample size so this comes out to be 301.16 and the t calculated value is 1.19. You need to keep in mind that this term µ1-µ2= 0 so X1 bar-X2 bar is simple you have this 92 – this so t value 0.05 degrees 0.05 at 25 degrees of freedom.

How come 25 this is n – this is n1+ n2 this is 27-2 so this is 25 this how we calculate degree of freedom n_1+n_2-2 because you have two samples over here right. In one sample test our degree of freedom was different one. Okay so let us look at this so calculated t value is 1.19 and the table value is 1.708 this is calculated so this is our calculated value t value. This is table value since our calculated value is in this region.

This region this side is this side so we will not reject Null hypothesis so at the end of day we will take decision that will not reject null hypothesis and we will conclude that both these teaching methods are equally effective. There is no significant difference in the effectiveness of these to my methods that is our conclusion right though the mean looks like that this method is better but it is not like that. It is statistically there is no significant difference in these two methods and of course how to look at this t value this t value it is 1.708 at 25 degrees of freedom.

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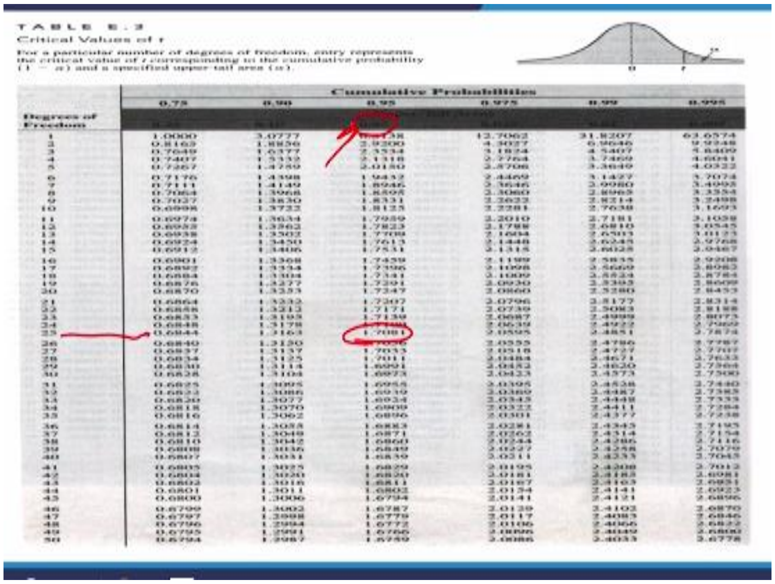

So, 25 degrees of freedom 1.7081 look at here so this is actually 0.05 this value so we can infact workout this example using mini tab software. So, let us use minitab and work out this example

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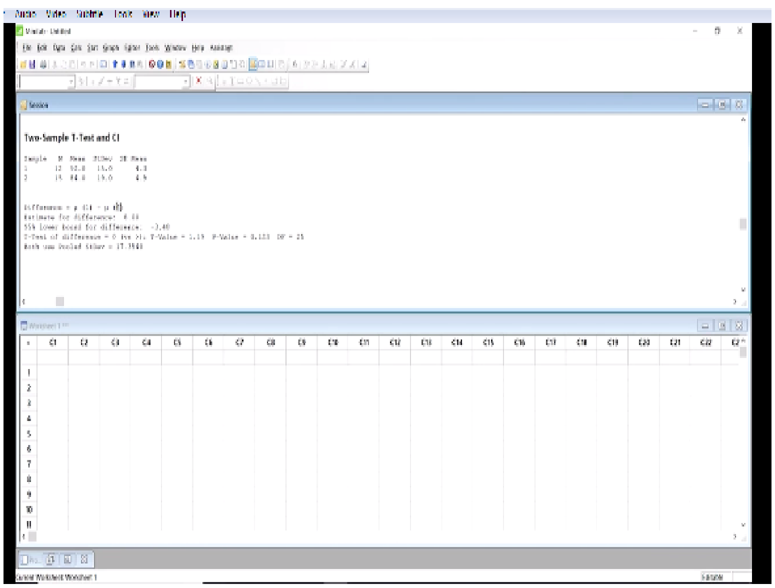

So, go to stat, basic statistics it is 2 sample t-test so 2 sample-t test now you can have values either in different columns and you can have summarize data. So, we will use summarize data option so sample size of first sample is let us say 1215, 1292, and 15 day is the standard deviation. Let us look at information about sample number 2 you have got 15, 84 and 19. click options, we need to test this hypothesis that what significance level.

