Business Statistics
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Lecture-33
Hypothesis Testing Examples

Hello friends, I welcome you all in this session as you are aware in previous session we were discussing about hypothesis testing and we have seen that there are different steps in hypothesis testing. We did workout couple of examples let us look at one more example on hypothesis testing. But before I take the next example, let me tell you what exactly we did in previous session. Si we tested null and alternative hypothesis.

In fact, we framed null hypothesis and alternative hypothesis and we calculated Z value and for a given alpha value we found critical value from table right and then we compared the calculated Z value with lower and upper limit is of critical table values right and then finally we decided whether our hypothesis was rejected or not rejected. So in previous case or in any case you should reject the null hypothesis when the calculated Z value is in rejection region rejected otherwise do not rejected right.

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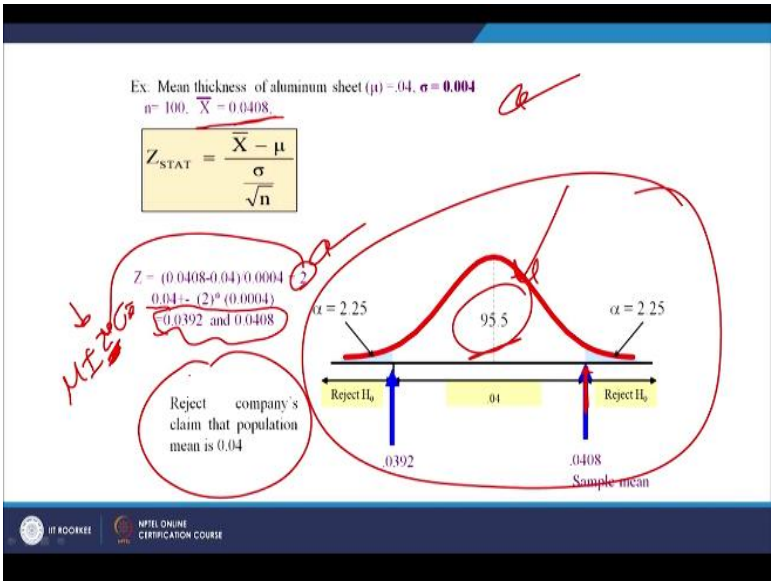

Now this is an example which we worked out in previous session as well, now here you not been given alpha value right ok, so what you did you just calculated Z value first, then population

mean right population mean ± whatever is Z value is that calculated value right not table value right. Because we have not been given alpha so we cannot have critical table value. So use this Z value over here, which is 2 and standard error right.

So you will get these 2 limit is and your sample mean is this which is not in this acceptance region which is just over here it just on the upper limit, so we will reject null hypothesis ok. In fact then how did you get this when alpha was not given. So what you did actually after looking at this Z value the probability value from the Z table is this ok. So this is how you can have a distribution like this ok.

But just for if you want to let say test a null hypothesis or alternative hypothesis then without having significance level you can get the answer ok, just look at more example let say the strength of the excel is 80000 pounds per square inch.

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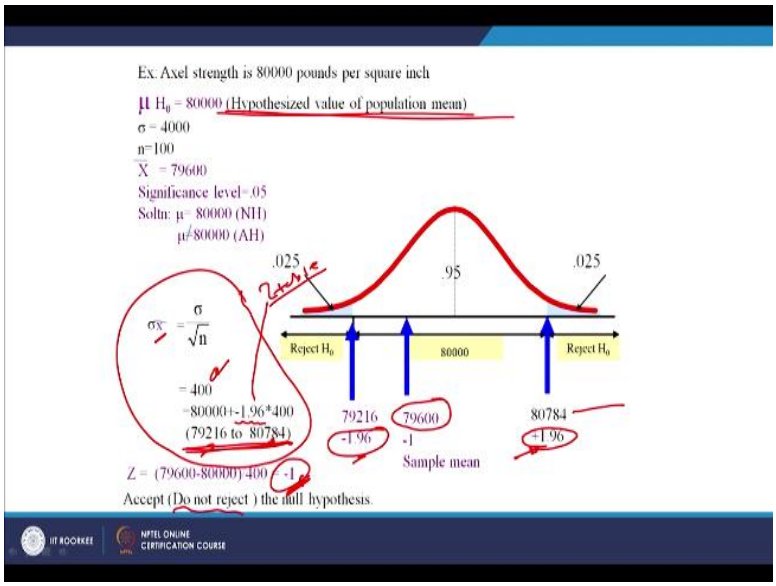

And standard deviation is 4000 when you take a sample if 100 excels then sample mean is 79600 1 square inch right, significance level is 1. So of course Z value would be 1.96 calculate Z value . So this should be -1 right. Now we just compare this -1 with this upper and lower limit is and we will see that this Z calculated is in acceptance region, So we will not reject the null hypothesis. and will not reject null hypothesis it means the excel strength is 5000 1 per square inch right.

Let us look at 1 more example in fact in this example it itself you can see this part carefully. So this is standard error of mean which is 400. Now population mean ± 400 into this is table critical value right or Z value from table right, z value from table. In previous example we were not given alpha, so what we did we just use the value of Z which we calculated right. Now we can get upper and lower limit is.

So upper limit is this, lower limit is this, and your sample mean is here, so we can say that since sample mean is in this limit you will not reject null hypothesis right. So there are 2 ways in which you can make a decision whether you are rejecting null hypothesis or you are not reject null hypothesis right, one is on the basis of Z value compare Z value with critical table values and one is time this interval just formulate this interval and check whether sample mean is in between these 2 arm right.

