Business Statistics

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Lecture - 39
Hypothesis Testing: Two Sample Test- I

Hello friends, I welcome you all in this session. As you are aware in previous session we worked out couple of examples using Minitab software. In this session as well we will continue our exercise and we will take up couple of examples not only related to hypothesis testing of proportion, but hypothesis testing of mean and there would be one tailed tests as well right.

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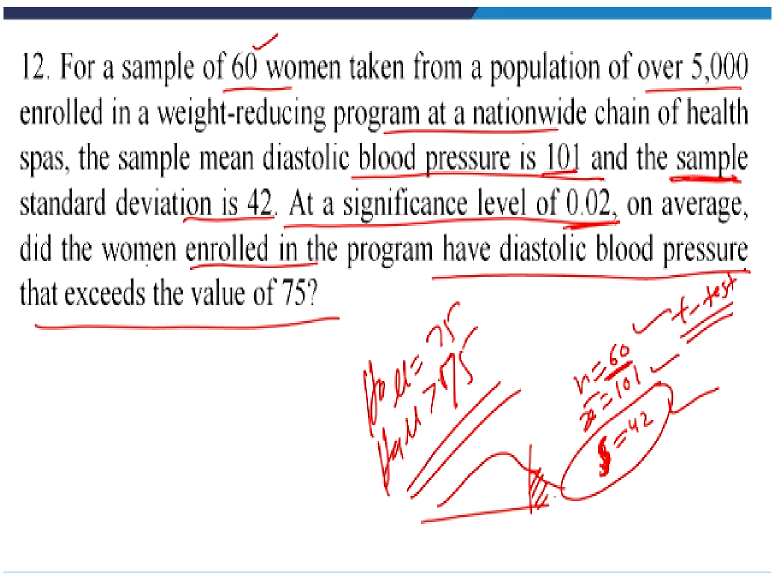

So let us look at this question. For a sample of 60 women taken from a population of over 5,000 enrolled in a weight reduction program at a nationwide chain of health spas. The sample mean diastolic BP is 101 and the sample standard deviation is 42 right. So first let us look at this n is 60 sample mean is 101 and sample standard deviation S right. You have not been given population standard deviation but the sample standard deviation right which is 42.

So by looking at this particular information you would have understood that this is what type test. Is this a case of t test or Z test? It is a case of t test because we have not been given though this sample size is not 30 or less than 30, but still this is a case of t-tail test. So let us look at this.

At a significance level of .02 on average did the woman enrolled, did the woman enrolled in the program have diastolic blood pressure that exceeds value of 75.

So what is the hypothesis here? So in a sample of 60, the mean blood pressure is this much and standard deviation this much at a significant level of 2% on an average, did the women enroll? So the women who were enrolled in the program have diastolic BP more than 75. So here mean is 75 and mean is more than 75. So this is your null hypothesis and this is alternate hypothesis with n=60, sample mean this, sample standard deviation this.

So let us work out this example. So this is a case of an upper tail test right this type of test.

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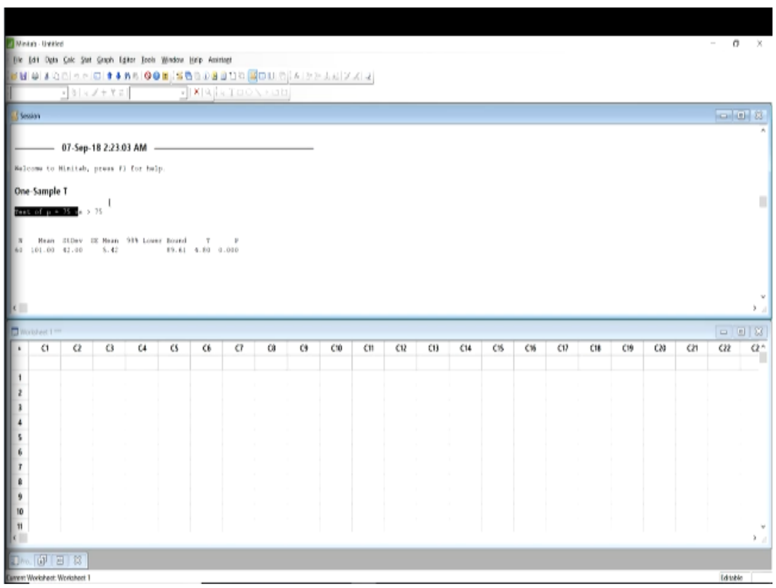

So let us look at this. We will go to stat, basic statistics. This one t test, this is one sample t test. Click over here. This is how you can enter data. So either you can enter data in these columns over here, we can know summarized data. So sample size is there it was 60, sample mean 101 sample standard deviation. 42 perform hypothesis test, hypothesized mean this is important. So its 75. Options, this is a case of before that we can enter confidence level.

