Business Statistics
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Lecture-19
Examples for Standardized Normal Distribution

Good morning friends as you are aware in previous session we discussed some of the characteristics of normal distribution and what are those applications. I have already said that is the most widely used distribution and you also seen how to convert a given variable into a standardized value. Let us workout couple of examples on standardized normal distribution.

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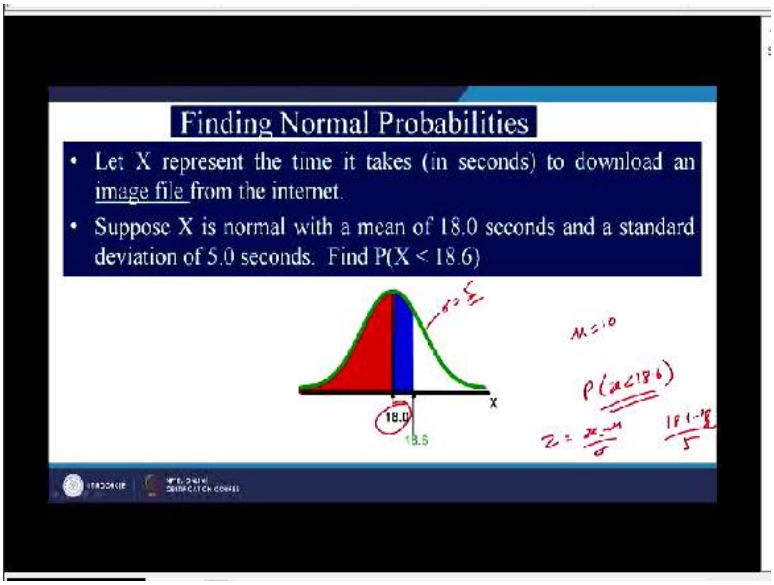

So, let say X represent the time it takes to download an image file from the internet. So, X variable is time, suppose X is normally distributed with mean of 18 which you can see here and standard deviation of 5. So, this standard deviation of 5 right, find probability that the download time for the 5 would be less than 18.6 seconds. Now this what we have to calculate, so for this first of all what you need to do is you just convert X value into Z value. So, you know X= -μ/ σ, X is known which is 18.6, (18.6 -18)/σ is 5 right.

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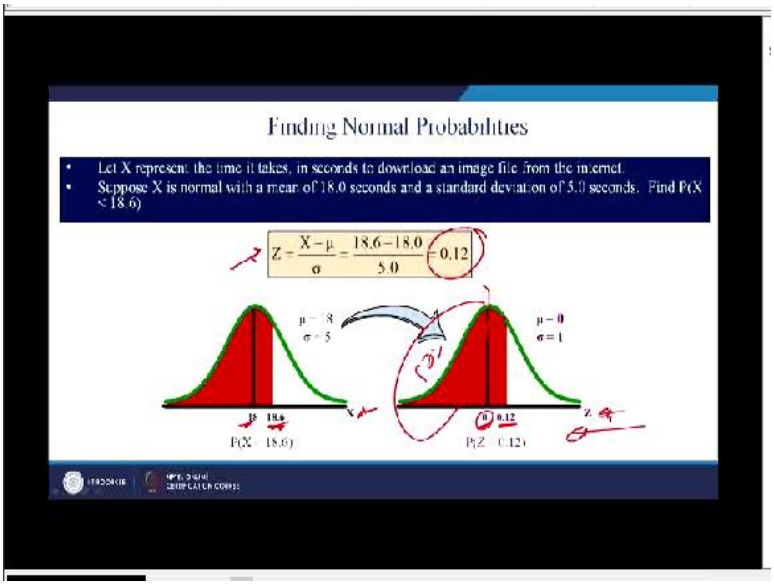

So, this how can calculate this is Z = 0.12, now you have to see the and area under the curve where Z is 0.12. So, we know that and this Z is standardized scale, so this is Z this is 0 Z is 0 and Z is 0.12 which is towards right side of 0. Now just by looking at this distribution itself you can know answer that the answer would be more than 50%. Because this area is 50% is not it and calculate more than that.

So, this is distribution wherein you have been given Z scale and this your original X variable, so mean is this and we have to calculate the probability of downloading the file in less than 18.6 seconds right. So, what we should do we should look at Z table right and will we should see what is the area under curve and Z=0.12.

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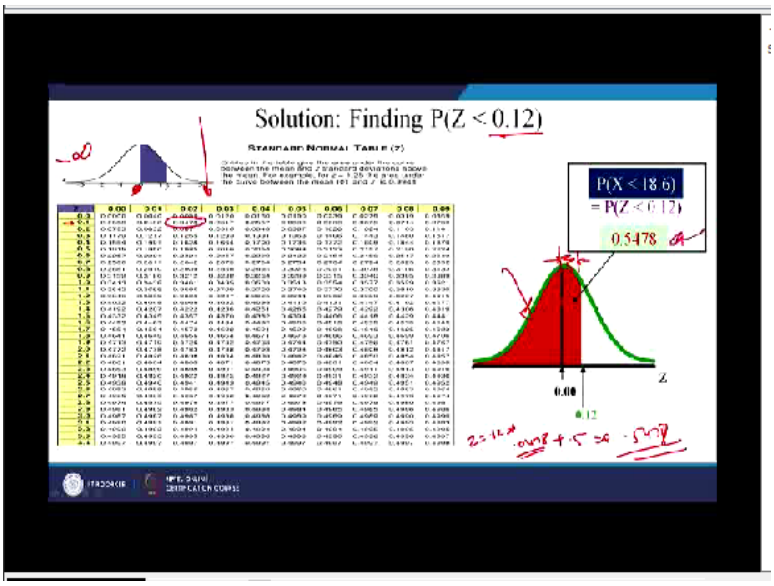

So, look at this table, in this table you have been given area from here right from middle of the distribution not from minus infinity ok. So, Z= 0.12, so this 0.1 and 2 is this right is not it. So, Z is equal to when Z is 0.12 what is the area under curve is 0.0478 right. Now this is the area wish you found from table this area is not it this area now you have to yet the remaining 50% area and this. So, this is +0.5, so this becomes 0.5478 is not it. Now if you had used the other table wherein you had area under the table from minus infinity to Z value then you would have directly got this answer.

