Business Statistics
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Lecture-31
Hypothesis Testing Process-I

Hello friends I welcome you all in this session as you were aware in previous session we discussed about hypothesis testing and we have seen what is hypothesis, what are different types of hypothesis, how do we frame null hypothesis and alternative hypothesis and so on. So, in today's session we are going to look at the same topic in little detail, so we know that hypothesis is something in which you are assuming about population parameter.

So, you are assuming let us say population mean or population proportion or population variance and whenever you assume something you would like to test it for the moment you test your hypothesis it becomes statistical hypothesis. So you can have as we have discussed in previous session that you have got research hypothesis right and we have seen that there is something called statistical hypothesis. So the moment you test research hypothesis it becomes statistical hypothesis.

Now statistical hypothesis can be again either null or alternative hypothesis. So we have seen couple of examples in previous session for example let us say the market share of your company is 18% and you have worked a lot in last let us say couple of months along with your employees on the to increase market share of your firm and now you think that the market share has gone up and it is not more than 18%, so how would you frame null hypothesis and alternative hypothesis. So, we have seen that hypothesis.

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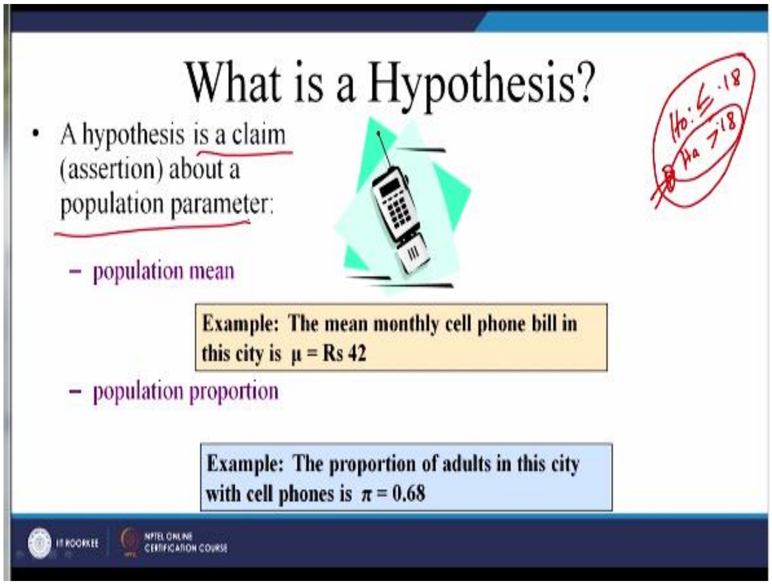

The null hypothesis was let us say H0 right, H0=0.18 right this 18% market share. Now you want to know whether this market share is more than 18% or not. So, you will say that alternative hypothesis is more than 0.18. Now there are couple of possibilities in market share may remain 18% or it may go down from 18%, so you can have this null hypothesis written in this way as well.

So market share is less than or equal to 18% and alternative hypothesis is market share is now more than 18%, so this is how you can run null hypothesis and alternative hypothesis and in this case you are testing what?, you are actually trying to prove alternative hypothesis. Generally, what happens whatever experiments you do you believe that, that experiment would be the truth one, so you want to prove your experiments right.

Let us look at something more about hypothesis testing, so we have said that hypothesis is it is claim about population parameter right. So population mean is let us say or let us say mean height of a group of students is let us say 5 feet 6 inches right. So, that is about the entire population or let us say the mean monthly sale cell phone bill in this city is 42 rupees right or population proportion is let us say 68% right. Then you have got null hypothesis, null hypothesis is this assumption about population parameter.

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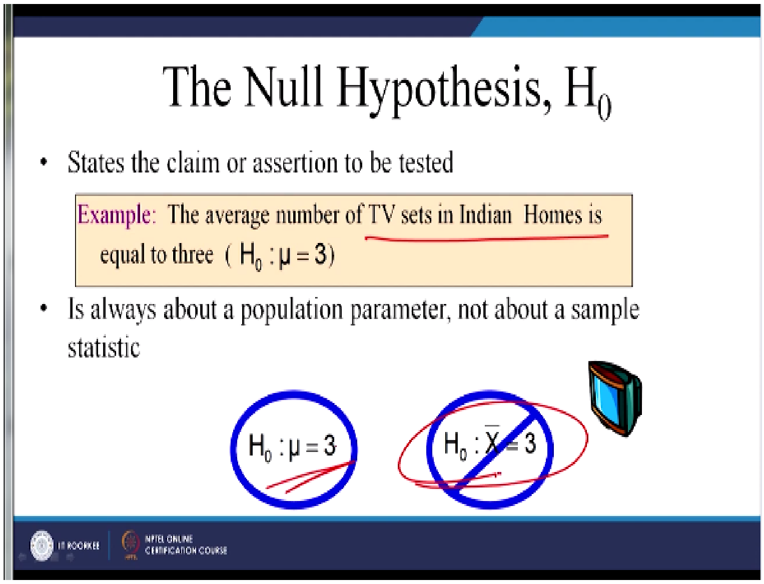

So, you can have an example like this, the average number of TV sets in Indian homes is equal to 3 right, it about sample mean, it is about population mean right.

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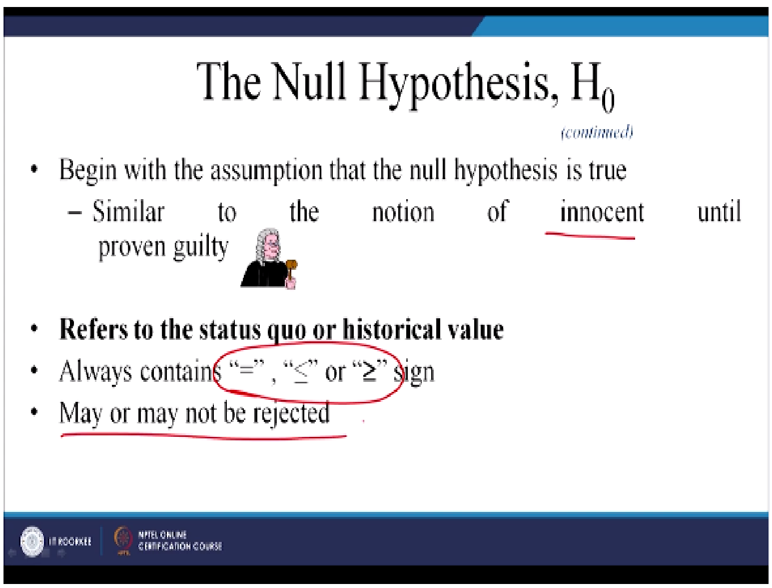

Null hypothesis is something which begins with the assumption that null hypothesis is true, so we always frame null hypothesis in such a way that we believe that null hypothesis is true. It is similar to the notion that a person is innocent until proven guilty right. So, you can use these types of signs while framing null hypothesis, so you may reject or you may not reject null hypothesis.