So confidence level is 98% and alternative hypothesis Mean> hypothesized mean right. Let us look at answer to this question. Click okay. So P is this 000 right. So null hypothesis and alternative hypothesis. So P is 0 here let us take a decision. So P is 0 alpha is 0.02. Is P < alpha?

Yes. P is < than alpha. So we will reject null hypothesis, reject H0 reject null hypothesis. It means that the Blood Pressure exceeded 75 value.

We are rejecting null hypothesis okay. So we can say that over a period of time the BP has gone up. Let us look at one more example.

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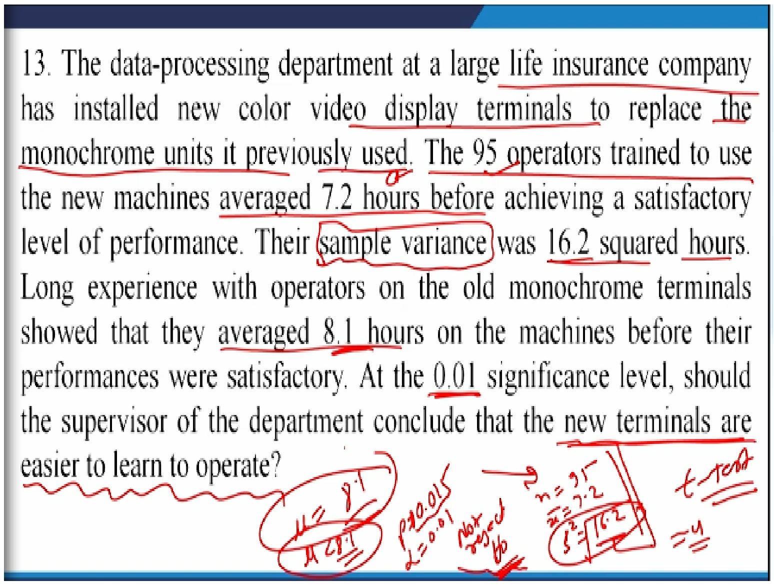

The data processing department at a large life insurance has installed new color video display terminals to replace the old monochrome units it previously used to have. The 95 operators trained to use the new machines average this much hours. So in 95 simple mean is 7.2 hours before achieving a satisfactory level of performance, their sample variance keep in mind this is sample variance.

This is S square right 16.2 was this much. So sample variance was 16.2 squared hours. Long experience with operator on the old monochrome terminals showed that they averaged this much hours on machine before their performance were satisfactory 8.1 hour. At 0.01 significance level should the supervisor of the department conclude that the new terminals are easier to learn to operate.

So this, this is the sample size and this is the average time, training time, sample variance is this much and from the past long experience the operator on old terminals showed that they average this much of hours right. So this is mean 8.1 and this is what we want to know whether the department concluded that new terminals are easier to learn or does it take less time? So this is alternative hypothesis is less than 8.1 right.

So this is a case of what type of test t test or Z test. This is again a case of t test right and n is of course 95. So let us work out this example and we will enter data. So one sample t test. So stat basic statistics, it is one sample t test right. Sample size just enter sample size it is 95, sample mean 7.2 and if you look at samples variance is 16.2. So we have to enter standard deviation. So under root of this so let us take it 4 approximately 4 point something, something right.

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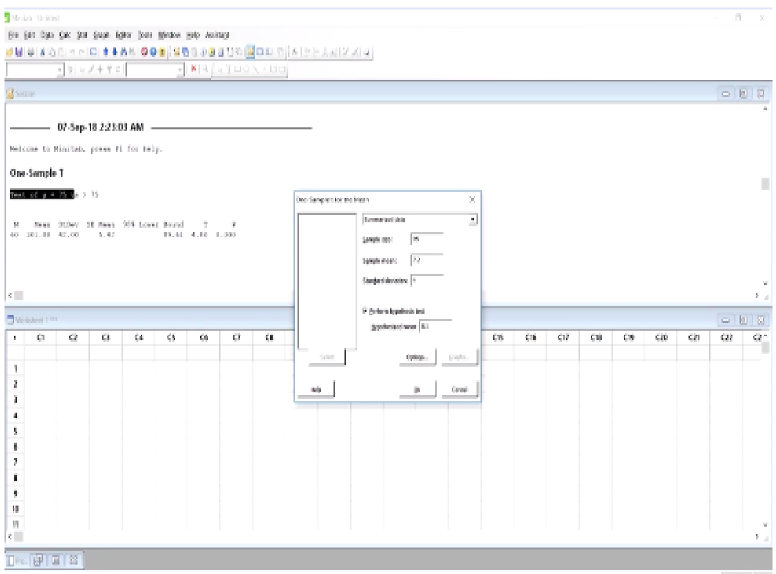

So we will enter standard deviation as 4, hypothesized mean would be 8.1. We will go to options and we will see what is our confidence level. So confidence level is 99% this is a case of upper tail test or lower tail test. This is a case of lower tail test right. So let us enter mean as less than hypothesized mean, we will click okay. Click okay and this is P value 0.015. 0.015 is the P value.