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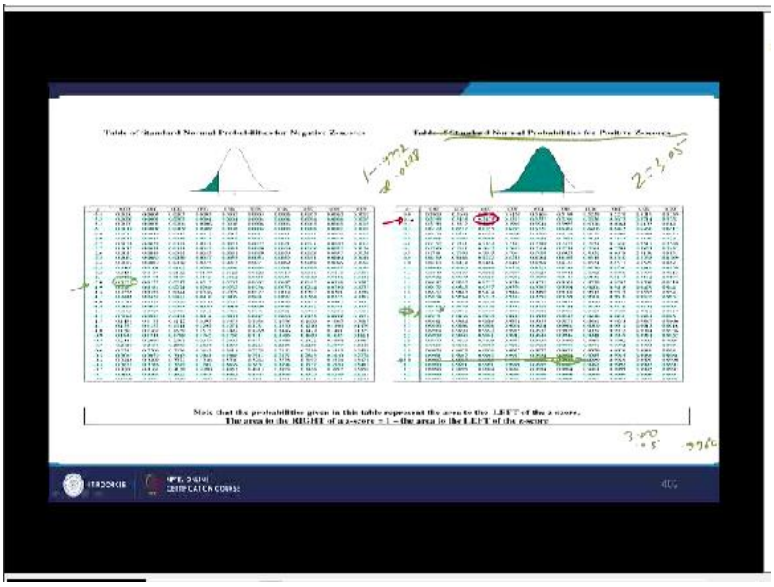

Let us look at that table as well yeah this table right Z=0.12 right, so this is 0.1 and is 2 just see the answer 0.5478, just once again. So, 0.1 and 2 right, so this is this divided 0.5478 which we obtained from this table as well.

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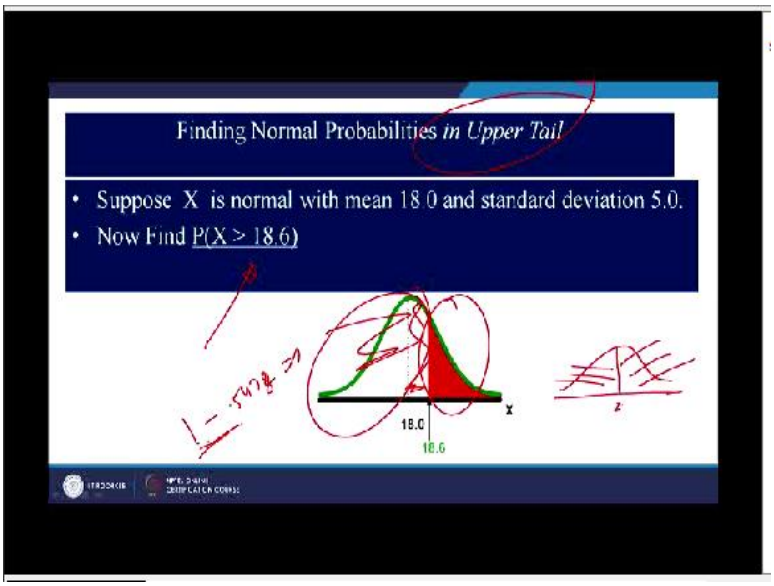

Now let us look at how to find probabilities when the Z values is in upper tail. So, it has got 2 tails right, so we call this one is lower tail and this one has upper tail right. So, where Z values are positive its an upper tail otherwise lower tail right. So, we will take the same example suppose X is normally distributed with mean this and standard deviation 5, find the probability that the image would be downloaded in more than 18.6 minutes.

So, what we just calculated we calculated how much time the image will take to download in less than 18.6, now it is more than 18.6. So, what was the our answer 0.5472, I think this was the answer yeah 0.5478, now if you want to know the answer you can easily just do it like this 1- this value. So, you will get this area because this area is already known to us, this one is 0.54 this one is 0.5478 and this can be calculated just by subtracting from 1.

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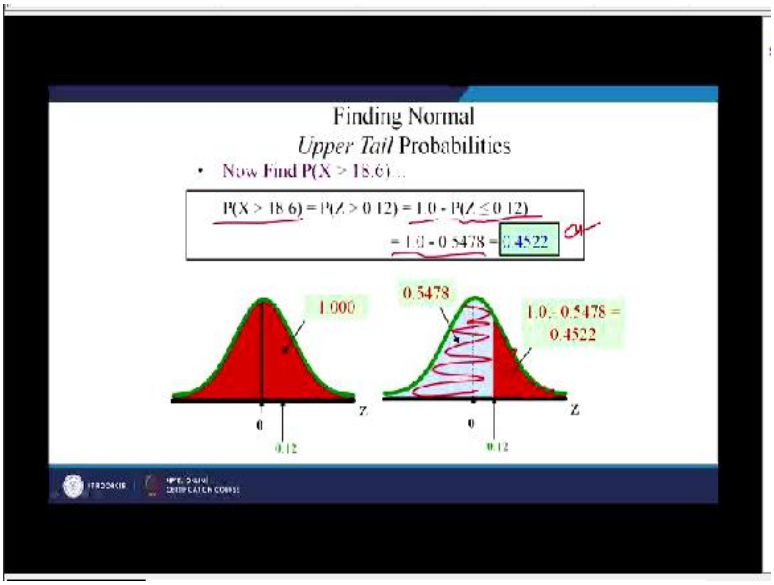

So, this how what we have done right, so probability that it will take more than 18.6 seconds would be 1-probability when z is less than or equal to 0.12, so this is 0.4522 right. So, this is what was the solution in earlier case and we calculated r we found out this area which has by subtracting this area from 1 ok.