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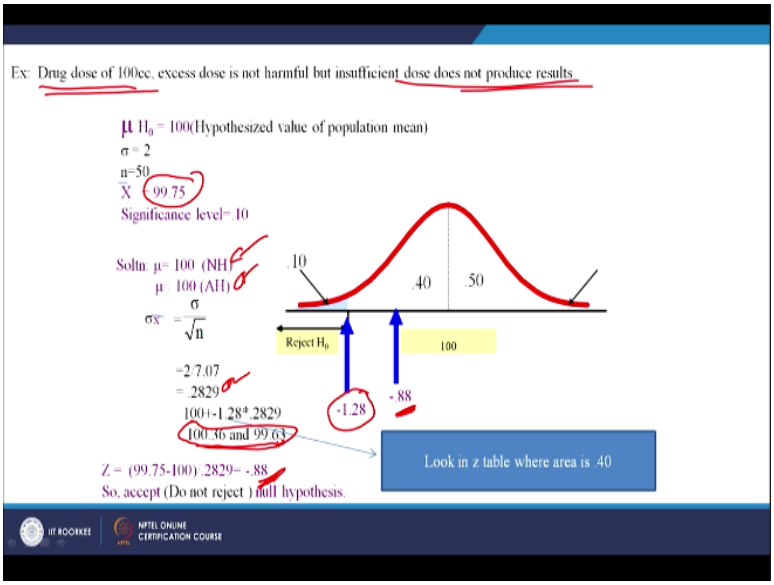

Let us look at one more example let say you have to give some dose to the patient and if excess dose is given it is not a harmful to the patient. But if less quantity is given then it would not produce sufficient reason right. So how would you frame the alternative hypothesis this how we can frame the alternative hypothesis right. So after this you just calculate standard error first which is this and lower and upper limit is.

Similarly Z is -0.88 right, so since this is just one-tail test, you will have just one Z value right ok. In fact you will have just one table critical value right, and you just compare these 2, now the calculated Z value is in non rejection region, So in not reject the null hypothesis, you not reject the null hypothesis and the other way of doing this is just find out upper and lower limit is and see whether the sample mean is in between these two yes.

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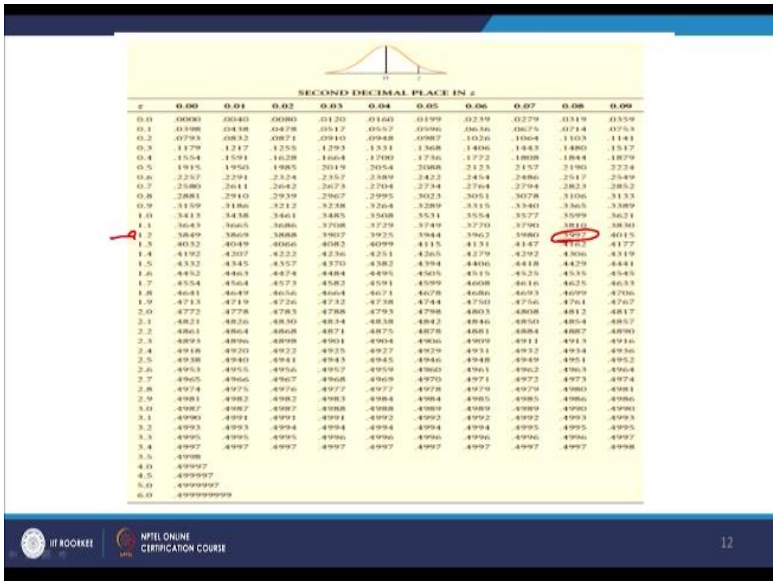

This is your sample mean so you will not reject null hypothesis right, let us look at in fact since this is a one-tail test and alpha=0.10. So from this area 50% area we are interested in we are interested in at this value right. So 50-10 right. So you just 0.4, so look at table the table Z table where area is 0.4. So that area would be at 1.28 right. So 1.2 and 8 is here right. This is 0.40 right.

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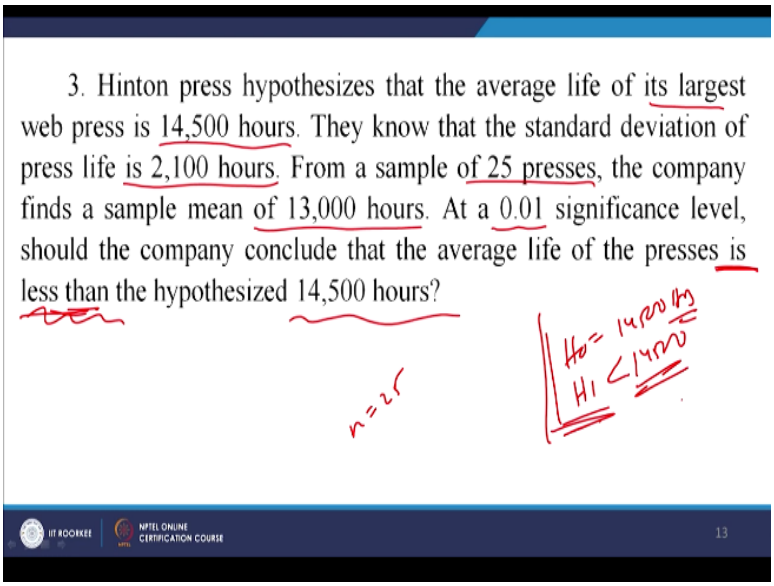

Let us look at one more example Hinton press hypothesizes that the average life of it is largest web press is 14,500 hours. They know that the standard deviation of press life is 2100 hours from a sample of 25 presses the company finds a sample mean of 13000 hours right, 0.01 significance level should the company conclude that the average life of the price is less than hypothesized 14,500 hours.