So let us look at whether we will reject or not reject null hypothesis. So point what it is 0.015 this is P value right. What is alpha value 01. Is P< alpha? No, so we will not reject null hypothesis. If P is < alpha we will not reject otherwise we would have rejected it right. So we will not reject null hypothesis. It means we will say that the training time on old terminals and new terminals is same.

There is no significant difference on training time. So with this we have finished examples using Minitab on one sample hypothesis testing and we have worked out several examples. We have worked out almost 15 to 20 examples.

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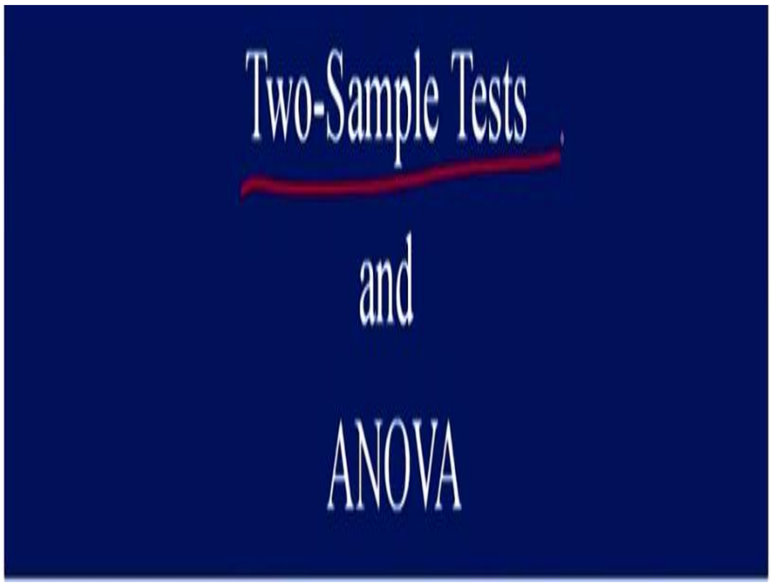

Let us move on to our next topic which is hypothesis testing of two sample tests. So we will have now two different samples and ANOVA it is called analysis of variance. So in coming slides, in coming lectures I will be talking to you about analysis of variance. So first let us look at what is two sample hypothesis testing. So far we have seen one sample hypothesis testing. So in two sample test what happens, you actually have two different independent samples.

And you take samples from both the samples and you try to find out is there any difference in their population means.

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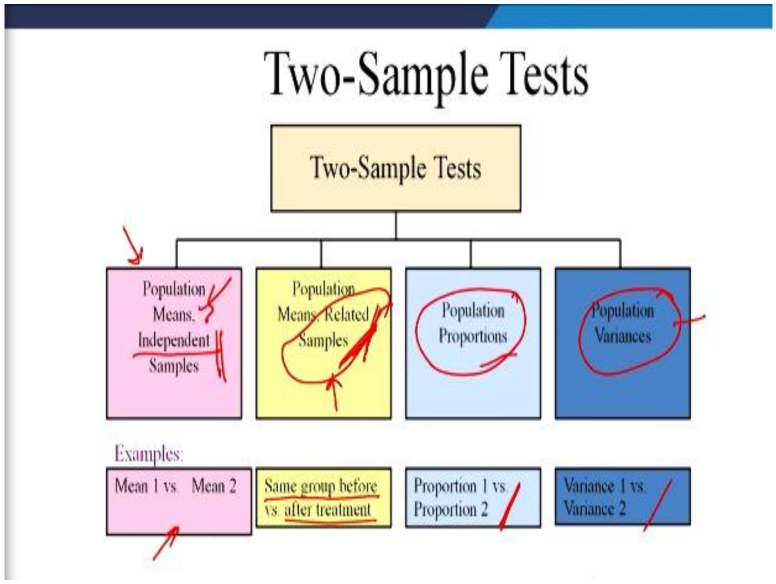

So you can have a situation like this, you can compare their population means, you can compare their population proportions, you can compare their population variances. Now when you when you compare population means you can have two different scenarios. In first scenario you can have just independent samples. So there are two samples. They would be totally independent of each other and the other scenario would be they would be dependent on each other.