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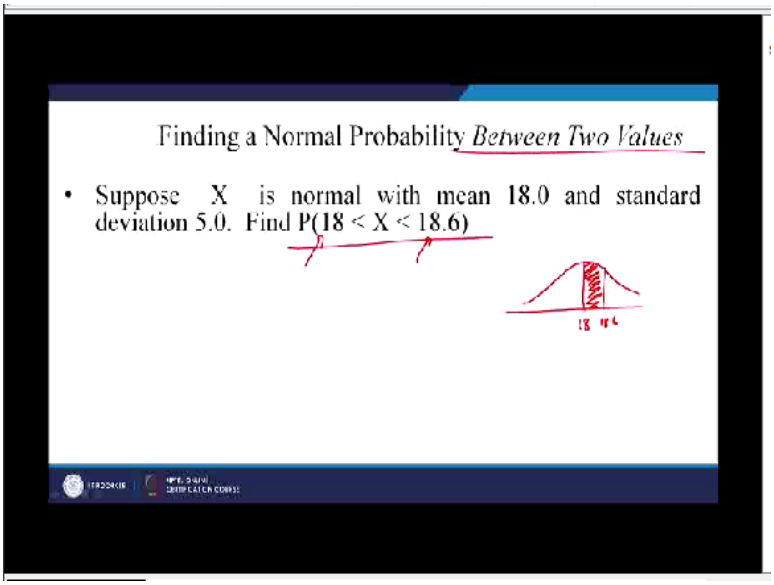

Let us take a situation where we want to find out probability between 2 values not in upper tail. So, we will the same example now was the probability that the image would be downloaded between 18 seconds and 18.6 seconds means more than 18 seconds but less than 18.6 seconds. So, this is your distribution, this is your mean 18 and this your 18.6 right, so we want to find out

this area ok let us so far this what you need to do. You need to calculate first of all convert this 18 and 18.6 into Z value right, so let us do it.

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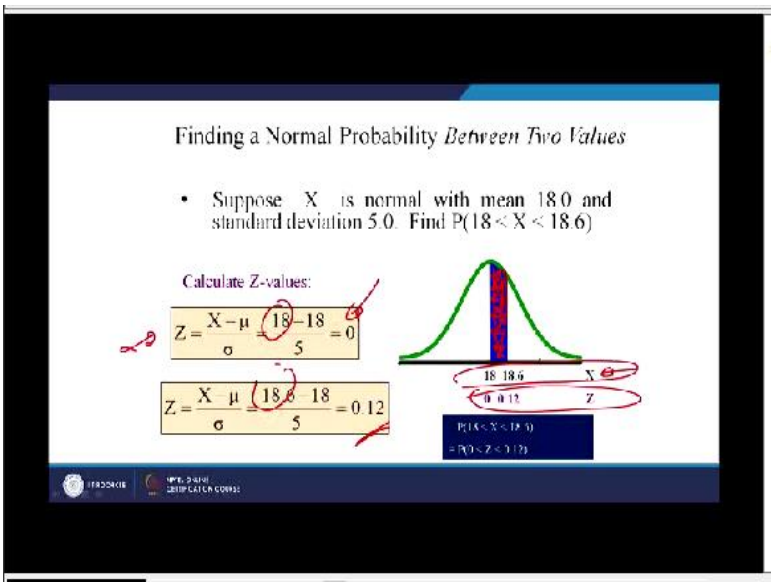

So, when X=18, this is 0 when it is 18.6 this is 0.12, so this is your X scale this one and this one is Z scale isn't it. So, what is the area under particular region, so area from 0 to Z is equal to 0.12, so look at one of the tables, so 0 to 0.12.

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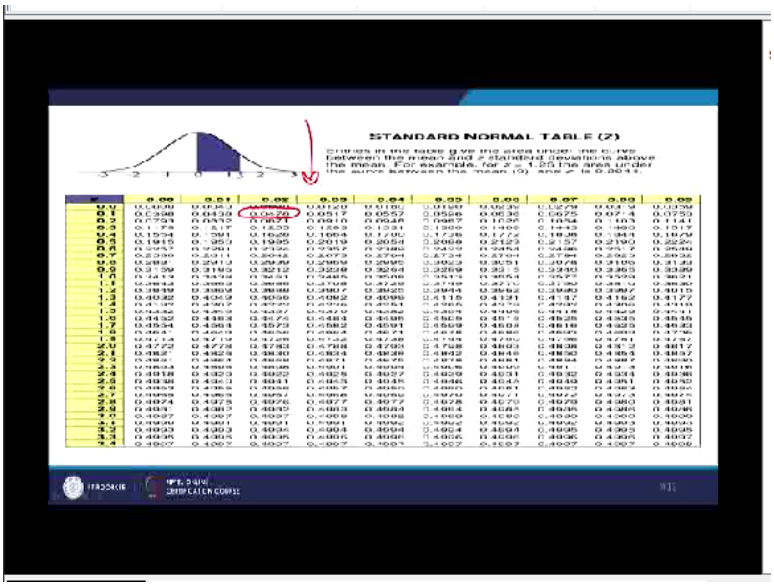

So, let us look at this is the I think right table is not it, so 0 to 0.12, so 0 to 0.12 is 0. right 0.0478 right.