We go back to the question it is 0.05 and this is 0.05 so it is 95 so hypothesize difference is 0. Alternative hypothesis has to be some more than right so the difference is more than hypothesize difference and you have to assume equal variances. This is one of the assumptions for hypothesis testing so okay again click okay. So, if you look at p value over here P value is 0.123 in fact other information is available here.

Sample 1 sample 2 there means standard deviation so we are saying that difference is 0 and this is our null hypothesis. So, p value is 0.123 so p value is 0.123 alpha is equal to 0.05. Is p value less than alpha? no. So, we do not reject null hypothesis so this is our conclusion using P value approach and using mini tab let us look at one more question.

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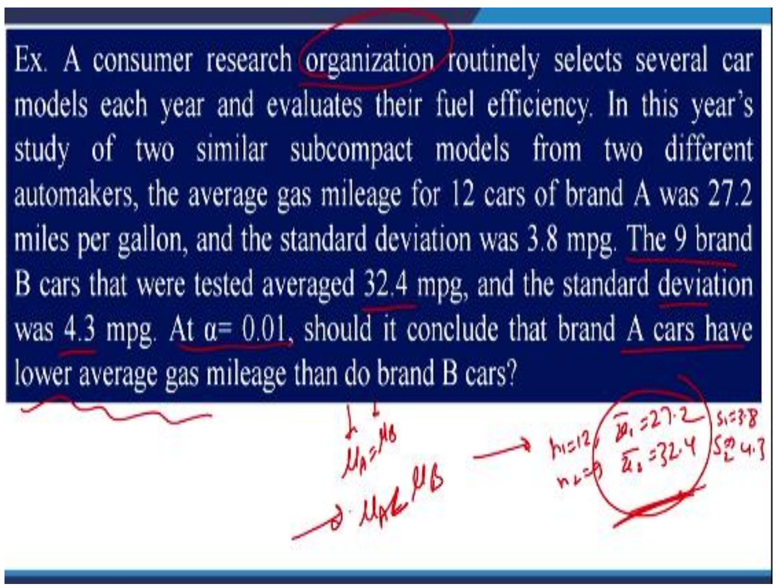

A consumer research organization routinely several car models each year and evaluates their fuel efficiency. In this year's study of 2 similar subcompact models from 2 different automakers the average gas mileage for 12 cars of brand A was 27.2 so n1=12 average is 27.2 and standard deviation was 3.8 this is for first sample The 9 brand B cars that were tested averaged this much so n2 is 9 and average = 32.4 and standard deviation is 4.3.

And now the question is at .01 significance level should it conclude that brand A cars have lower average gas mileage than do brand A so what will be the null and alternative hypothesis so let us $\mu_A = \mu_B$ right average of A average B these 2 are not these 2 infact sample means we will say that let us say the population means of A and B are same and can this organisation conclude brand A cars have lower average. So, $\mu_A < \mu_B$ this is our alternative hypothesis so this is the case of again.