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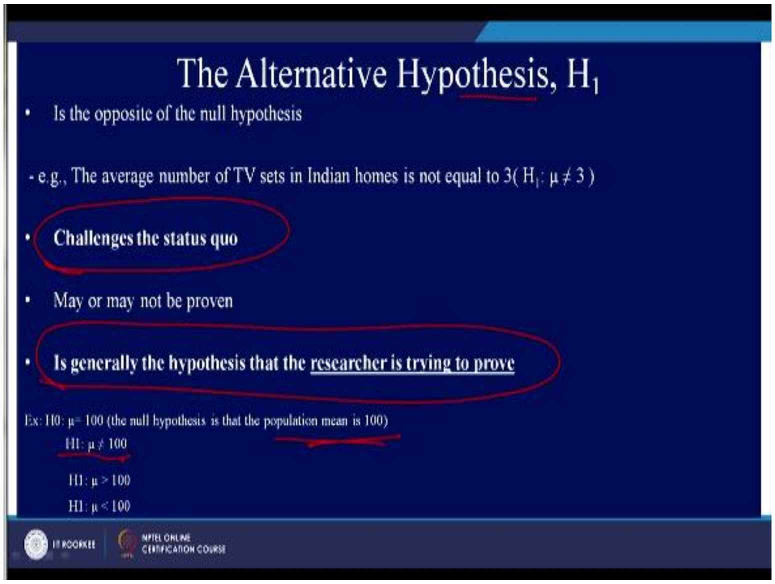

Similarly alternative hypothesis, alternative hypothesis always challenges the status quo, you will always try to say that the new theory is in place the system is now changing right, you can say that the process is now out of control. So it is basically challenge to null hypothesis, so this is what generally the hypothesis that researcher actually tries to prove. So let us say null hypothesis is that the population mean is 100 and alternative hypothesis it is not equal to 100.

So when I say not equal to 100 means in other words I can always say H1 greater than equal to 100 or H1 less than 100 right.

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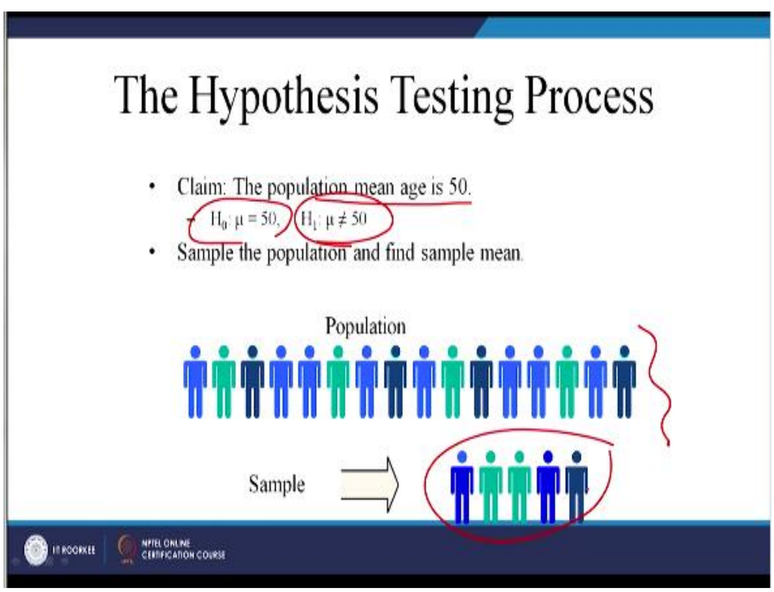

So, this is the testing process, so let us say population mean age is 50, so this is your null hypothesis and alternative hypothesis it is not equal to 50. So, there is a population you need to take a sample from this population and find out the sample mean or age of the sample which you have selected from population.

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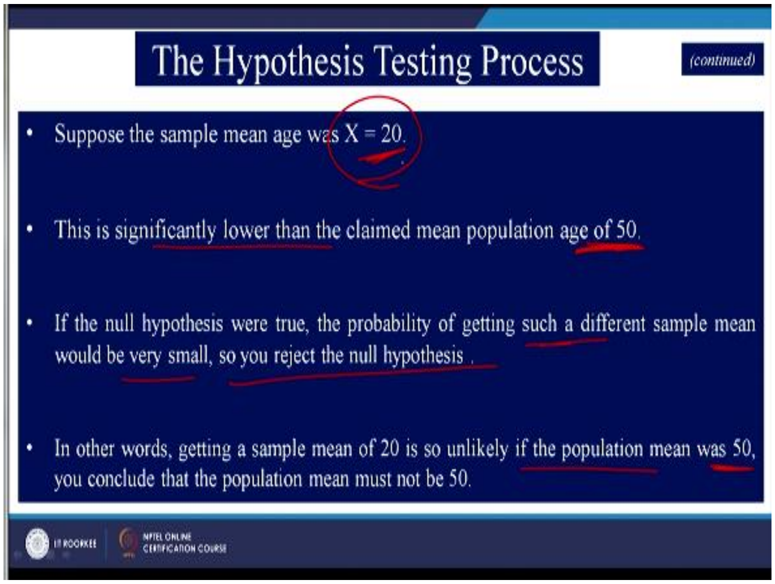

Suppose the sample mean is or sample mean age was 20 and hypothesized population mean in this case was 50 right. So, we will say that this is significantly lower than the claim age of 50, so if the null hypothesis were true the probability of getting such a different and such a small sample mean would be very low. So, you will reject the null hypothesis, in other words getting a sample mean of 100 is shown likely if the population mean was 50. So, what we are saying is if we assume that the population mean was let us say 50 then how it is possible to have sample mean of 20 right, so we will reject this null hypothesis.

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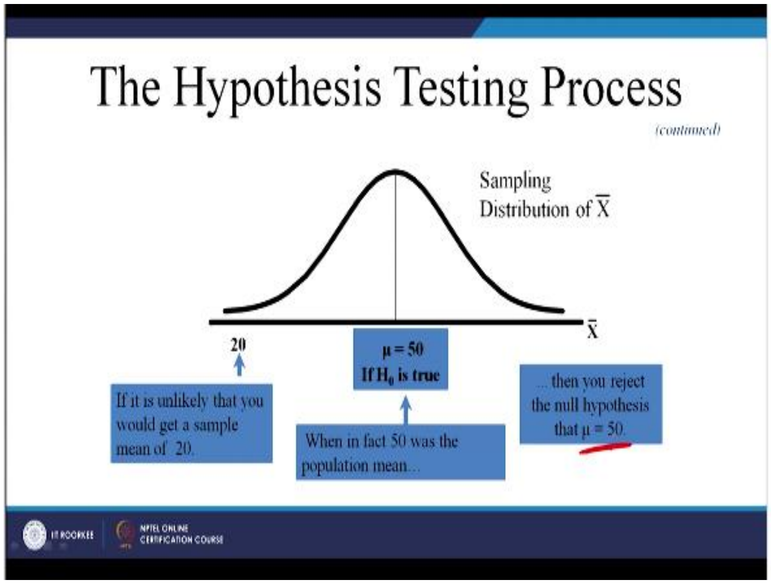

So, the same thing can be said in this way, if it is unlikely that you would get a sample mean of 20 when in fact 50 was the population mean. So, then you reject the null hypothesis of population mean 50.