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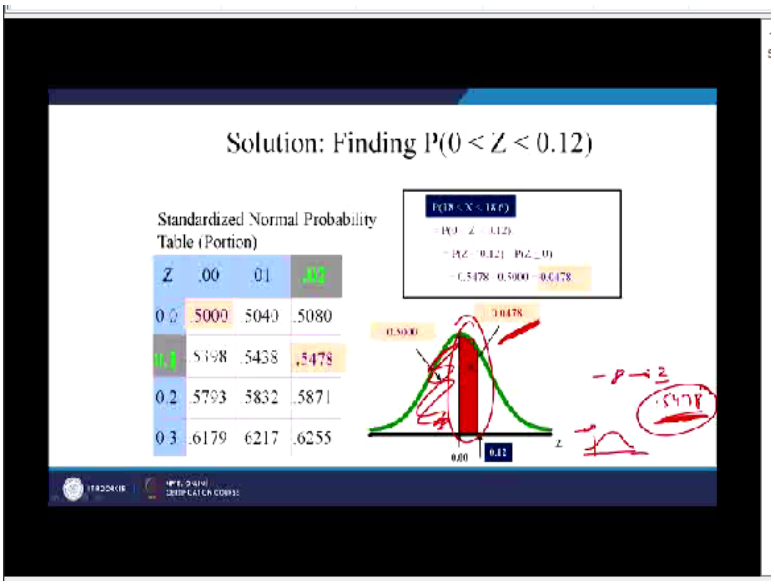

So, this is 0.0478, now if you wish you can do it this manner as well you can have a table from -infinity to Z wherein you will find that the total area would be 0.5478. Let us look at this ok its not there next slide, so if you look at table wherein you have been given Z value from -infinity to some Z value right. You will get area under curve 0.5478 since you know this area and you know this area, so subtract this area then you will know this area right.

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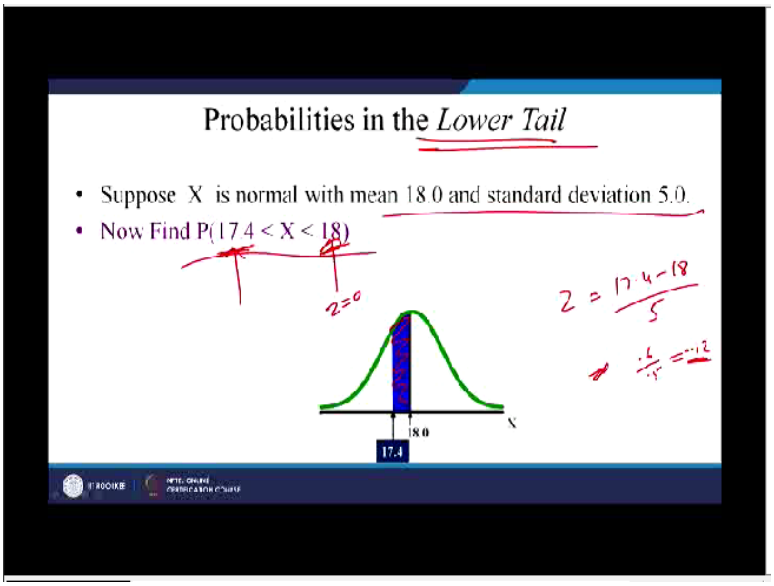

This is very simple example, now we have to calculate probabilities in the lower tail, so you have seen probability in upper tail probability between any 2 points. And now let us look at probability in lower tail, so will take the same example what is the probability that the time

image will take to download between 17.4 to 18 seconds, more than 17.4 but less than 18 seconds. So, this is the area we have to calculate right.

So, first of all what we should do convert these 2 values into Z right, so we already calculated Z for this X=18. So, this was 0, for this what it would be Z = x-μ right. So, (17.4 – 18)/5, so it would be some negative value right is not it. So, let say this 0.6/0.5, so -0.12 is the Z value.

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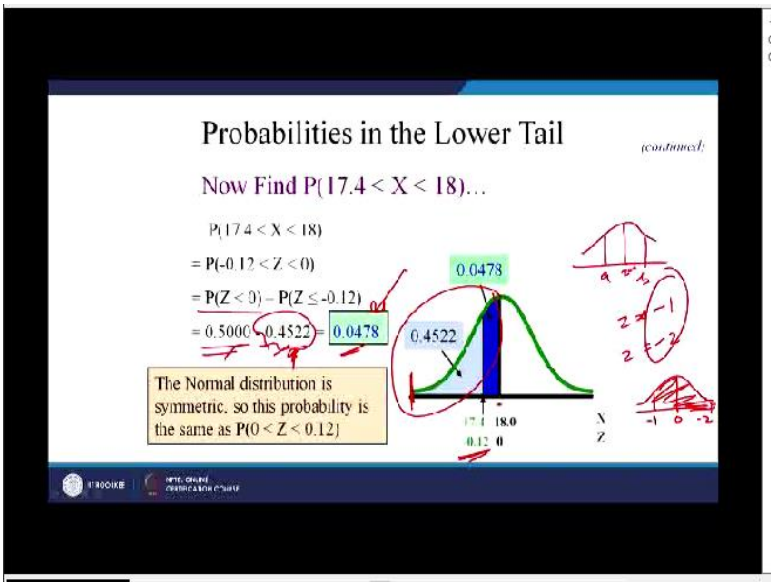

Here you will get -0.12 right, now how to calculate this area, so from 0 to -1. -0.12, so we know that this total area is 50%, this total is 50%. So from this from infinity to 0 areas 50% from 0 to -0.12, this is the area in fact this area is from again -infinity to 0.12. And if you subtract this value from this value will get remaining area, keeping mind that the answer would always be more than 50% in upper tail.