So they are called related samples or dependent samples. They are also called paired samples right. So basically we are looking at broadly 3 categories. Population means, proportion and variances, but when we look at population means we can have independent samples or dependent samples. So here we compare two means again in when we talk about dependent samples we will have same group before and after.

So we will collect data from one sample first and then after some time we will collect data on the same simple because we are saying they are it is a related sample right of course we can compare proportion and variances.

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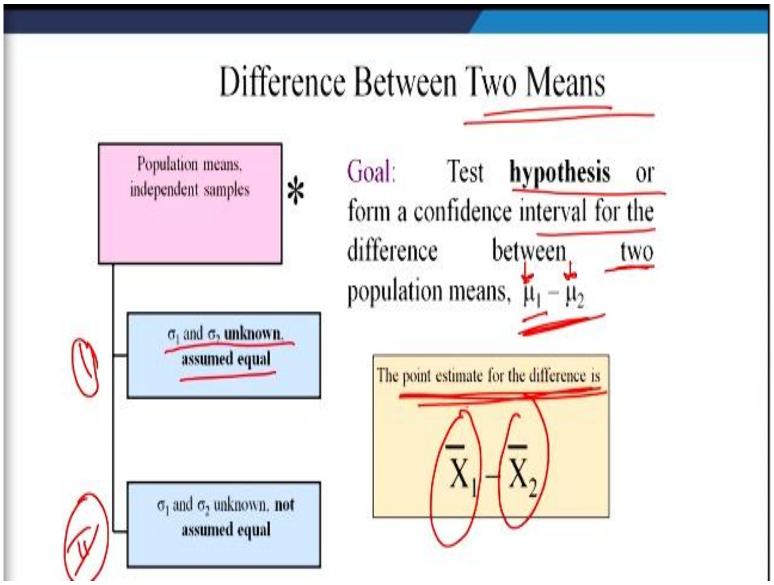

So difference between two means you can have two situations standard deviations unknown but equal unknown but unequal. This is second case right this first case this is case number 1, this is case number 2 right. So they are unknown but equal assumed equal. They are unknown but they are not equal right. So you want to test hypothesis or we want to form confidence interval for the difference between two population means.

So this mean of population 1, mean of population 2 and we will do it using, using point estimate right or we will take sample this is the sample mean of the samples you have taken from first population and is from second population right. So we will compare population proportions of two samples using samples from those two populations.

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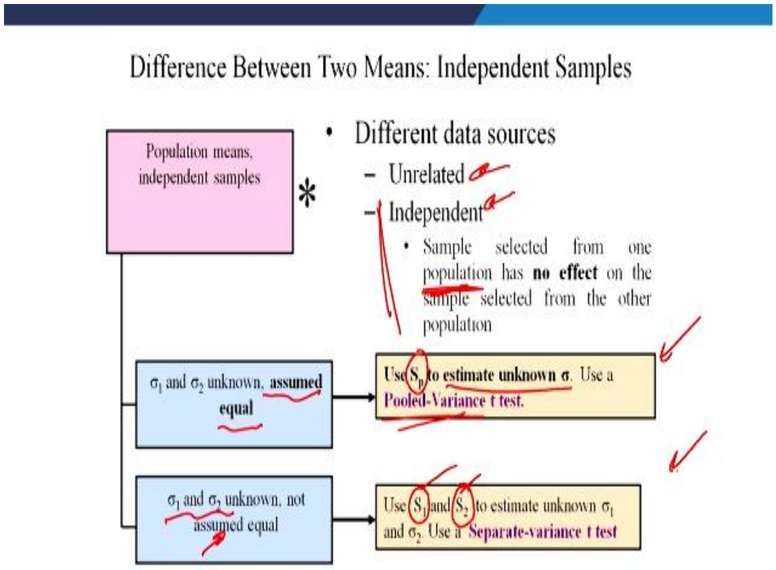

So as I said, you can have independent samples. The population one is totally independent of population two or you can call them unrelated samples right. So sample selected from one population has no effect on sample selected from other population that is the meaning of independent sample which I have already explained. So when you assume these two equal right standard deviation are unknown, but you are assuming them equal.

So there is something called we will use pooled variance test Sp this is nothing but pooled variance. This we will use two estimate unknown standard deviations. In the second case we will use standard deviation of sample one, standard deviation of sample two to estimate population standard deviations. In this case pooled variance and in this case sample standard deviations.