So question is very simple, in fact you need not move focus much on framing none and null and alternative hypothesis because hint is there in question it itself. So what is null hypothesis. Null hypothesis is that that the average life of the press is 14500 hours and alternative hypothesis is this less than 14500 hours right. So this how we can frame null and alternative hypothesis right. So you have been given n + 25, so this is a clear cut case of one sample or one sample one-tail test right and one-tail is what is an upper tail or lower tail test, it is a lower case of lower tail test right.

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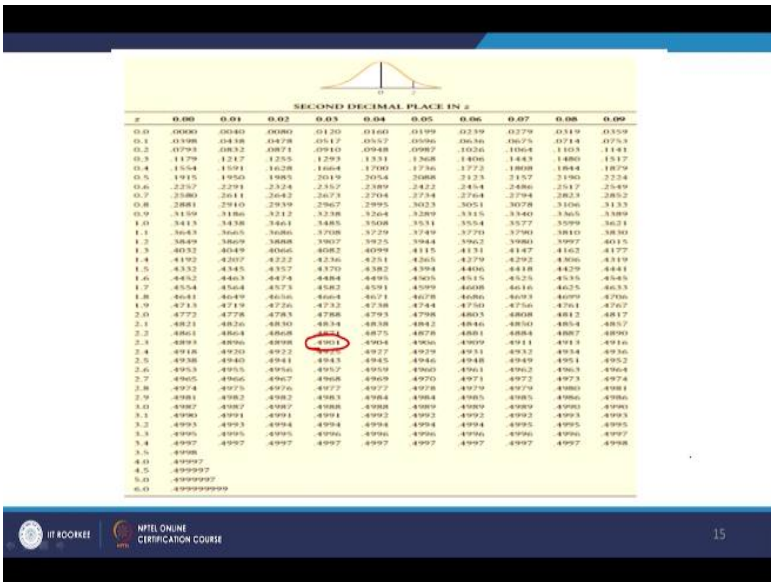

So just calculate Z value which is sample mean minus population mean divided by standard error right which is -3.57.So how would you draw distribution of this problem right, so this is less than type of so this is your rejection region right, rejection region and this is your mean right, hypothesized mean so we calculated Z values this -3.57 ok. Now the table value is -2.33, so when you look at table value how to find out table value this area is 0.50, this area is 0.50 right.

So this 0.50 - .01 right is 0.49 , now look at the Z value when area is 0.49, just look at this 0.49 this is right so table value is S, so we should reject null hypothesis the average life of average life is significantly less than hypothesized value ok. Now let us move on to next question.

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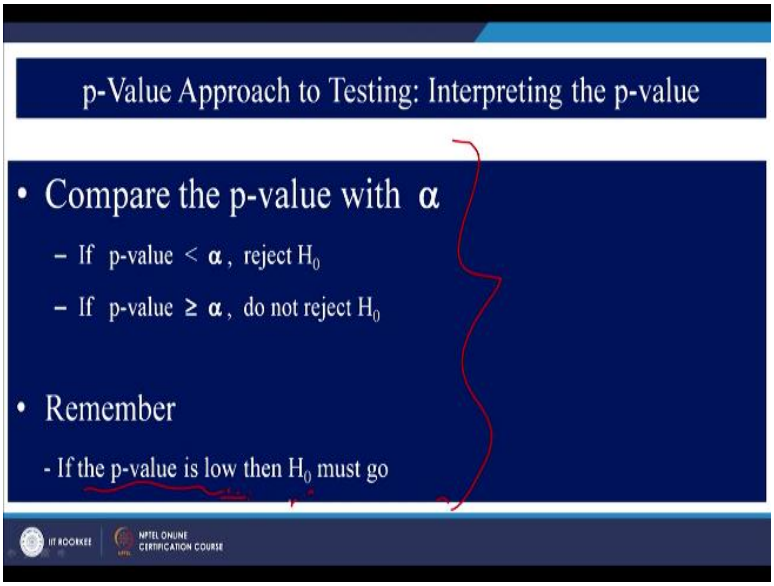

Let us solve a question using P value approach, so far we have solved our questions by comparing Z value with the critical value or by seeing whether the sample mean is falling within upper and lower limit is or not right. Just look at another approach for testing hypothesis it is called P value approach, P value is nothing but probability of obtaining test statistic equal to or more extreme than the observed sample value.

Given null hypothesis to right, so is basically a probability P stands for probability right, probability of obtaining test is equal to observe sample value given null hypothesis true, it is also called observed level of significance. So you will in every question in most of the questions you will have significance level given right and this is P value nothing but observed level of significance.

So we will compare observed level of significance and given significance level. So it is basically the smallest value for which H0 can be rejected by null hypothesis can be rejected. P value approach for testing null hypothesis is this. So we always compare P value and alpha value, if P is less than alpha reject the null hypothesis. Otherwise, do not reject the null hypothesis.

So just keep one point in mind in P value approach that if p is less than alpha reject null hypothesis otherwise not right. So remember if P is less than alpha is less than significance level H0 must go right or null hypothesis rejected right.