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$s_A = 3.8 \quad n_A = 12 \quad \bar{x}_A = 27.2 \quad s_B = 4.3 \quad n_B = 9 \quad \bar{x}_B = 32.1$

$H_0: \mu_A = \mu_B \quad H_1: \mu_A < \mu_B \quad \alpha = 0.01$

$$s_p = \sqrt{\frac{(n_A - 1)s_A^2 + (n_B - 1)s_B^2}{n_A + n_B - 2}} = \sqrt{\frac{11(3.8)^2 + 8(4.3)^2}{19}} = 4.0181 \text{ mpg}$$

reject

α

The lower limit of the acceptance region is $t = -2.539$, or

-2.53

2.76

$$\bar{x}_A - \bar{x}_B = 0 - ts_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}} = -2.539(4.0181)\sqrt{\frac{1}{12} + \frac{1}{9}}$$

$$= -4.499 \text{ mpg}$$

Because the observed t value $= \dfrac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)_{H_0}}{s_p\sqrt{\frac{1}{n_A} + \frac{1}{n_B}}} = \dfrac{(27.2 - 32.1) - 0}{4.0181\sqrt{\frac{1}{12} + \frac{1}{9}}}$

$= -2.766 < -2.539$ (or $\bar{x}_A - \bar{x}_B = -4.9 < -4.499$), we reject H_0. Brand B delivers significantly higher mileage than does brand A

A simple t test example so we have got all this information given to us so just calculate pooled variance first of all. So, this is pooled standard aviation 4.0180 lower limit of the acceptance region would be this and how the distribution look like. It would be like this right this is your left tail test right so this is your rejection region and this is your acceptance region so calculate t value. t value is this which is -2.766 and when you compare it.

So, this is your table value -2.53 and this is your calculated value somewhere here -2.76 so this value falls in rejection region so we will reject null value hypothesis and when we reject this hypothesis we will say that yes mileage of brands A cars is lower than mileage of brand B cars or in other words brand B delivers significantly higher mileage than brand A and the same thing let us work out this example using many type.

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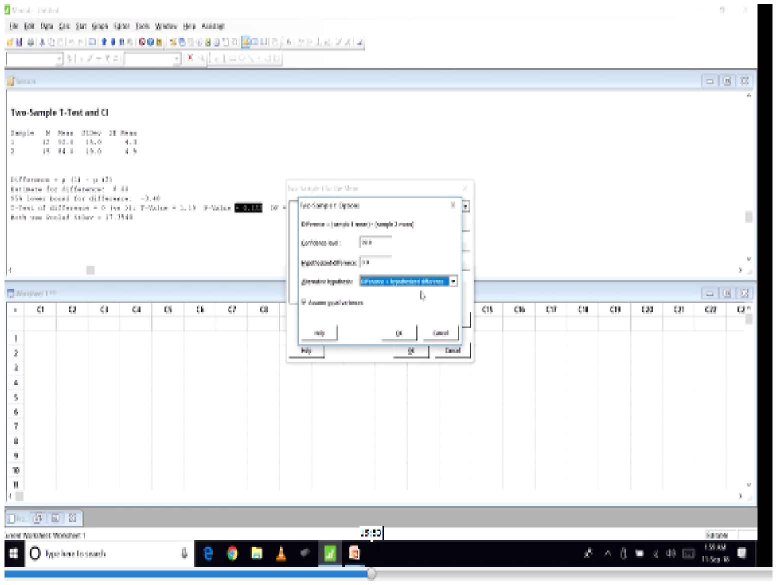

So, we will go to stat, basic statistics this 2 sample t test so let us look at once again data so you have got 12, 27.2 and 3.8 so this is how should you should enter data about centre one. Let us look at sample 2 it is 9, 32.4 and 4.3 options this is a hypothesis system tested at .01 it means 99% confidence level so hypothesis difference level will remain same. So, this is in case of left tail test so difference is less than hypothesised difference right. click okay and okay.