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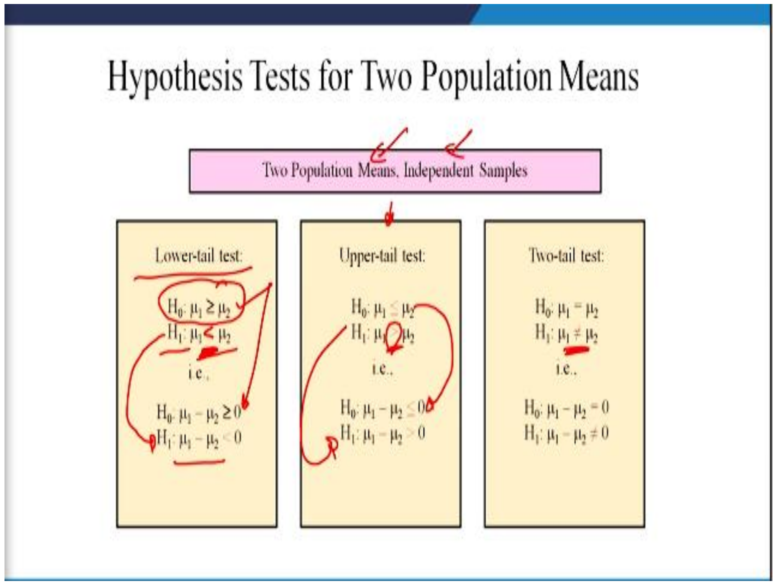

So this is how the two sample tests works. So you can have two population means and independent samples. So this is let us say this is your null hypothesis where you are saying that the mean of a population 1> or= to mean of population 2 and alternative hypotheses is it can be just you can have only just one sign over here now right. Alternative hypothesis mean of first population is> mean of second population and alternatively we can write this as like this.

So mu1-mu2>= 0 and alternative hypothesis can be written as mu1-mu2 < 0 right. So this is lower tail test. How did you come to know about lower tail test? Why this is a lower tail test because this sign off alternative hypothesis which is less than type (18:12). So this is lower tail test. Just similar to this you will help upper tail test right just see this sign. So sign in alternative hypothesis is>type.

So this is an upper tail test and of course you can write null hypothesis like this and alternative hypothesis like this. Now since you know what is, what would be this, what would be lower tail test and upper tail test? If the sign is not equal to type, it will be two tail tests similar to what we have seen in case of one sample hypothesis testing.

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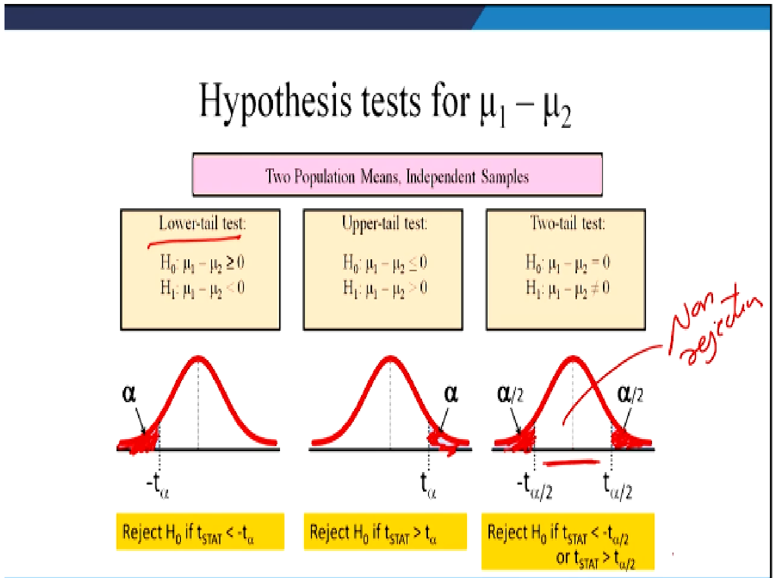

So this is how the distributions would look like. So this is a lower tail test or left tail test, this is your rejection region is not it. And in upper tail test or right tail test this is your rejection region. And in two tail test you have two rejection regions right. So this is one rejection region and this is second rejection region right and this is non-rejection region, non rejection region.

So if the sample statistics and hypothesized population parameter are in this region, then you will not reject null hypothesis.

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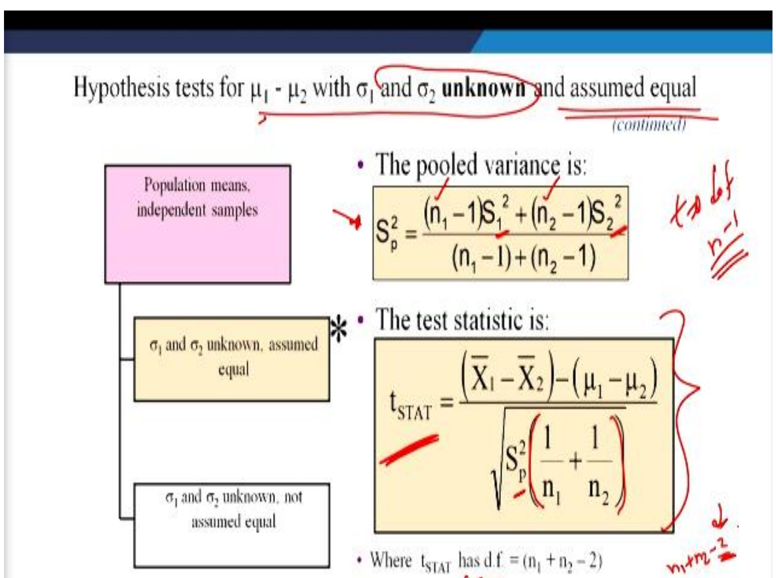