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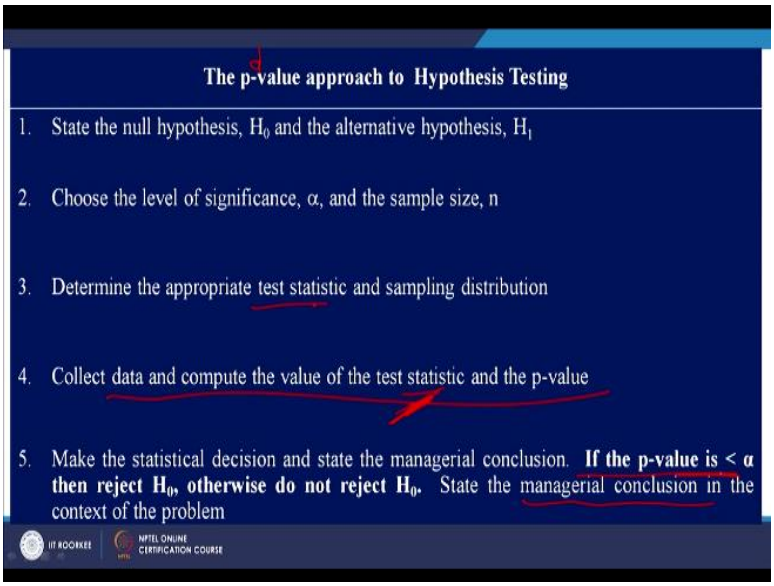

So these are couple of steps in P value approach almost similar steps which we have seen in another approach. So first is state null and alternative hypothesis, choose significance level in sample size, third is determine appropriate test statistic right, whether you want to use Z or t statistics right. Collect data and compute the value of test statistic only the point here is what we are missing in fact earlier in earlier approach there was 6 steps right.

So one of the steps which we do not have in this approach is that we do not look at critical table values right. So that is why there are now 5 steps instead of 6 steps. So finally you have to make a decision right, if P value is less than alpha reject null hypothesis otherwise not and then manager conclusion right.

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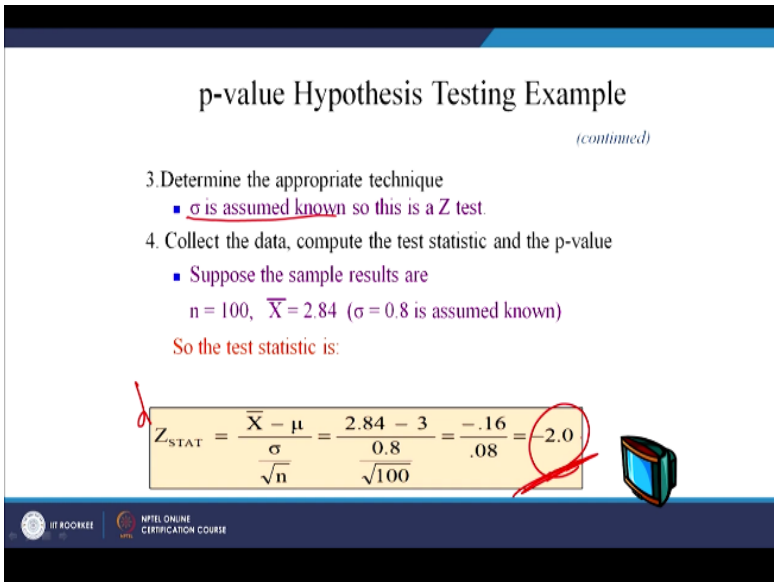

So let us look at the same question which we have flowed earlier, so we want to test the hypothesis that the number of TV sets in Indian homes is equal to call 3 right, so this is null hypothesis, this your alternative hypothesis right. Since this is not equal to 3, so this is a case of two-tail test right. So this is first step, second step, specify significance level which is given alpha and n and for this alpha what is critical value 1.96 right.

So we would be using of course Z test because sample size is this right 100, and we have also been given standard deviation of population that is why we will use Z test right, collect data and calculate Z statistic, so which is -2, the same which we were getting in earlier approach right. So Z is -2. Now in this point the earlier approach and this approach are same right.

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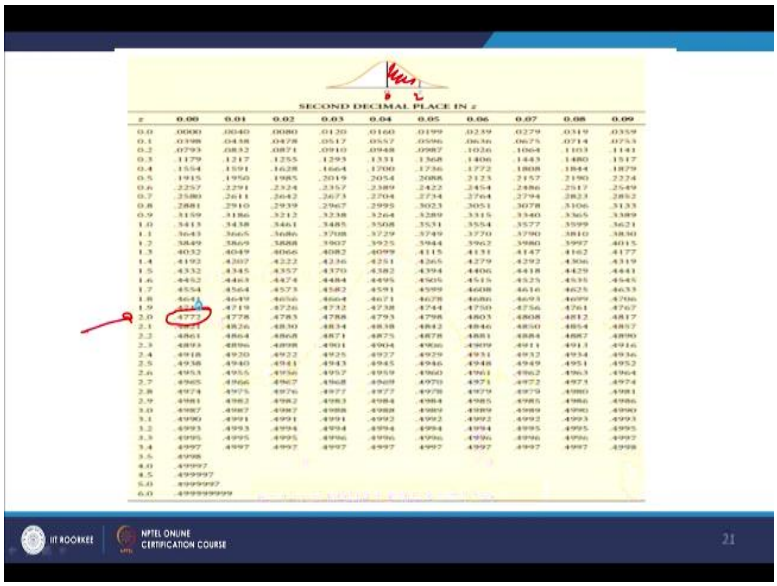

Now calculate something and P value right, now this is Z is 2 right ok, now what is the probability when Z is 2 in Z table right, let us look at so Z=2 and the probability under curve is this right so let us say this is 0, this is 2 and this area is 14772 right.