It will always be less than 50% in lower tail and it would be now it can be any probability if it is between a and b right. So, you can have a situation like this where you're a value is **is** towards left **size** side of Z = 0 and your b value can be towards positive side right. So, you can have a situation where Z = -1 and Z = -2. So, area this would be somewhere like this.

So, Z = 0, Z = -1, Z = -2, so this area you can calculate ok, now so the answer to this question would be what is 0.0478.

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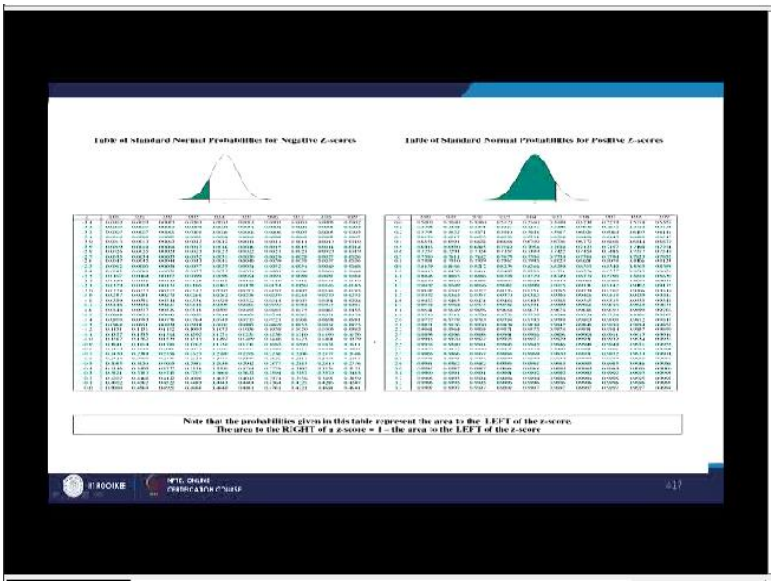

And this answer you can see over here as well in table where Z is equal to what was the Z value- 0.12, so -0.12 this is -0.12 is this 0.4522 and subtract 50 this 2 is subtracted from 50% area right. So, this how you should you get the answer, so this how you should get the answer to this question.

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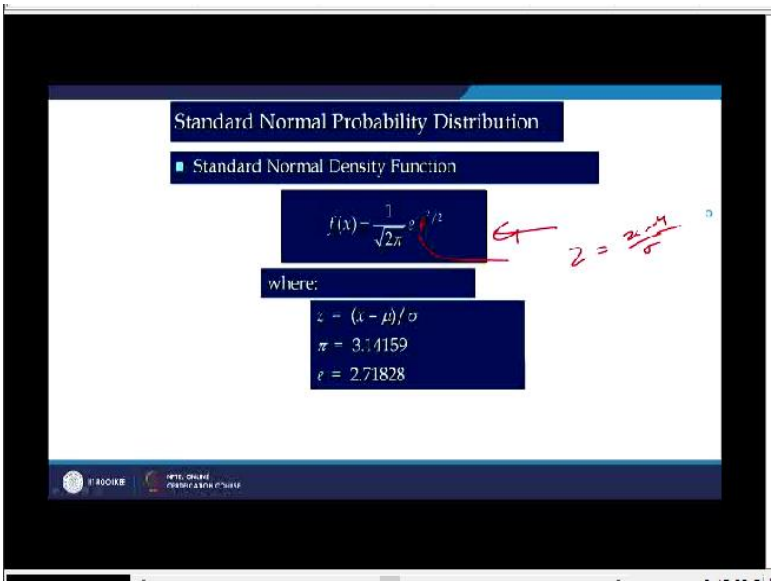

Now let us look at 1 more example on normal probability distribution, so this is the normal density function, I have already told you that this Z is nothing but what is Z, x- mu/sigma right.

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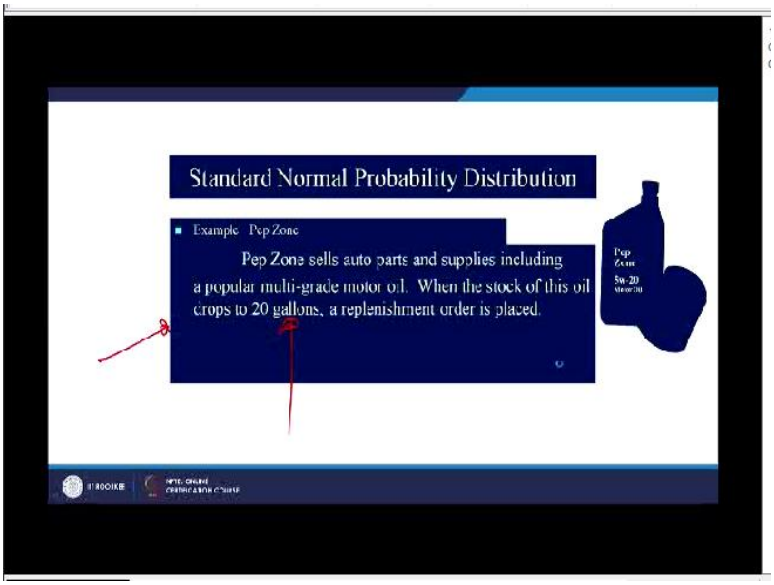

Let us look at this example pep zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallon drops to 20 gallon replenishment order is placed.