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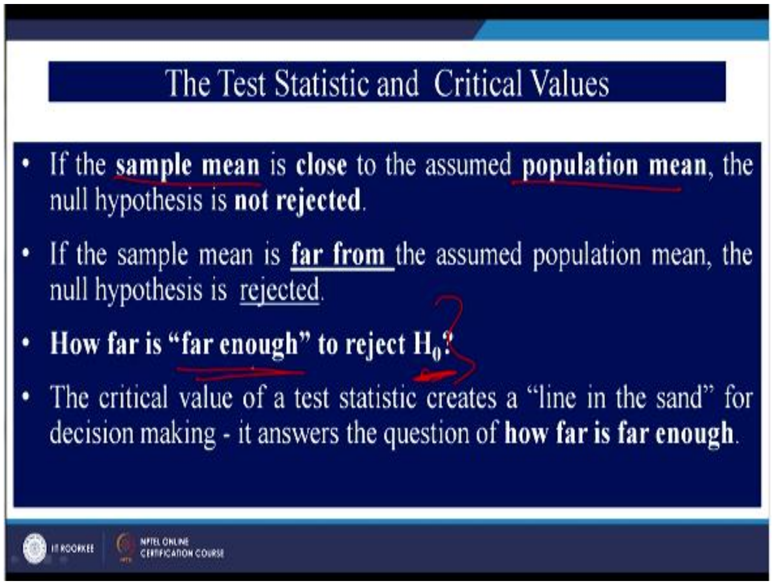

Let us look at how to test hypothesis testing or how to test a hypothesis, so if sample mean is close to the population you will not reject the null hypothesis otherwise you will reject it right. So, we have to take a decision how far the sample mean should be from population mean, so how far is this far enough to reject null hypothesis right. So, to help us in a situation like this probability will come into picture and there is something called confidence level or alpha right.

So, on the basis of alpha we will decide how far this sample mean should be from population mean to accept or to reject a null hypothesis.

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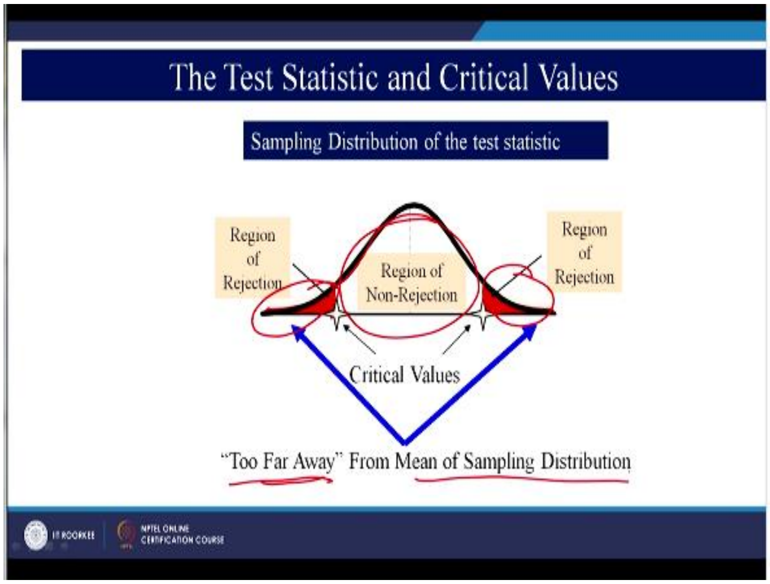

So, let us look at this distribution wherein this area is a rejection of it is a non rejection region. So, if sample mean falls somewhere in this region, we will not reject the null hypothesis. Otherwise if sample mean is somewhere here or somewhere here we will reject the null hypothesis. So, there are 2 rejection regions right, so these 2 limits are actually too far away from mean of sampling distribution.

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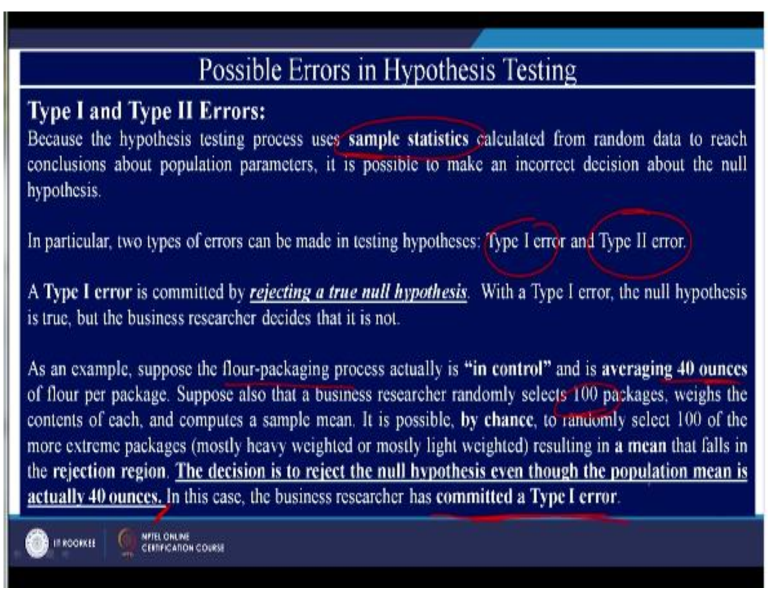

Now after looking at hypothesis let us look at different types of errors in hypothesis testing. So, basically there are 2 types of errors first is type 1 error and the other one is type 2 error, why these errors do occur. Because of sample statistics because we always take sample to know about population right. So, there are 2 types of errors type 1 error and type 2 error right, what is type 1 error.

Type 1 error is an error when you reject a null hypothesis which is true, so rejecting a true null hypothesis is type 1 error. So, let us look at the same example which we have discussed in previous session as well, so there is four packaging process and the process is in control.in control means the average weight of the bag is 40 ounces right, now you want to check whether the process is under control or not ok.

So, what you have done, you have taken sample of few bags, let us say 100 packages you have chosen. And after calculating mean of let us say those 100 samples we found mean to be let us say 42 ounces right, then you will reject the null hypothesis. So, the decision is true reject the null hypothesis even though the population mean is actually 40. So, the process is in control but the sample you have chosen tell you that the samples have**.**

After let us say finding mean of the samples you have decided to reject the null hypothesis while the process was in control, so this is known as type 1 error right. Process is in control but still you are saying process is out of control on the basis of sample this is known as type 1 error.

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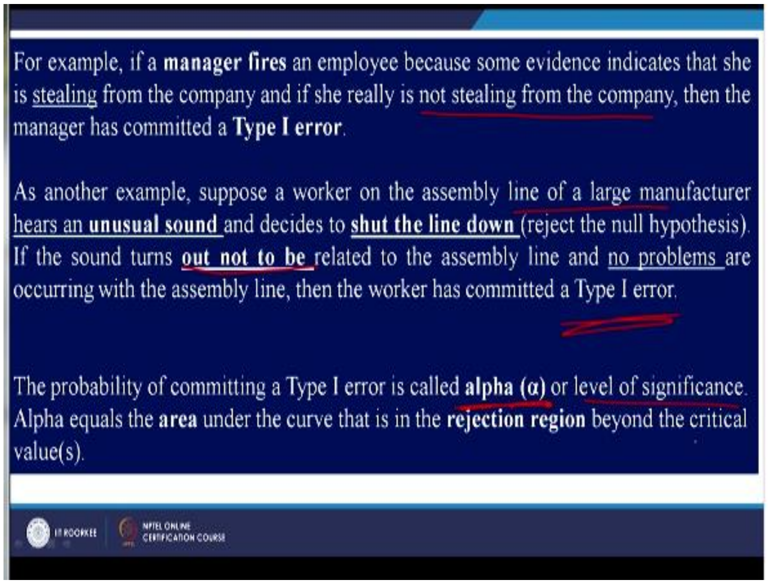

Let us look at one more example, now there is a manager in a company and the manager fired an employee because manager thought that the employee was stealing something from the company. But later on but in actual situation the employee was not stealing anything, so this a situation where everything was alright but still employee was fired, so this is known as type 1 error.