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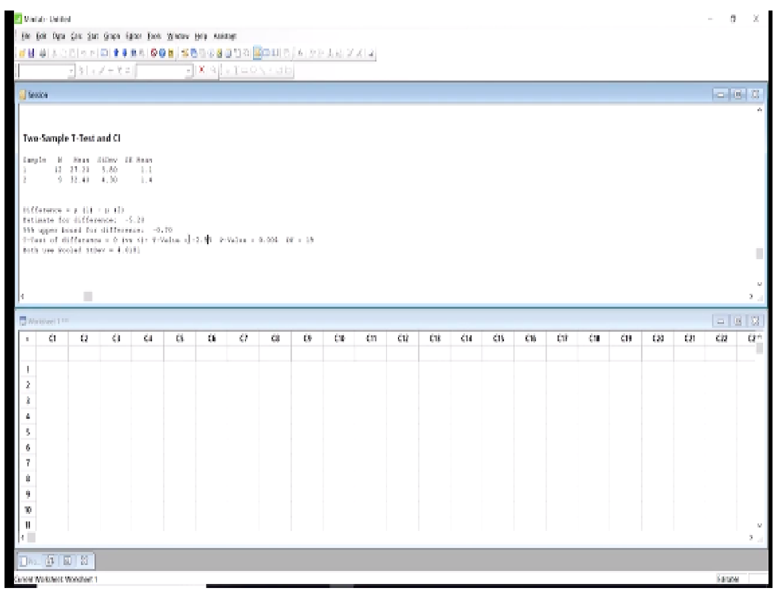

So, this is how you should be getting P value .004 of course your null hypothesis and this is what you calculated t value – 2.93 and it is .004, so .004. so p value is 0.004 alpha is equal to 0.01. so p value is < alpha so will reject null hypothesis. Is not it? This what we have said when we used p value approach for hypothesis testing. So, using these 2 methods we are getting same answer so far we have talked about hypothesis testing of independent samples.

Let us look at hypothesis testing of dependent samples. Dependent samples means you are collecting data from a sample after some time you are again collecting data from the same sample of course let us say you have collected height of students in the month of January you find out it is mean and you collect again the height of all these students in let us say month of December. So, we can see is there any significance in these 2 heights of the same group so this is the case of dependent sample we call it paired sample.

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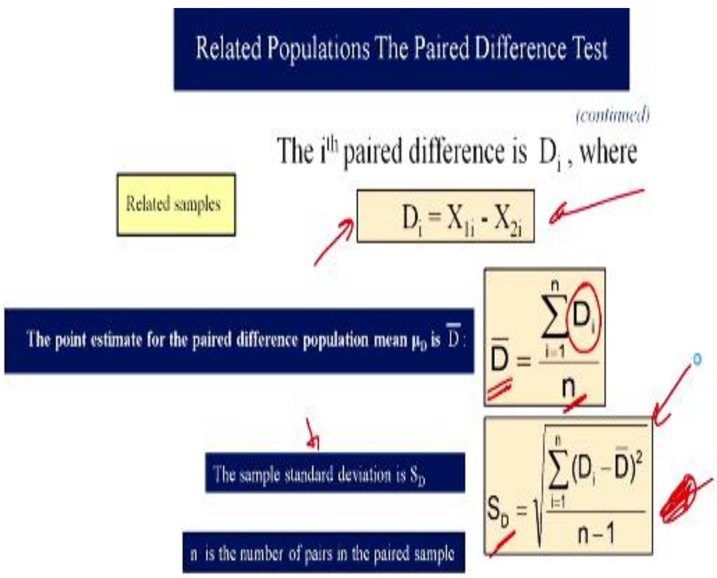

We also call it related sample right either you call it dependent related or paired one and the same thing only the thing is you just collect repeated measures happen over there before and after so the difference in mean would be this so Di is the difference so X_{1i} and X_{2i} is similarly you can have X2 and X3 and so on. So, you will have some data said and then after said time after say giving certain stimulate to the group of student.

We will again measure height or weight or some other variable. So, you need to keep in mind 2 important assumptions that both the populations are normally distributed. And if it is not normal distributed then we will take large samples. So, this how you should be calculating paired difference. We have already talked about this the mean of paired difference is this.

So, this is summation of differences/ number of pairs and the standard deviation of the paired sample Di – D bar the whole square summation of this and the root of this is similar to what we have seen in one of very few classes. In the beginning we have seen how to find out standard deviation and variance right.