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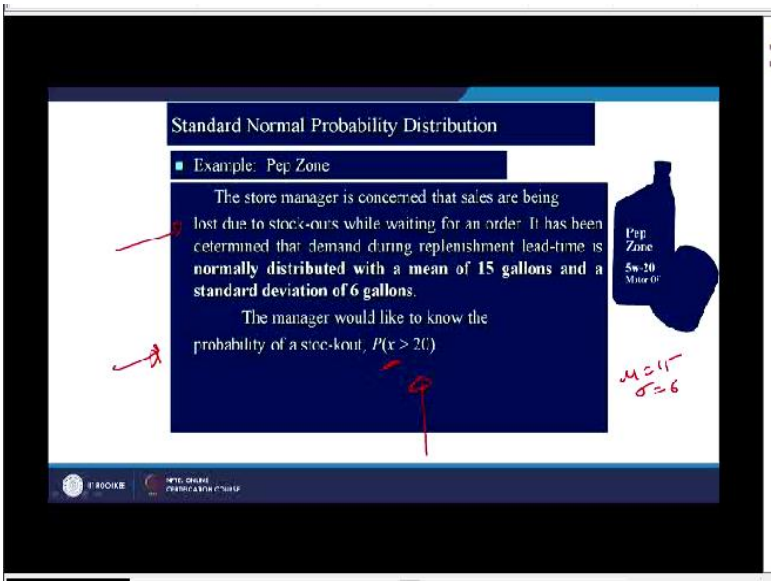

The store manager is concerned that the sales are being lost due to stock out while waiting for an order. Now it has been determined that demand during replenishment lead time is normally distributed with mean of 15 gallons and standard deviation of 6 gallons. So, what is the concerned here whenever you place the order and when you receive the order, so that time is known as lead time.

So, the there is a possibility of a customer not getting oil during this lead time due to stock out. So the manager would like to know the probability of stock out, what is the probability of stock out when the oil is more than 20 gallon right or when the so in this case we need to find out that probability wherein the X value is more than 20, it is very similar to what we have seen in previous one of the examples.

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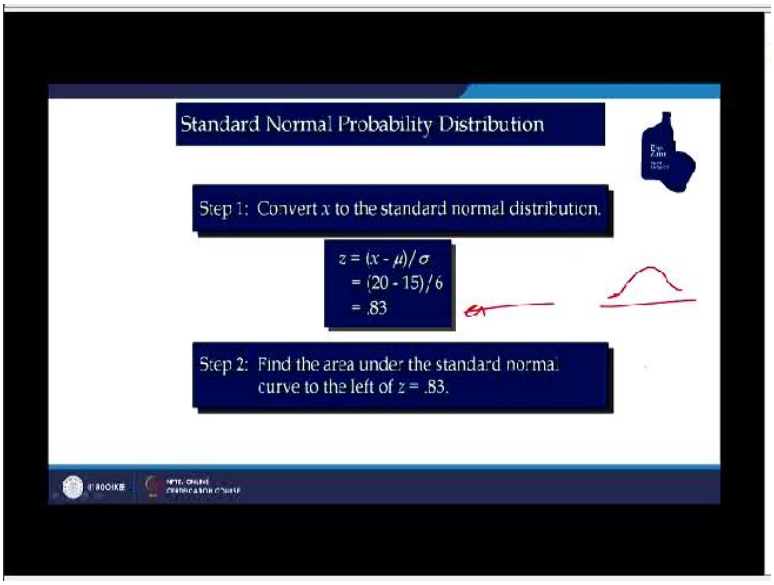

So, first of all convert X into normal distribution, so this is how you should convert, so Z value is 0.8 right. So find area under normal curve with the left of Z right.

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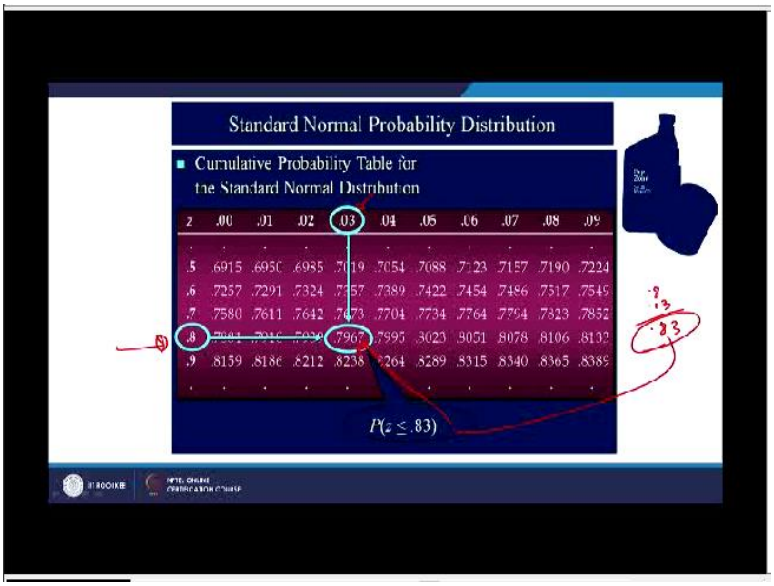

So this how you can look at table, so what is the Z value once again it is look 0.83, so this how you can calculate area under the curve. So, Z=0.8 and 3 right, so just once again 0.8 and 0.03. So it becomes 0.83, this how you should calculate probabilities under curve when Z is 0.83, so this 79.67.