Let us look at one more example of type 1 error suppose a worker on the assembly line of a large manufacturing. Here is an unusual sound and decides to shut the line down right, so he heard some sound unusual sound and he decided to shut the line. But actually these sound was not related to assembly line but he stop the line, so this is a case of type 1 error actually the assembly line was perfectly working but he heard some sound and he stops it.

And at the end of the day the sound is not of assembly line it was some other sound. So, this is a case of type 1 error, so assembly line was perfectly working well but it was stopped, so a case of rejecting a null hypothesis which is exactly true right. Generally we denote type 1 error by alpha, so the probability of committing a type 1 error is called alpha or level of significance. So, this is the area under the curve that is in rejection region beyond critical value, so if there are 2 rejection regions then each one would be alpha/2 and alpha/2.

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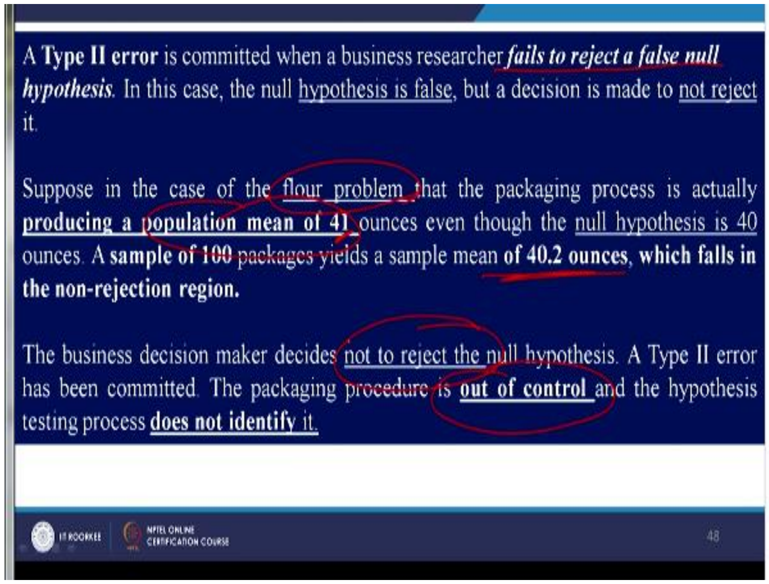

Let us look at the opposite to type 1 error which is type 2 error right, now type 2 error is a situation where a researcher or a decision maker fails to reject a false null hypothesis. So, null hypothesis is false but still accepted or not rejected ok. So, let us look at the same flour problem, so in this problem we have seen that the let us say that the population mean is now 41 ounces.

But the sample which you chosen has got mean of 40.2 ounces, so you are still saying that the process is in control but the process is now out of control. So, type 2 error is what there is something wrong happening and you are still not able to detect is type 2 error. So, this is a case where the process is out of control but you are not able to reject this hypothesis right.

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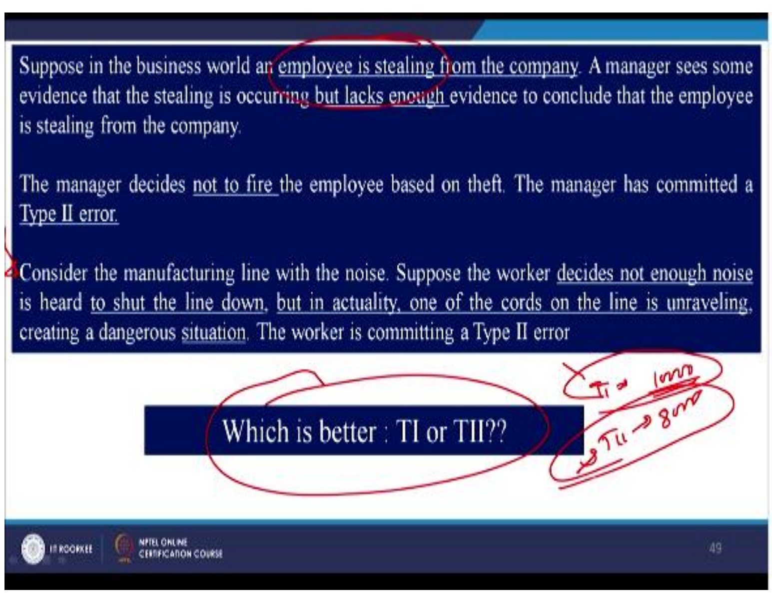

One more case, so the employee stealing something from the company but manger does not have in a proof to fire him. So, this is a case of type 2 error, so something wrong is happening but still you are not able to detect it right ok. So, and one more example, so again on assembly line suppose the worker decides not enough noise to is heard to shut down the line and actually he does not stop the assembly line and production is continued.

So what is happening there is some fault in the assembly line and he did not stop it and production is continue, so it is type 2 error. So which one should be preferred or should be which one is better type 1 error, type 2 error, in fact you should not have any error at the end of the day why to have error, the type 1 or type 2. So, try to minimize these errors ok let us take the example where employees was stealing something or was not stealing something right.

Let us take a case, let us say the employee was fired but he was not stealing anything, so this is type 1 error right. An employee was fired but he was not stealing anything and that actually will cause some loss to the company right. Let us say that loss is 10000 rupees ok, so this is loss of type 1 error right, on the other hand let us say you have not fired the employee and he was stealing something.

So, let us say at the end of the month the loss to the company due to theft was let us say 8000 rupees. So which is better, which type of error would you prefer type 1 or type 2, so type 2 error is better right. So, you can have different situations where sometimes you would like to prefer 1 type of error over other type of error.

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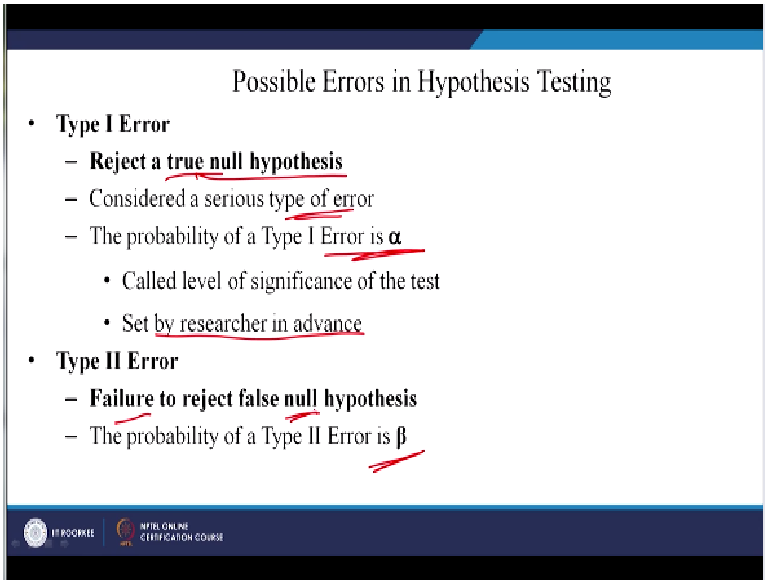

So, type 1 error reject the null hypothesis or reject the null hypothesis which is true right, it is a series type of error it is also called alpha, type 1 error is also called alpha it is said by the researcher in advance. Type 2 error is called beta and what is type 2 error something wrong is happening but we have still not rejecting right, so failure to reject false null hypothesis.