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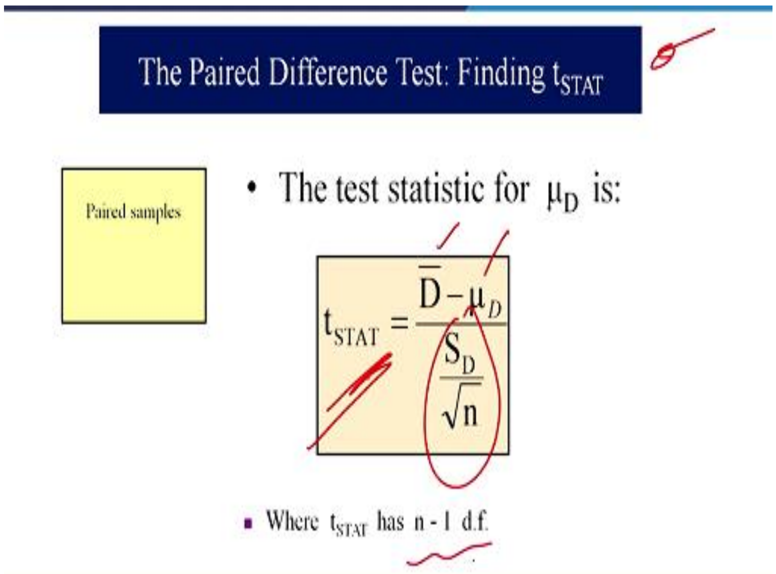

So, paired difference test this is how you be using t test if you are using paired hypothesis testing so $\frac{\bar{D} - \mu_D}{\frac{S_D}{\sqrt{n}}}$ and it will have n-1 degree of freedom in fact when we worked out questions on 2 sample test independent samples the degree of freedom was n1+ n2-2 but since here and there is only one sample you are just collecting data from one sample before giving certain stimuli and after giving certain stimuli so this is how the distribution would look like.

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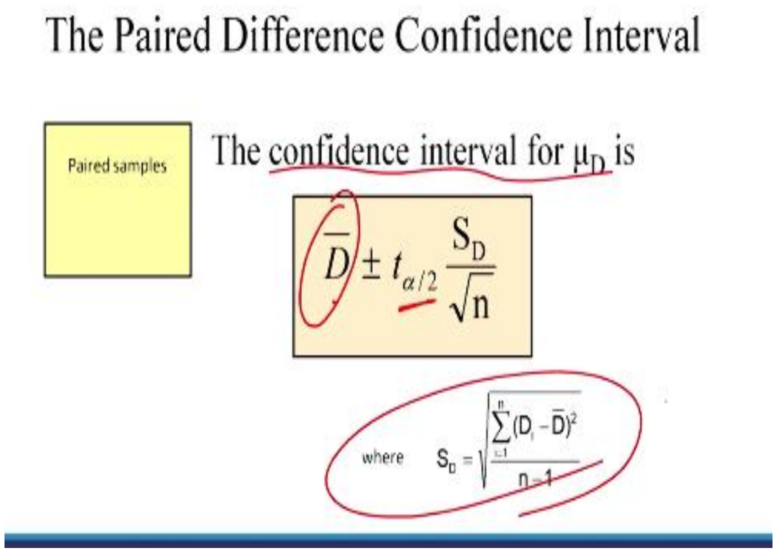

So, the direction of sign over here we will decide the direction of test whether it is a upper tail test or lower tail test so this is a left tail test and this how the distribution will look like. This is your rejection region this is acceptance region or non rejection region. This is an upper tail test and this is your rejection region. And this is 2 tail test case so there are 2 rejection region. so you need to again compare the calculated t value with the critical table value.

Of course, you can solve this kind of problem not only with the help of t value approach but confidence level approach as well. So, you can use you will have this mean of differences you have t alpha/2 this and just check whether the after you will get 2 limits over here upper limit and lower limit and see whether you are rejecting or not null hypothesis using confidence interval approach.