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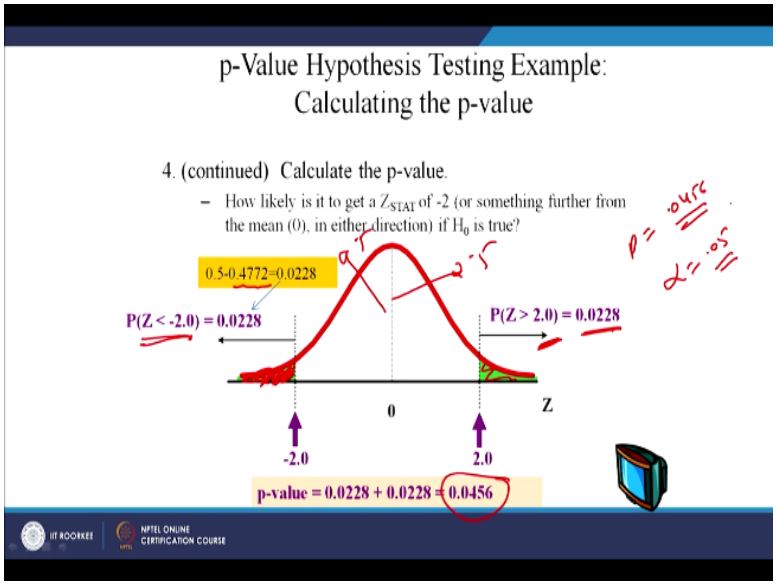

So we are interested in let us look at this we are interested in this area right, so this area is this side is 0.5, this side is 0.5 and we are interested in this area right, so from table we have got 0.4772 when Z was 2 right ok. So probability Z less than 2 is 0.228 and similarly this side same value right 0.0228. So just add these 2 values so probability of Z between +2 and -2 is 0.40 0.0456 right.

And you have to compare and this is known as P value right, so just compare this with alpha value, what you what was your alpha value 0.05. So is P value less than alpha this P value is less than alpha so we will reject the null hypothesis right.

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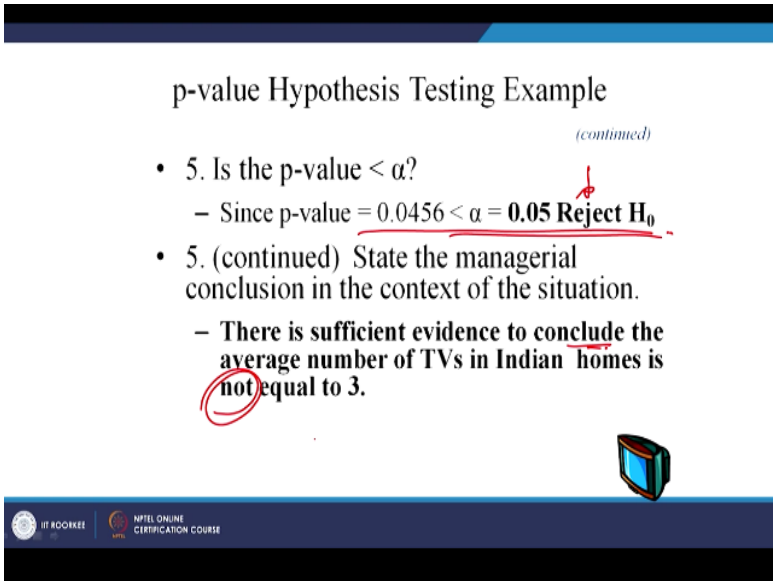

Since P value is less than alpha, so we will check null hypothesis, so what was about null hypothesis, let the average number of TV sets in Indian homes is equal to 3 right. So there is a sufficient evidence to conclude that the number of TV sets in Indian homes is not equal to 3 again when I say not equal to 3 means either more than 3 or less than 3. So what it is, is more than 3 or less than 3, it is less than 3 because our sample mean is less than 3 right.

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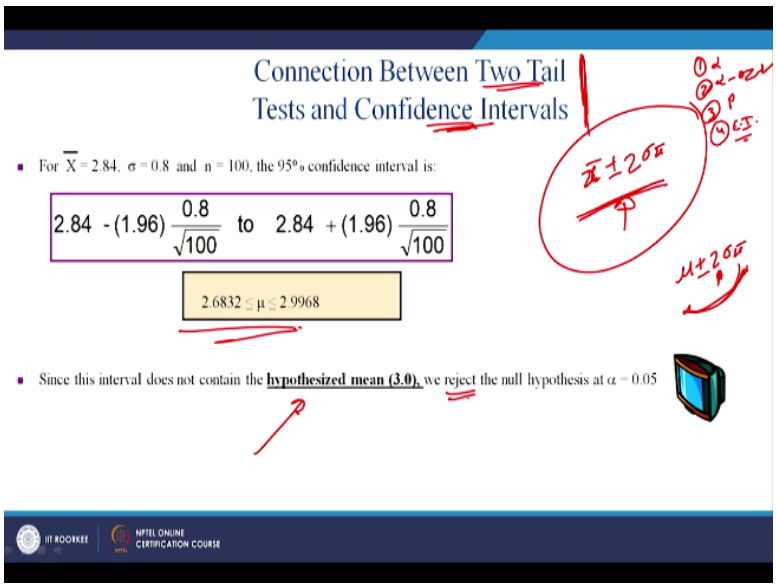

And there is a relationship between two-tail test and confidence interval, in fact this another approach of testing hypothesis, in fact we have seen couple of methods earlier and apart from P value approach is another way of testing hypothesis. So just find out confidence interval. So confidence interval is mean ± Z right into standard error and see whether population mean falls in this range or not.