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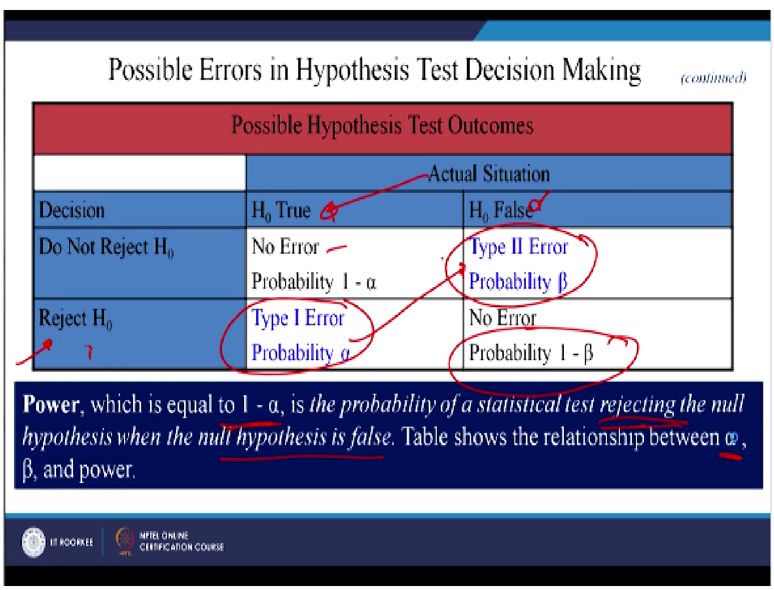

So, this is the relationship between the actual situation and decision you are taking ok, let us look at this. In actual situation your null hypothesis is true right but decision you have taken is to reject it, so this one is type 1 error, so 2 null hypothesis is rejected is type 1 error, the extreme and what you have the actual situation is false but you have do not reject it. So, null hypothesis is actually false but you do not reject is the type 2 error and this is something where actual situation is null hypothesis is true and we do not reject it.

And in the actual situation null hypothesis is false and we reject it right, so there is no error in these 2 cases. So, what you want we want to have this is known as power of test 1-beta which is equal to 1-alpha right, is the probability of statistical test rejecting null hypothesis when the null hypothesis is false. And this table is 1 a relationship between alpha, beta and power right. So, what you want at the end of the day you want a false null hypothesis to be rejected and a true null hypothesis not be rejected right.

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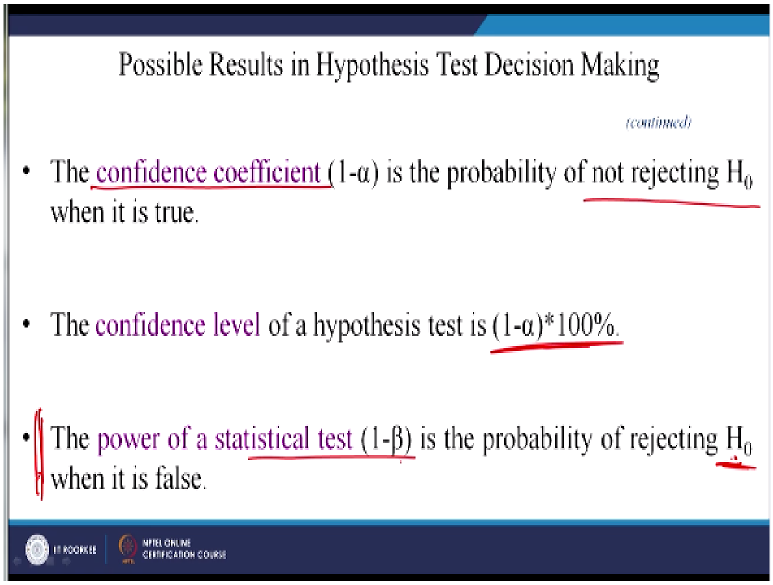

So, this one in the same thing you have got confidence coefficient is the probability of not rejecting null hypothesis when it is true confidence level. The confidence level of a hypothesis test is 1-alpha*100 right, power of test 1-beta is the probability of rejecting a null hypothesis when it is false right, so this is equivalent definition power of test**.**

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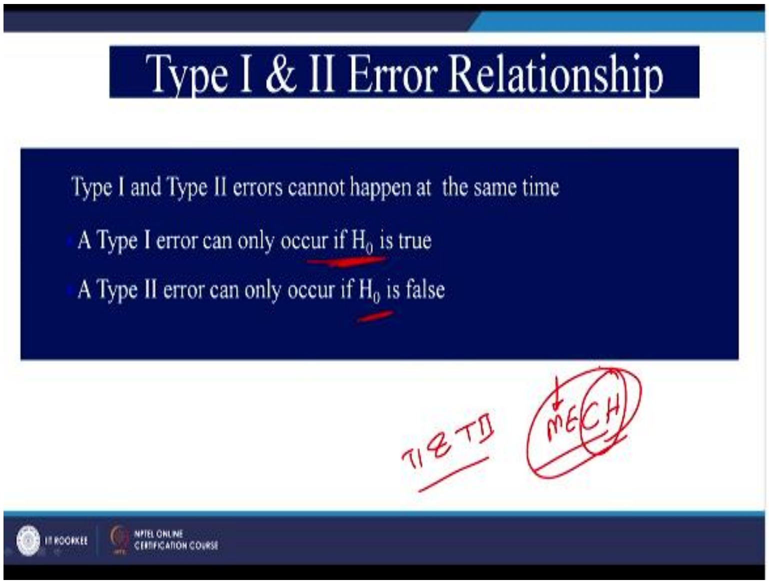

So, after looking at these 2 errors we should know that these 2 errors simultaneously cannot happen. If one error happens the other will not and vice versa right, so a type 1 error can only occur if the null hypothesis is true. And if null hypothesis is false then only type 2 error can happen right, so these 2 errors t1 and t2 are mutually exclusive and collectively exhaustive right. Mutually exclusive because one can occur at a time and correctively exhaustive means there are only 2.

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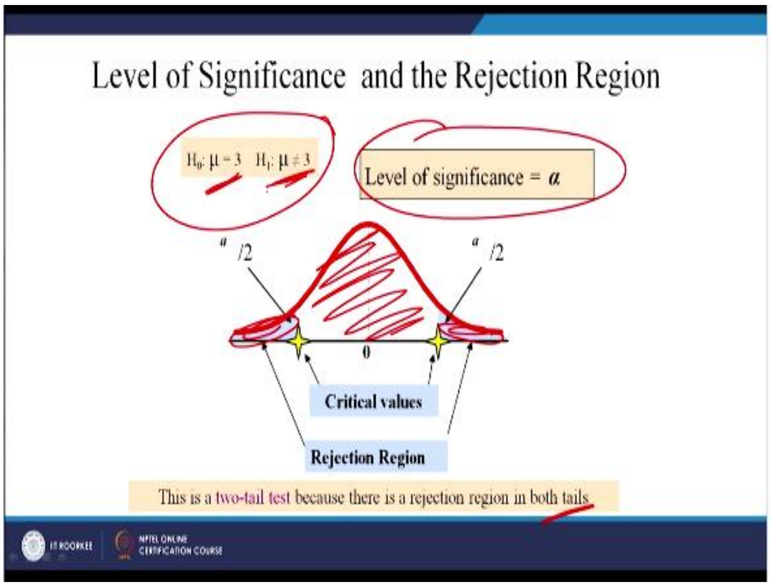

So, let us look at this slide wherein you have been given critical values and rejection region, so this is your rejection region and other rejection region. So there are 2 rejection region, first rejection region and second rejection region. So, this is your known rejection region, so if sample mean is in this region then hypothesis will not be rejected otherwise till the rejected, so this is a level of significance is alpha.