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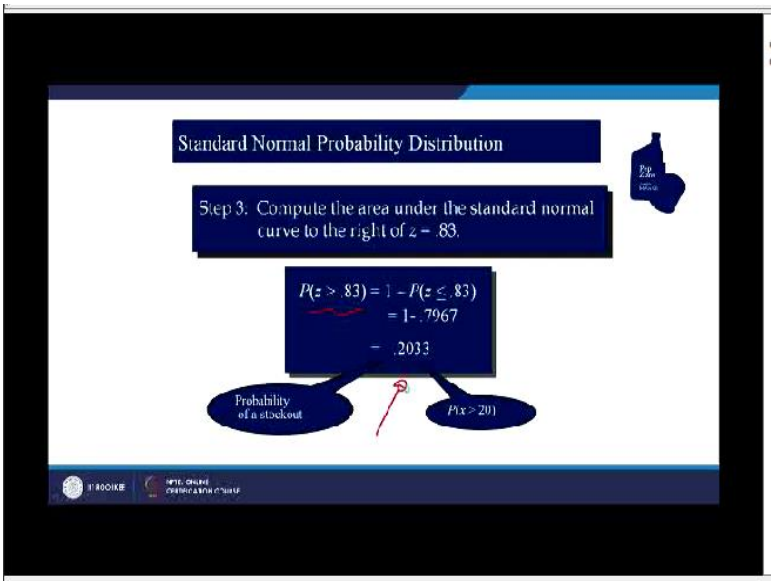

So the area under standard normal curve to the right of Z=0.83 is 1- 0.7967, this is the table value when Z=0.83 when Z is for Z more than 0.3 it is 0.2033. So, probability of stock out is 20.33% right, so this how we have answered our question.

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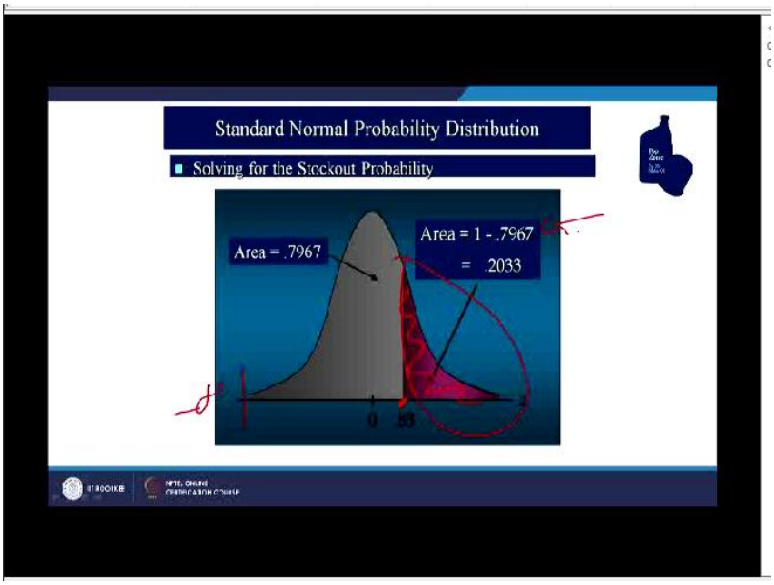

The graphical representation of the same, so what we wanted in fact, we wanted this Z for more than 0.83 right. So, this is this area is more than 0.83 and table value we calculated from minus infinity to this area to do this point right. But we wanted this area right, so 1-0.7967.

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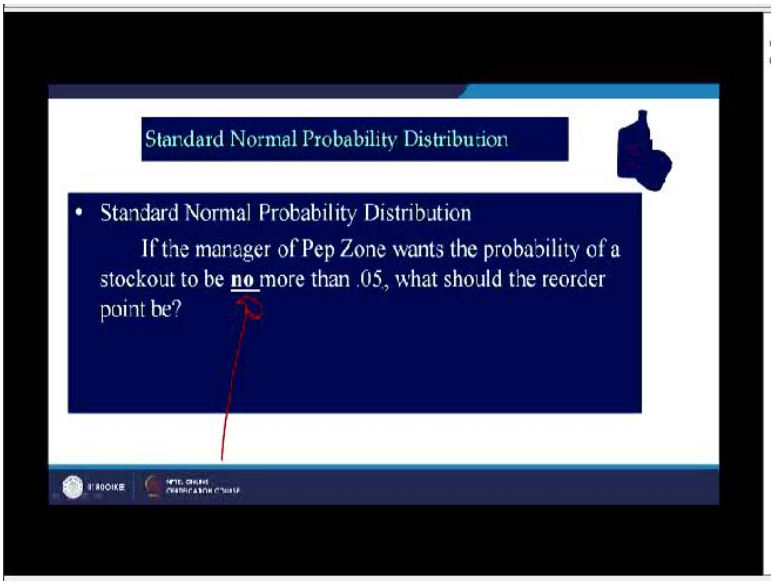

Now if the manager of Pep Zone wants the probability of stock out to be no more than 0.05 here was the probability of stock out is 20% is not it 20.33%.

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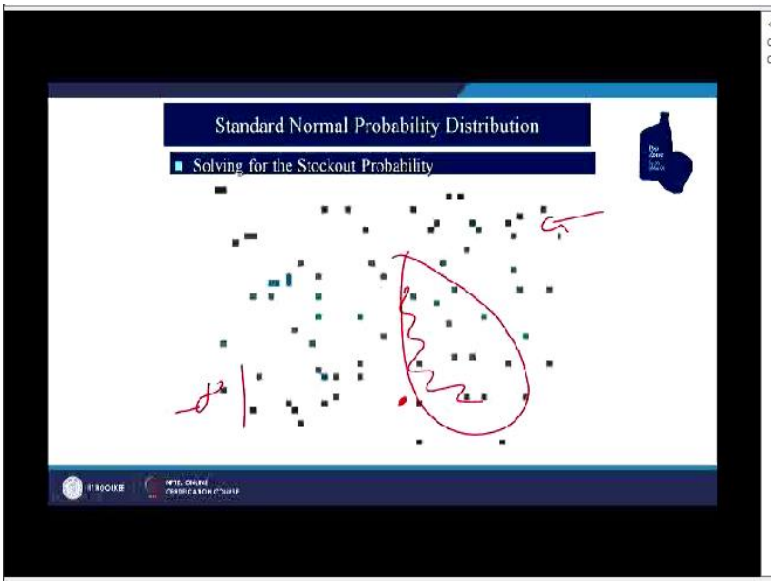

Now the question is if the manager of Pep Zone wants the probability of stock out to be no more than 0.05 what should be the reorder point should reorder point be more than the previous one or

it would be less than that. So, let us calculate that value, so what is the question, what is the probability that the stock out probability becomes 0.05 ok.