In fact we have seen one method wherein we wrote μ ± Z, when alpha was given we used table value and alpha was not given we used calculated Z value ok and we check whether sample mean was in this range or not, here we are doing similar to that but we are finding confidence interval and we are checking is population mean falls in this range. So let us find out this range are confidence interval, which is 2.68 to 2.99 right, do you have your hypothesized mean in this range no.

So will reject the null hypothesis ok, so we have seen how many methods, first when you are given alpha, second when you are not given alpha but we used Z value in third case the third one was P approach, the fourth one is this confidence interval approach right. So we will see 4 methods will be side are to test hypothesis testing right. Let us look at next slide.

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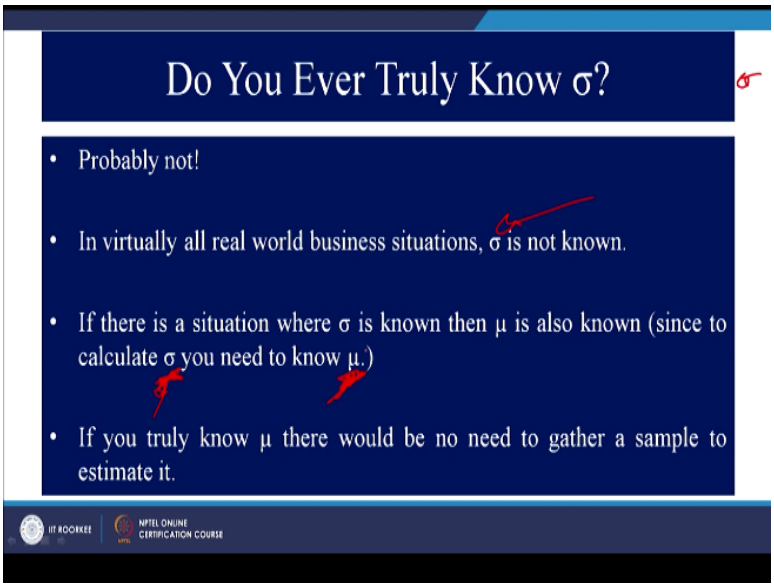

Now in real life we really do not know standard deviation of population, in fact in case of sampling and sample distribution when we studied that topic. We have seen that in reality we

really do not know the population, standard deviation or population proportion. Similarly, in hypothesis testing as well you will have a situation where standard deviation of population is unknown.

So we really do not know standard deviation of population, so what we did in fact earlier we used sample standard deviation t represent population standard deviation right, same thing will do here as well right. So as I said you really do not know the population standard deviation because if you know the population standard deviation then you also know mean right population mean.

Because to calculate you need mean right, since to calculate standard deviation you need population mean. Since we do not know this we really do not know this but let us say if we know this then we also know this right and if you know this then what why are you doing this exercise. Because in this exercise we are trying to know about population mean from sample right.

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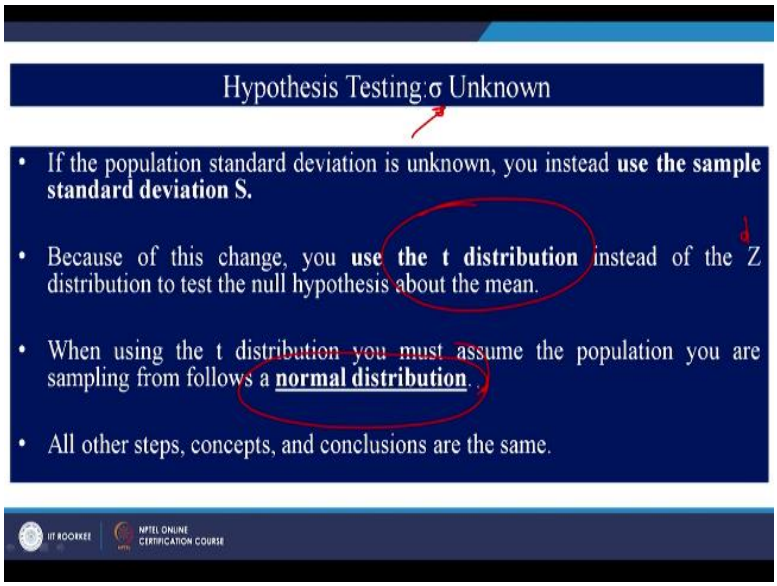

So in real life we really do not know standard deviation of population so will replace standard deviation of sample for standard deviation of population. Now what changes you should do when standard deviation is unknown. So first is instead of Z test will use t test right and even in this situation when standard division is unknown we assume that the population is normally distributed.

New sampling follows normal distribution in fact this is what this CLT right central limit theorem right ok and all other states will remain same, only we change would be what instead of Z test we will be using t test right. So this how you should be calculating t statistics is very similar to what we calculated when we knew this ok.

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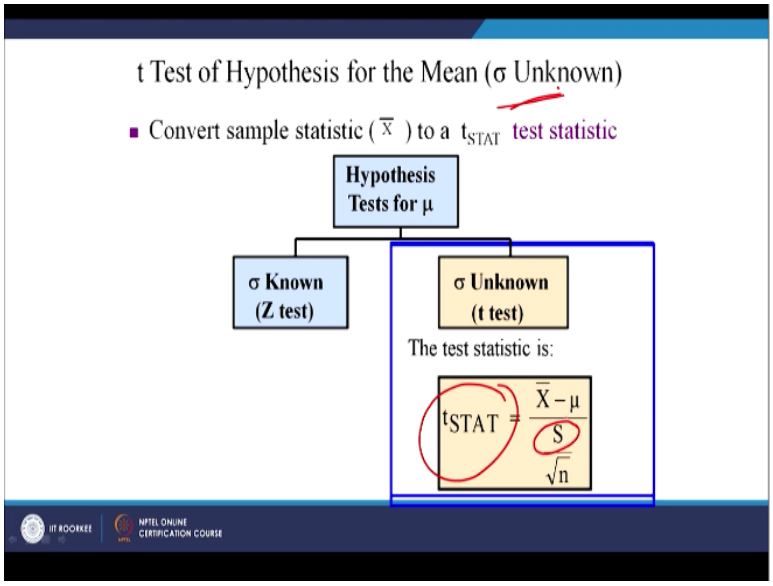

So it was standard deviation earliest now here this sample standard deviation right earlier it was population standard deviation right.