So these 2 areas are alpha/2 and alpha/2, so this is a two-tail test because there is a rejection region in both tails right. So this is the case wherein we have said that the average TV sets in Indian homes is 3 right, so null hypothesis is this alternative hypothesis is this right.

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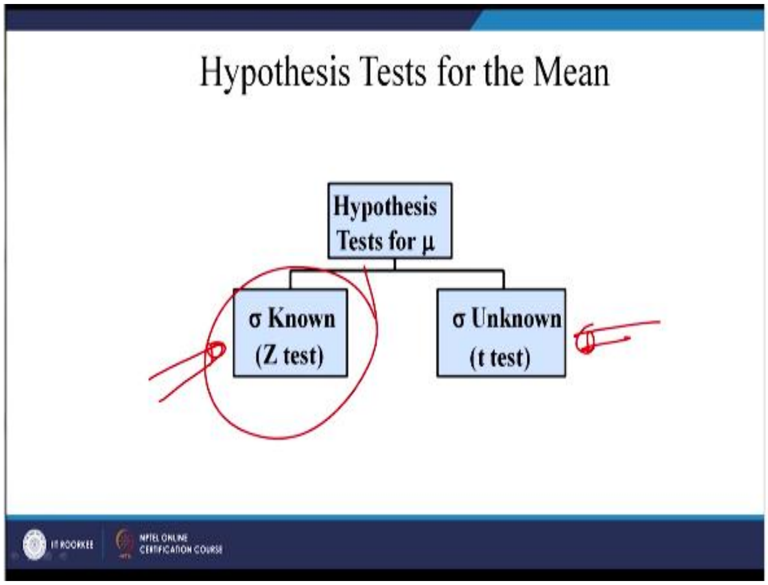

Let us look at hypothesis test for the mean, hypothesis test for mean can be found in these 2 different situations where when standard deviation is known when standard deviation is unknown, let us look at this case first.

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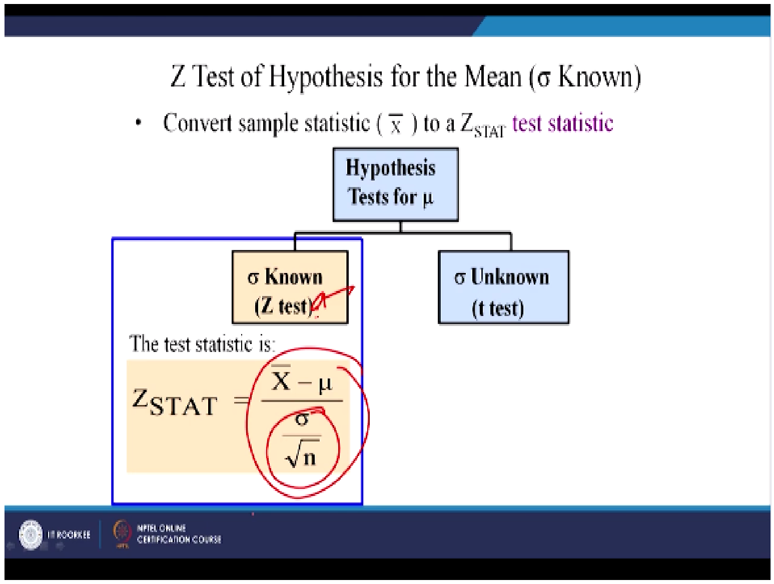

So, when standard deviation is known we will be using z test, irrespective of sample size even a sample size is let us say 20 and standard deviation is known we will go for z test right. So $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$.

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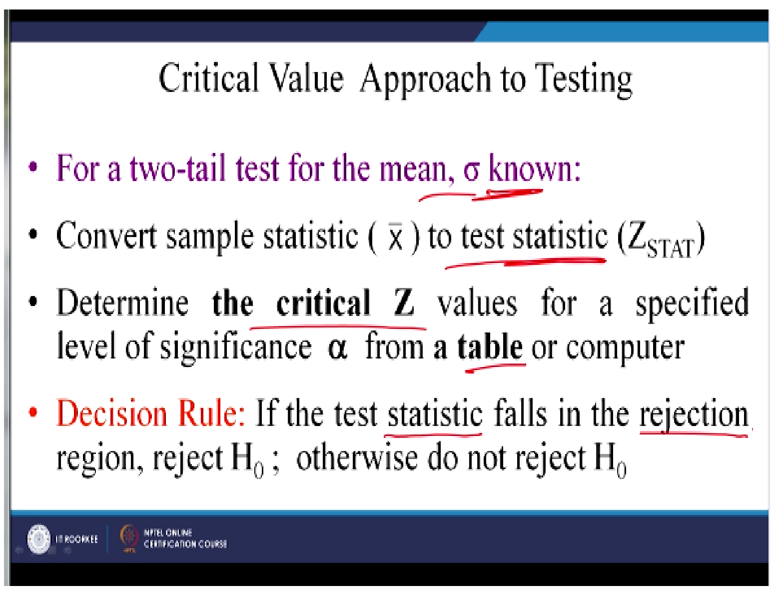

So, let us look at hypothesis testing approach there are basically multiple approaches, so there is something called critical value approach in which we will take the value of Z from table and will take decision whether to reject null hypothesis or not right. So, for two-tail test for the mean and sigma is known, converts sample statistics to Z statistics first, determine the critical value from table and if the test is statistic falls in the rejection reject null hypothesis otherwise not reject null hypothesis.

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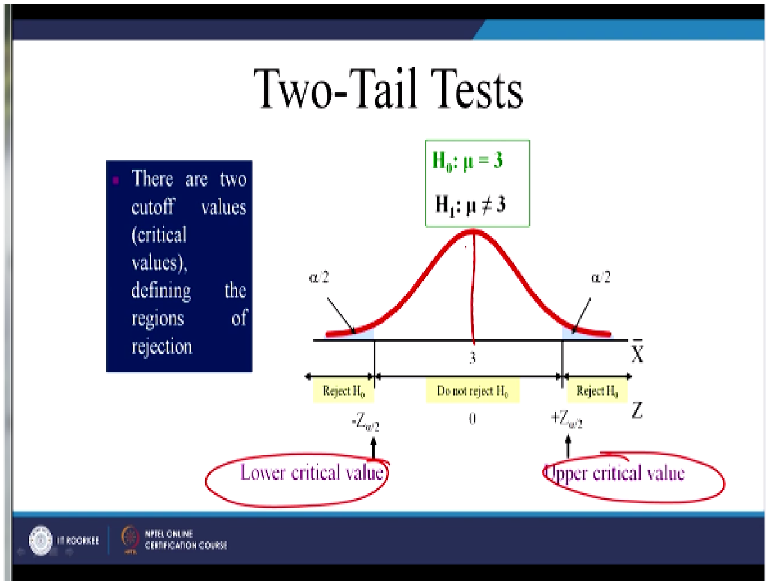

So this is lower critical value and upper critical value from table you will come to know right, so there are 2 cut-off values defining the regions of rejection, so this is your mean value right average number of TV sets in Indian homes 3 right.