So let us look at this case when these two are unknown and we assume them equal right. So when we assume them equal we will use pooled variance of both the populations. So we will take sample from first population and that sample size is n1, n2 is the sample size of a sample which you have taken from population two. S1 is standard deviation of first population, standard deviation of second population right.

So n1, n2 and S1, S2 right. So these two are variances right. So this is pooled variance and the t statistics which you would be calculating is this X1 bar - X2 bar. So first sample mean-second sample mean - population mean first population mean second population mean/under root of pooled variance right. So this Sp square pooled variance and multiplied by this. So here we have seen in case of one sample test that whenever we used t statistics.

We always used degree of freedom as n-1 is not it, but since here you have got two samples. So the degrees of freedom would be you just add those two samples-2, so this is -2 because there are two samples right.

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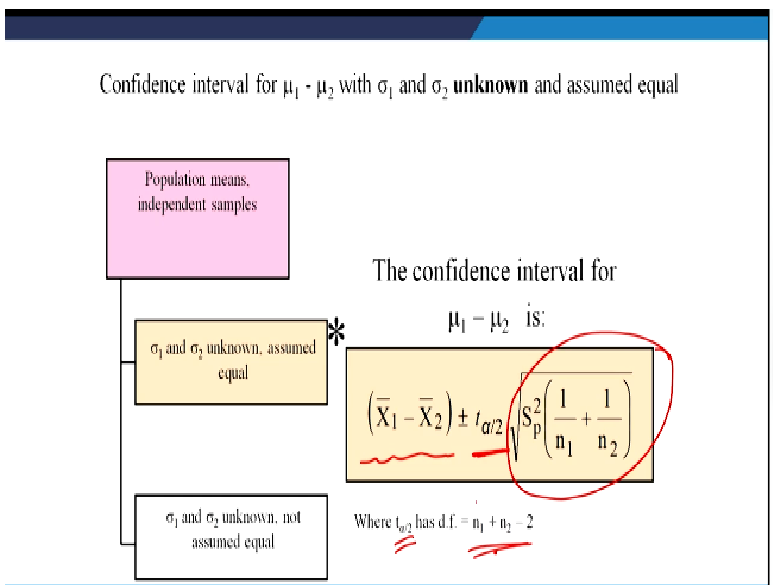

Now the confidence interval for this would be X1- X2 ± $t_{\alpha/2}$ this value which we have already discussed. Again t value at n1 + n2 - 2 degrees of freedom. So this confidence interval approach can be used or you can use this critical value approach for testing hypothesis.

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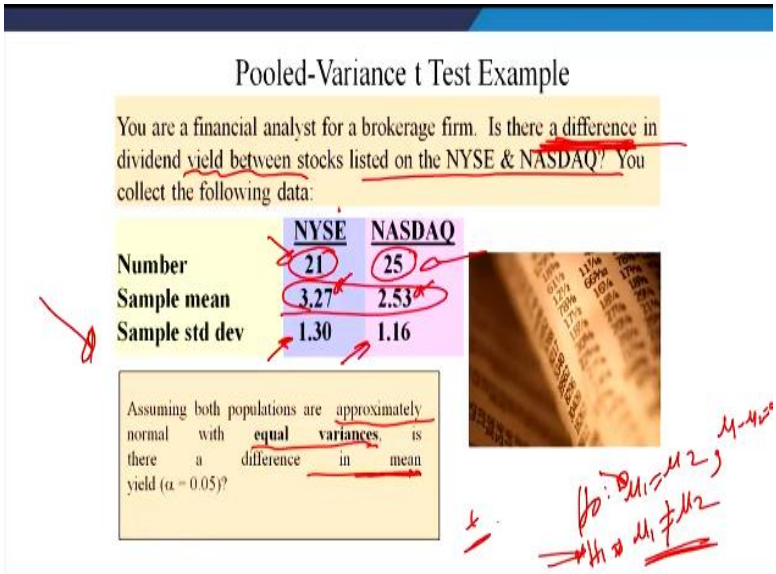

So let us look at this question, very interesting question. You are a financial analyst for brokerage firm. Is there a difference in dividend yield between stocks listed on New York Stock Exchange and NASDAQ? You collect the following data. So you collected 21 stocks from New York Stock Exchange and 25 stocks from NASDAQ. You found that the sample mean as 3.27 and NASDAQ it is 2.53.