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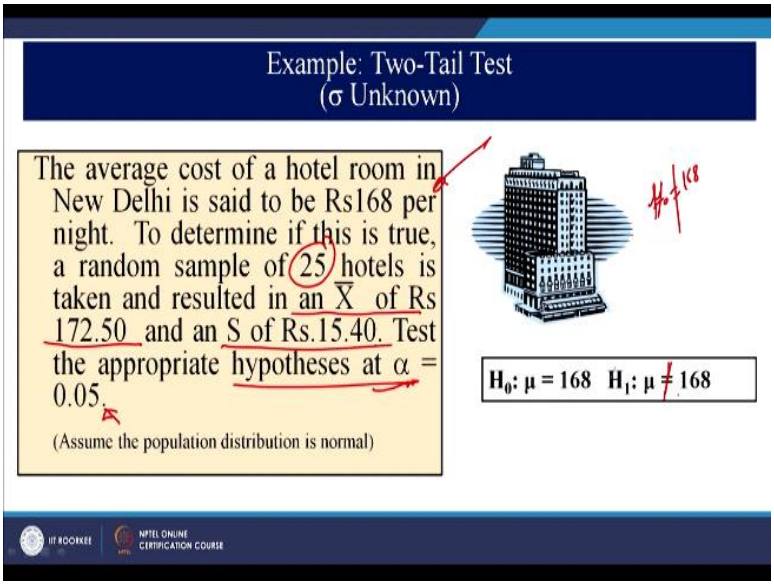

Let us look at an example, the average cost of hotel room in New Delhi is said to be 168 rupees per night to determine if this is true a random sample of 25 hotel taken which resulted in an average of 172.5 rupees and sample standard deviation 15.4, test the appropriate hypothesis at significance level alpha which is this. Now the question is what is the null and alternative hypothesis.

So null hypothesis is this, you said that the population mean is 168 and the other one is not 168, so it is a case of one-tail test or to two-tail, this is not equal to right, so this is the case of one-tail test or two-tail test, this is the case of two-tail test right because this symbol is not equal to type right. So you know two rejection regions right ok.

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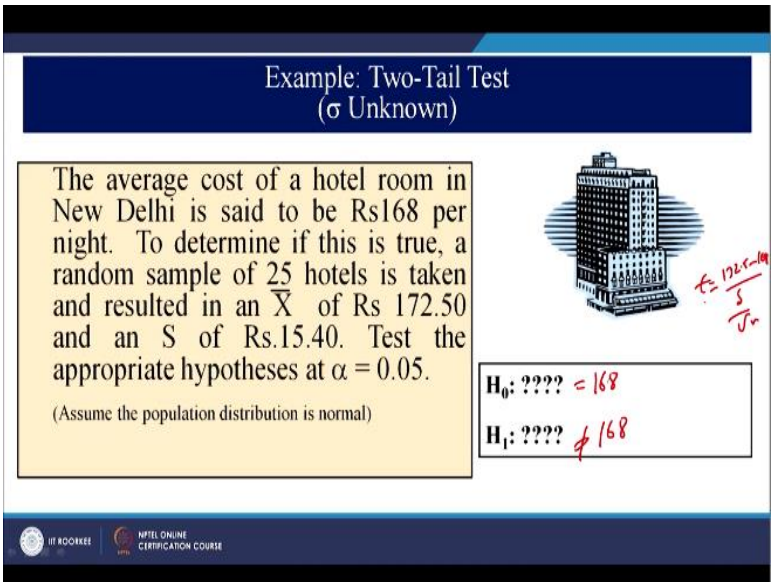

Let us look at this, so we have already seen this is 168 and this is not equal to 168 right. Now how to proceed further calculate t statistics night X-mu right 172.5-168/S under root of n let us calculate that value right. So that value is 1.46 ok.

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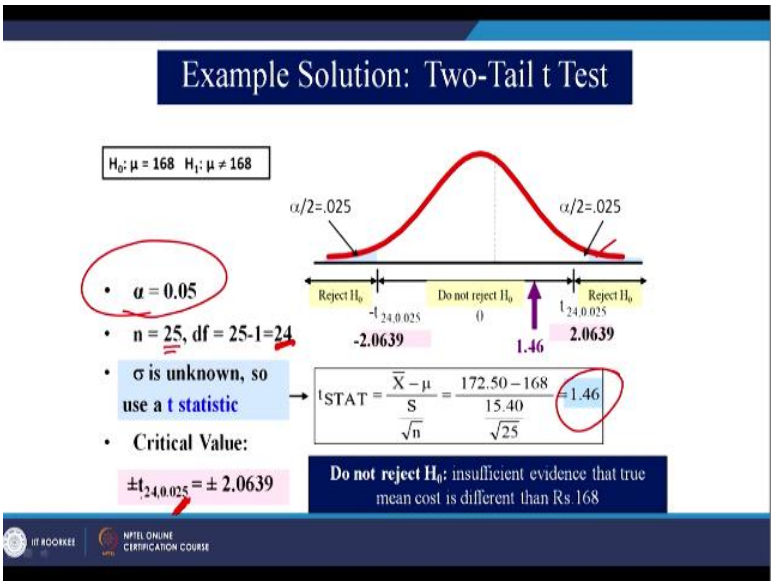

Now we have been given alpha=0.05 right, so alpha/2 is 0.025 degrees of freedom since n is 25 at 24 degrees of freedom we have to see the critical value ok. So at 0.025 at 24 degrees of freedom let us see what is t value.