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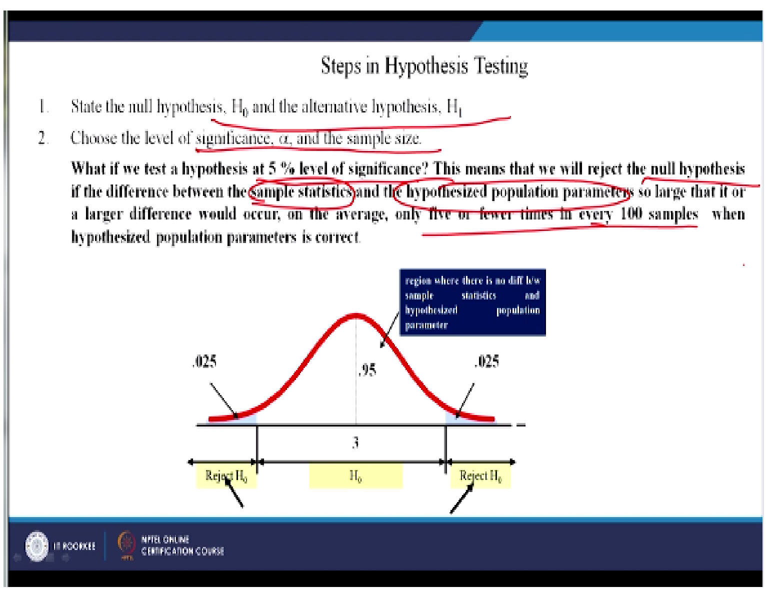

Let us look at steps involved in hypothesis testing, there are 6 steps and of course it depends on which study material you refer, sometimes you can have 4 steps, sometimes you can have 6 steps and so on right and in the thing is what decisions you are taking in each of the steps right. So, the first step in hypothesis testing is lead to frame null and alternative hypothesis or you need to propose null and alternative hypothesis.

The second one is choose significance level and sample size, so let us say if you test a hypothesis of 5% level of significance. So, what is the meaning of 5% level of significance, this means that we will reject the null hypothesis if the difference between sample statistics and population parameter or population mean or proportion or variance right. So, we will reject the null hypothesis if the difference between sample statistics and population parameter.

So, large that or a large difference would occur on when the average only 5 or fewer times in 100 samples when hypothesized population parameter is correct. So, what did you understand out of this, so let us say if population mean is 3 then if you take sample find out it is mean then out of 100 times the only 5 times the null hypothesis would be rejected if it is true ok.

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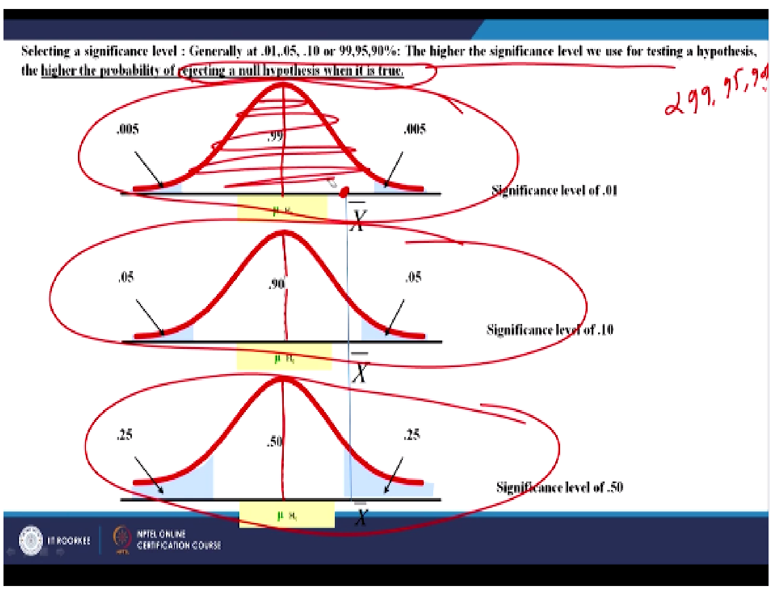

Let us look at the same step number 2, so we generally select hypothesis or we test hypothesis at significance level of 99, 95 and 90%. The higher the significance level we use for testing hypothesis higher the probability of rejecting null hypothesis when it is true. So, this is a case of 99% and this is the case of 50% and this is the case of 90%. So these are your population mean and these 3 distributions and this is your sample mean.

So you are testing hypothesis at 99% and even not rejecting null hypothesis, this is your non rejection regions will detect.

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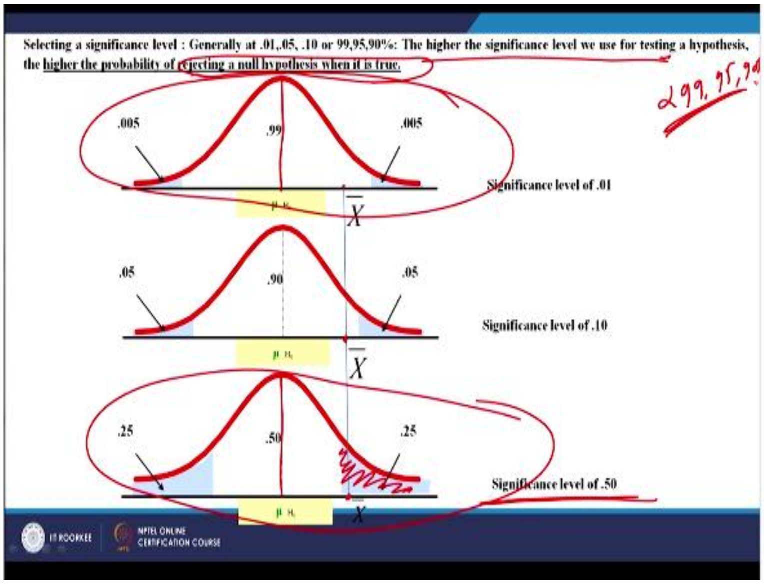

Now let me erase all these ok, if you look at second one let us say when you paste the same hypothesis at 90% significance level this is your sample mean again you are not rejecting an hypothesis. And but when you your significance level is 50%, this sample mean is in now rejection region, this is your rejection region, so you will reject the null hypothesis, so what we have said the higher the significance level mean is for pasting null hypothesis.

The higher the probability of a rejecting a null hypothesis when it is true**,** so it is always suggested that the null hypothesis or hypothesis is to be tested at these significance level. So, let me summarize what we did in today's class we have seen hypothesis what is null hypothesis, what is alternative hypothesis, we have seen what are different rejection regions what is critical value how to get Z value for the given value of n and sample mean and sample standard deviation, in next class we will continue the same in and the after looking at these 2 steps you get.