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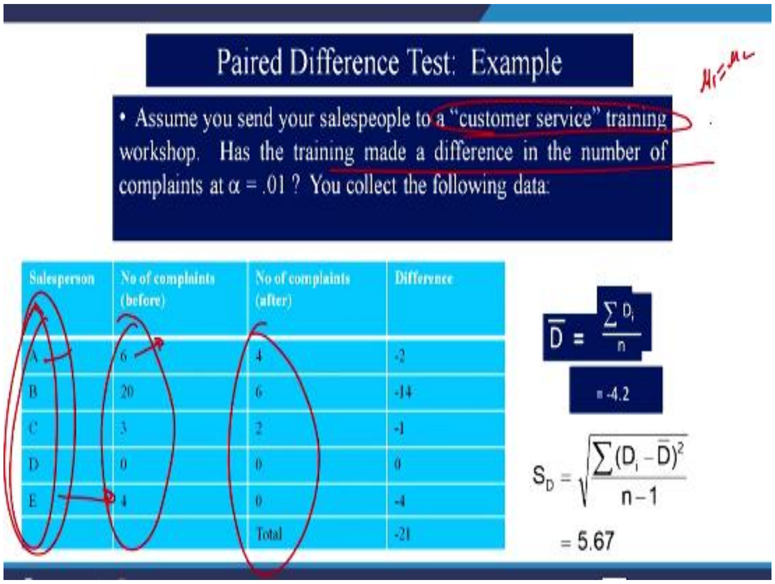

So, we will take up this question, so on paired difference test or related sample are dependent sample. Let us say you are running a company let us say a service a service type of business or service station of automobile for simplicity. So, you are sending your sales people to customer service training workshop so you are saying you are employees to a training center where they would be learning how to interact with the customers.

So, you have sent 5 of your sales people for training program and before training before sending them on training program. You just measure the number of complaints they were solving so this fellow used to solve 6 complaints E fellow use to solve 4 complaints and after attending the training program. You have noted down the number of complaints being solved by these sales person or sales people.

Now this is in the end you want to know what is there any difference in these 2 set of data or has is the training program made a difference in number of complaints whether your complaints are gone down or it has gone up. So, what kind of null and alternative hypothesis would you frame. You need to have null hypothesis as let us say there is no significant difference right. So, say mu1 = mu2 and the other one would be because you want to know has the training program made any difference? So it is not equal to type so this is a case of two tell test.

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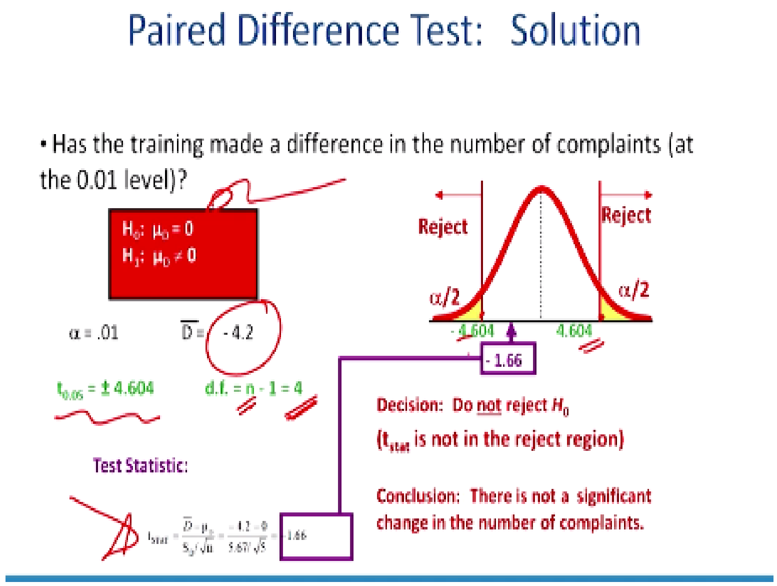

Okay, so μ_D infect this is how you should be writing your null alternative hypothesis so the difference mean difference is 0 and mean difference is not equal to 0. So, you just calculate the t value first of all so D bar would be– 4.2 how did you get - 4.2. So, -4.2 this is the difference 4-6 and 6-20, 0 – 4 so this average is -21. Some of all these differences divided it by 5 so you will get this value as – 4/2.