Sample standard deviation of these 21 stocks 1.3 and it is 1.16. Now what is the question is there the difference in dividend yield? It means this mean is there any difference between mean of these two, these two stocks, these two stock exchanges. So 21 stocks you have taken from this stock exchange, 25 from this stock exchange and you are comparing mean of these stocks. Assuming that both populations are approximately normal with equal variances is there any difference in mean, yield.

So if I asked you in which stock would you like to invest whether in New York stock exchange or NASDAQ by looking at this data. You have the mean and standard deviation of a couple of stocks of these 2 stock exchanges. So how would you proceed? What kind of example is this? Is this so first of all this is Hypothesis testing of two samples and since we want to know the difference right.

Is there any difference of these two? So initially we will say that µ1= µ2 right. So this is our null hypothesis H0 and alternative hypothesis. In fact I can write this as the same mu1-mu2=0 is not it? It is up to you how you are writing it right okay. So this is null hypothesis and alternative hypothesis would be what µ1≠ µ2. So this is a case of what? It is a t test or Z test case?

So in fact by looking at this you would have understood that this is a two tail test first of all right, but what about T or Z test? Since we have been given sample standard deviation right and here in fact sample size in both of these is< 30. So in fact we have seen a case when sample size was 60 and sample standard deviation was given. So we applied T test. So here we will apply T test right.

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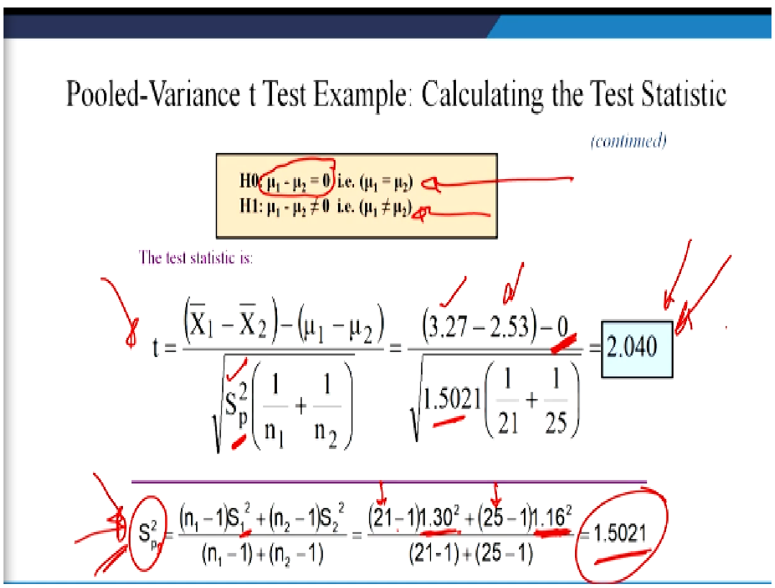

So this is how you should write null and alternative hypothesis, this is what I have already written mu1 and mu2 are same alternative hypothesis they are not same. So T is 2.04. So this is mean of first 21 samples just see this 3.27 and X2 bar is 2.53 right X2 bar and µ1-µ2 is equal to 0 because we are saying that they are same there is no difference okay. So this is µ1-µ2 =0 right. This is what we have said initially right.

µ1-µ2 =0 right. This is pooled variance so first calculate this pooled variance. This n1, n1 is what n1 is 21, n2 is 25 right. So 21 n2 is 25, so 21-1 and 25-1 this is standard deviation right not standard deviation this is variance right. S1 square is variance. So this is sample standard deviation. So you just, you need to take square of this. So you have just taken square of this 1.3 square.

This is standard deviation of all the stocks off all 25 stocks of NASDAQ Stock Exchange of course divided by $(n_1-1) + (n_2-1)$ right. So this is pooled variance. So before calculation of t statistics in fact you should calculate a Sp. So this is variance right pooled variance. So this 1.501 which you can write over here in this formula, 1 by 21 + 1 by 25. So this is how you should be getting t value calculated t value. Now what would you reject or you will not reject null hypothesis.

Again you should look at your t value right.