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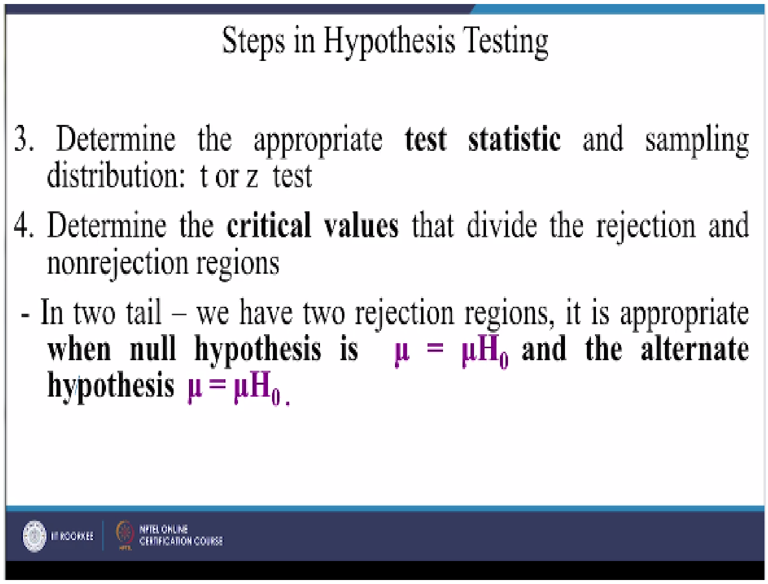

The next one is need to determine the appropriate test statistic, so first step is frame null and alternative hypothesis. Second state choose alpha, third is determine which test we want to apply whether t test or z test. Then find out critical value and compare these sample statistics with the hypothesized population parameter and take a decision whether the hypothesis is to be rejected or not to be rejected.

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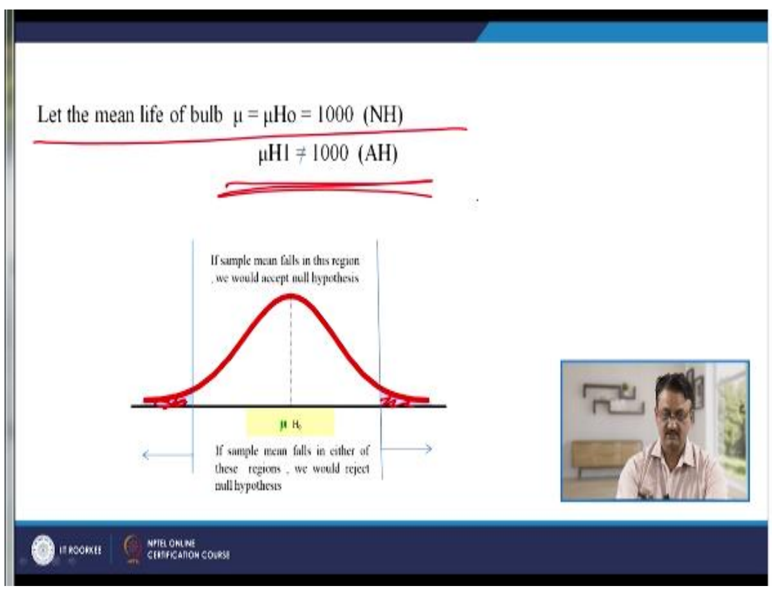

Let us look at this slide wherein this is basically a two-tail test the why we are saying a two-tail test. Because there are 2 rejection regions right, so let us say mean life of the bulb is 1000 hours, so manufacturer of bulb is claiming that the mean life of the values 1000 hours. In alternative hypothesis is no it is not 1000 hours, so we will take a sample and we will measure life of the bulb and we will take a decision accordingly.

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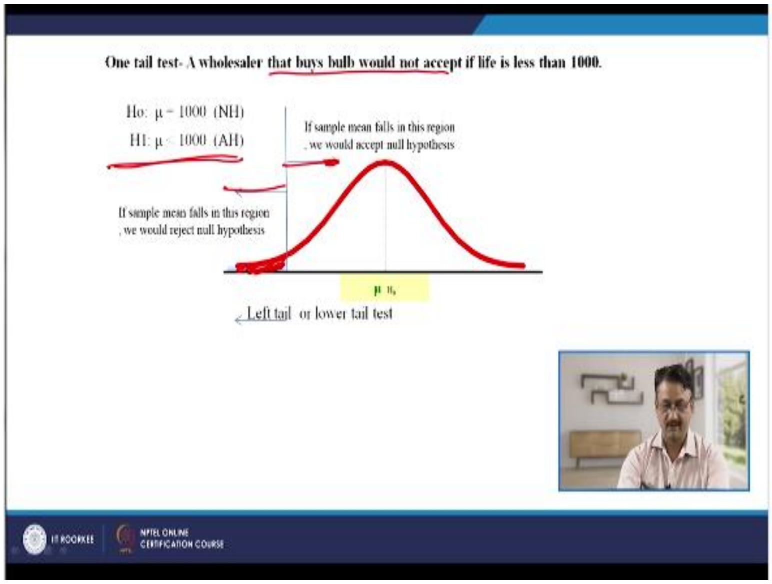

So, this two-tail test, in one tail test this is if sample mean falls in this region we will reject the null hypothesis otherwise not. So, this is rejection region, this is non rejection region, so let us say a whole seller that buys bulbs would not accept if the life is less than 1000 hours. So, the manufacturer is claiming that the life of the bulb is 1000 hours but a buyer he would not select those bulbs for which life is less than 1000 hours, so this is one tail test.

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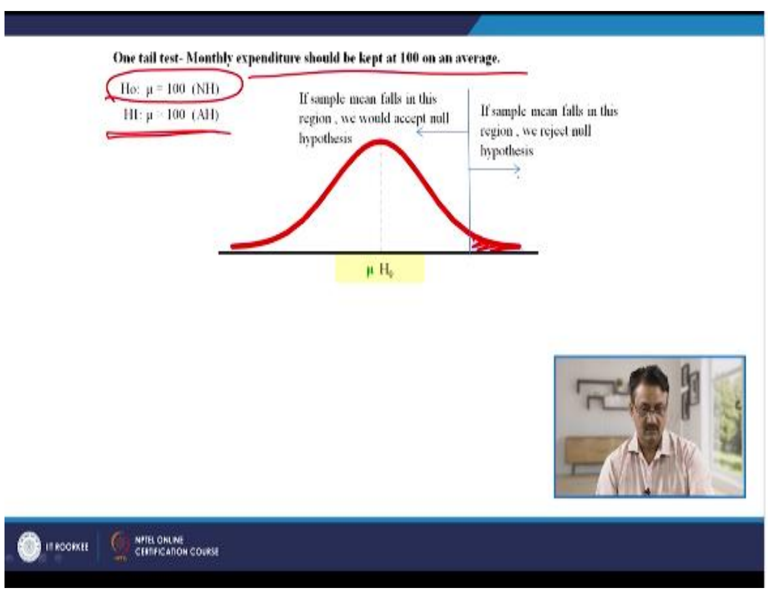

Let us look one more example of one tail test, so let us say monthly expenditure should be kept 100 on an average on certain activity right. So, null hypothesis is that mean expenditure should be 100 and if the mean expenditure goes more than 100 then an action is to be taken right, so alternative hypothesis is this mu greater than or equal to 100 right. So, this is rejection region, so if sample mean falls in this region we will reject the null hypothesis otherwise not, so the remaining steps will see in next class, thank you.