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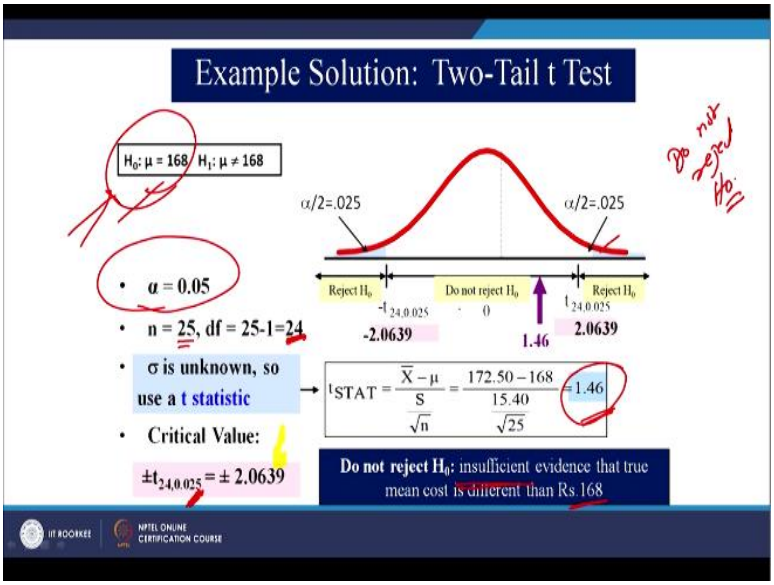

24 degrees of freedom and 0.025, this is 0.025 right here this one, in fact this is 0.975, so 1- this is 0.025 right and 24 degrees of freedom is this, this is 2.0687 2.06, so where is that mean is 2.0693, this is 0.06 right 2.0639 right, which is right ok. So what is the conclusion now, you have calculated in fact t value and critical value from table. So compare this calculated t value with table value.

So we will say that this calculated t value falls in this region right in non rejection region, so you have designs you do not reject the null hypothesis right, do not reject null hypothesis it means the average price of Hotel in Delhi 168 rupees right. So there is insufficient evidence that the true in cost are the true price of Hotel in Delhi is this much 168 right. So we will it is 168, it is not more than this or it is not. Let us see whether it is more than this or less than this right yeah.

In fact we have not rejected it right so we will say that the new prices this much right, had we rejected it then we would have said that the mean price has gone because our averages 172.5 right. But we are saying that will not reject null hypothesis and the price is 168 rupees right. Let us look at same example you can in fact solve using different tools right.

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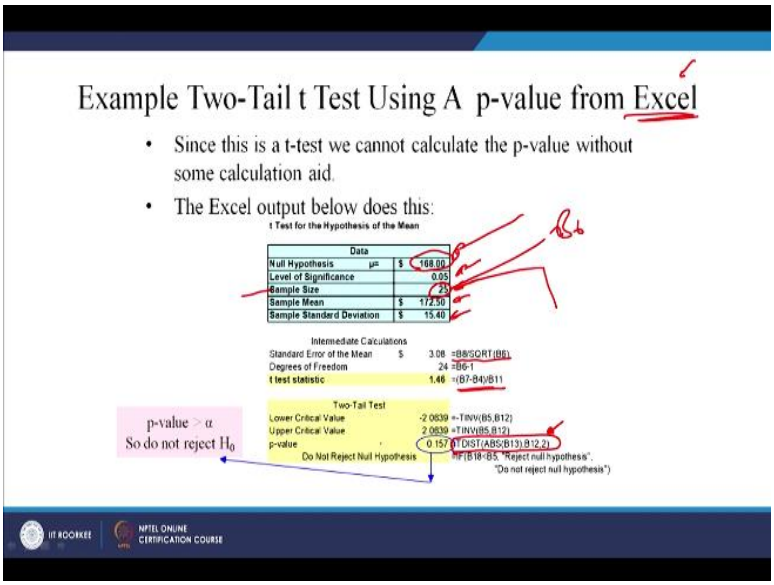

Either we use let us say a SPSS software or Minitab software or let us say if you are using excel sheet or let us say if you are using size software or any other software for that matter right. So this same example can be solved using excel sheet right only the point is you need to let say you just enter data in one of the columns as let us say 168 is the null hypothesis significance level is 0.05, sample size 25, sample mean is this, sample standard deviation.

You just apply the formula for sample standard right, sample standard deviation, then standard error of mean this let us say B8 square root of B6 then you degrees of freedom of course sample size -1 right. So this is let us say this is B6 right, this cell is B6 right. So similarly you can have

B6-1 test statistics is B7-B4/B11 right and then upper and lower limit is right and if you apply this particular let us say formula you will get P value.

And finally see this is a case wherein we are using P value is P value less than alpha no, P value is more than alpha so will not reject null hypothesis right ok. So this how you should solve a question on using t test right when sample standard deviation is being used for population standard deviation, thank you.