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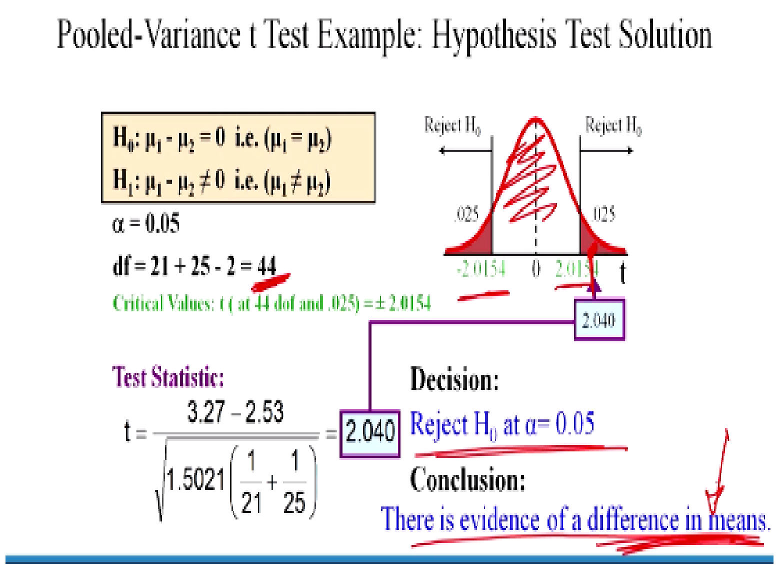

So t value is to be looked at 44 degrees of freedom right. So this is 21, n1+n2-2=44 degrees of freedom.

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TABLE E.3

Critical Values of t

For a particular number of degrees of freedom, entry represents the critical value of t corresponding to the cumulative probability $(1 - \alpha)$ and a specified upper-tail area (α)

Degrees of Freedom	Cumulative Probabilities 0.75	0.90	0.95	0.975	0.99	0.995
	Upper-Tail Areas 0.25	0.10	0.05	0.025	0.01	0.005
1	1.0000	3.0777	6.3138	12.7062	31.8207	63.6574
2	0.8165	1.8856	2.9200	4.3027	6.9646	9.9248
3	0.7649	1.6377	2.3534	3.1824	4.5407	5.8409
4	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
5	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
6	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
7	0.7111	1.4149	1.8946	2.3646	2.9980	3.4995
8	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
9	0.7027	1.3830	1.8331	2.2622	2.8214	3.2498
10	0.6998	1.3722	1.8125	2.2281	2.7638	3.1693
11	0.6974	1.3634	1.7959	2.2010	2.7181	3.1058
12	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
13	0.6938	1.3502	1.7709	2.1604	2.6503	3.0123
14	0.6924	1.3450	1.7613	2.1448	2.6245	2.9768
15	0.6912	1.3406	1.7531	2.1315	2.6025	2.9467
16	0.6901	1.3368	1.7459	2.1199	2.5835	2.9208
17	0.6892	1.3334	1.7396	2.1098	2.5669	2.8982
18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
23	0.6853	1.3195	1.7139	2.0687	2.4999	2.8073
24	0.6848	1.3178	1.7109	2.0639	2.4922	2.7969
25	0.6844	1.3163	1.7081	2.0595	2.4851	2.7874
26	0.6840	1.3150	1.7056	2.0555	2.4786	2.7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1.3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1.3055	1.6883	2.0281	2.4345	2.7195
37	0.6812	1.3049	1.6871	2.0262	2.4314	2.7154
38	0.6810	1.3042	1.6860	2.0244	2.4286	2.7116
39	0.6808	1.3036	1.6849	2.0227	2.4258	2.7079
40	0.6807	1.3031	1.6839	2.0211	2.4233	2.7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
43	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
44	0.6801	1.3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896
46	0.6799	1.3002	1.6787	2.0129	2.4102	2.6870
47	0.6797	1.2998	1.6779	2.0117	2.4083	2.6846
48	0.6796	1.2994	1.6772	2.0106	2.4066	2.6822
49	0.6795	1.2991	1.6766	2.0096	2.4049	2.6800
50	0.6794	1.2987	1.6759	2.0086	2.4033	2.6778

So when you look at t table which is there it is 2.01 at 44 degrees of freedom. Now this is 44 degrees of freedom. This is 2.040 right so 2.040. Let us look at once again, at 44 degrees of freedom, 2.01 this is the one right. So these are your critical table values right. So -2.0154, so is our calculated value falling in this region. It is falling in rejection region right. So 2.015 and this 2.04 so this is somewhere here right in rejection region.

So we will reject the null hypothesis. So you will reject the null hypothesis and conclude that these two means are not same they are different. The evidence is that there is difference in mean so difference in mean right. It means they are not equal. So which has got higher mean? Where would you invest? Would you like to invest after looking at this result? You would like to invest in that share market where mean is higher right.

So mean is higher where it is in New York Stock Exchange. So you would like to invest in New York Stock Exchange. So this is just an example on two sample hypothesis testing. In next class we will work out couple of more examples and then we will use Minitab software to work out examples. Thank you very much.