Once this is known you can calculate standard deviation as well so 5.67 is the standard deviation now let us look at the solution to this question. So, t value is -1.66 and table when you need to look at 4 degrees of freedom because 5 is the you have got 5 pairs. So, 5- 1 = 4 and t value if you look at table then it would be ±4.60 so this + 4.60 is – 4.60 these are your critical values and calculated t value is in this region.

This is the region, this region is non rejection region we do not reject null hypothesis. It means what? There is no significance difference in the mean number of complaints before training program and after the training program in other words the training program has not produced any result. It was just a you can say a fertile exercise he did not reduce number of complaints. so we will work out the same example using mini tab software. So, here what we have done we are not rejecting the null hypothesis right. So, let us look at data and these are the data set.

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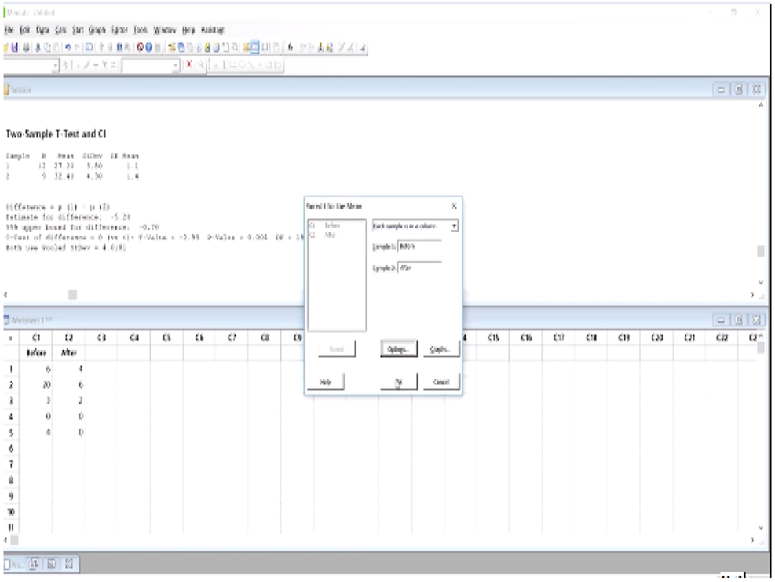

So, let us work out this session using minitab software. so this is first column is before the number of complaints before sending employees for training program. And after a number of complaints after sales persons have received training. So, let us enter data, so this is 6, 20, 3, 0, and then after 4, 6, 2, 0, 0. So, you just do this much data entry and then go to stat basic statistics so there is an option here it is paired test.

This is paired test so click over here paired test simple in each column we do not have summarized data so we will have a sample in each column as it is. Okay, so now we have got sample 1which is before. So, click over here and select before and then select. Then again after select after select this go to options. we have to test this hypothesis at what significance level it is .01.

It means 99% so hypotheses difference is 0 and we are taking it and alternative hypothesis is not equal to type right so it is the case of 2 tail test. So, we will click it. Okay again Okay so P value 0.173 yes just look at this 0.173 let us compare with p values 0.173, alpha is 0.01. so p is not < alpha. So, we do not reject null hypothesis H0 this is the decision we took over here when we used critical value approach.

So, let me summarize what we have done in todays session we have used high fast testing for two samples where in we first framed null hypothesis the alternative hypothesis. Then we decided on the type of test to be used then we calculated t value or z value whatever was the case. Then we compared the calculated value the critical value and then we decided whether hypothesis was to be rejected or not to be rejected.

And we have worked out and all those questions using mini tab as well and when we used the mini tab package we in fact used p value u approach and we found that the answer in both the methods what exactly similar and it have to be similar. If you are not doing any you know wrong data entry and then you will get the answer you have to get the answer because many types software, it is just a desk calculation.

What you can only solve a problem if you entered data properly then software will give you the same answer so with this let me complete todays session. We will have some more questions on hypothesis testing of 2 samples in next lecture. Thank you.