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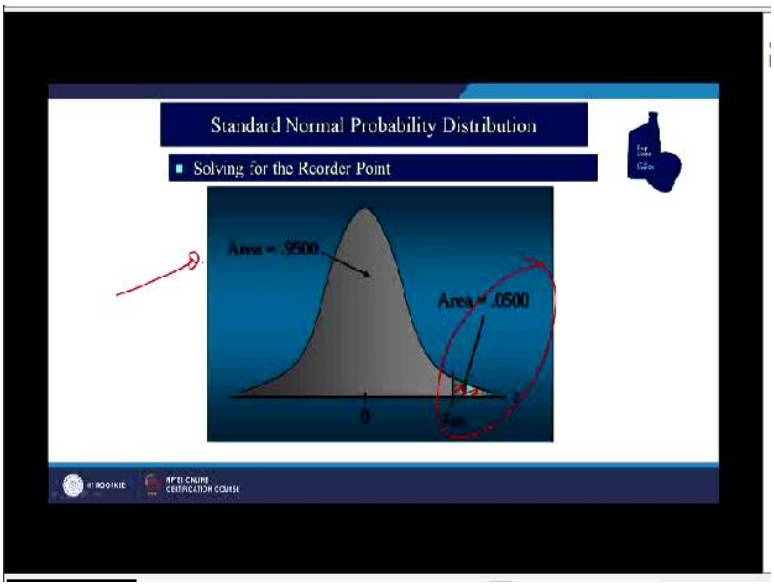

So, area under this curve is unity we are interested in this area right, this is 0 05.

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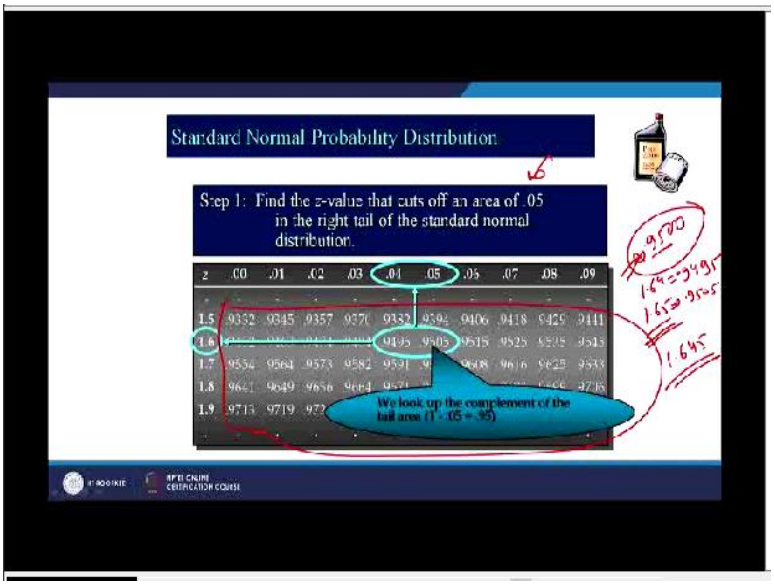

Now find the Z value that cuts of an area of 0.05 in the right tail of the standard normal distribution. So, you need to look at what is the Z value when area is 0.9500, so if you look at this table now you do not do not do not look at the value of Z for the time being looked at in this table and see where area is 0. 0.9500. So, if you look at this it is somewhere here right 1.64, what is the area at 1.64? it is 0.9495 and area at 1.65 it would be 0.9505 what we want?.

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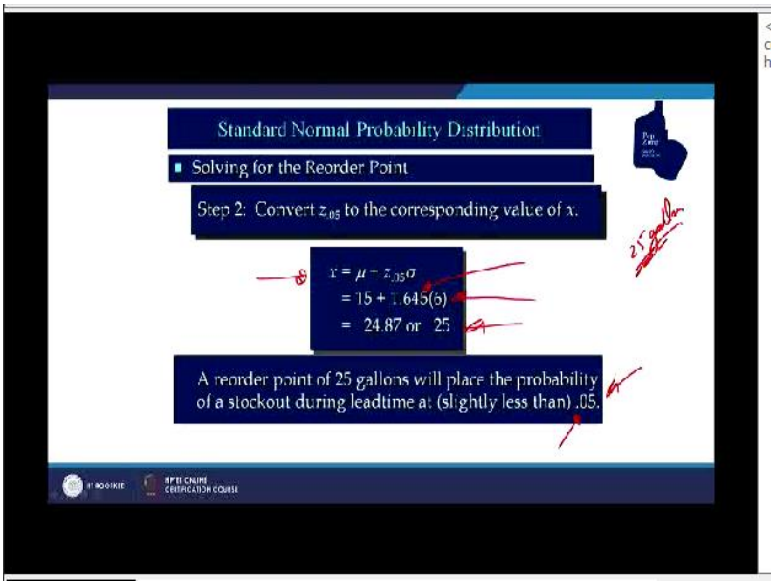

We want get value of Z where area is this ok, so we will convert so we will see that that Z value is nothing but you just take the average of these 2 right. So, that would be 1.645 right, so this is how you can calculate value of Z, just by looking at area, so you can use normal distribution curve normal distribution table for calculating area for a given value of Z. And if you know the area then you can get Z value as well.

So, this how we got Z value, so X now we want to convert this X value is this Z value*X value, so $X= \mu + z\sigma$. We know sigma standard deviation is equal to 6. So, this X value becomes 25, so the reorder point of 25 gallons will place the probability of stock out during lead-time at it should be less than the previous case right. So what is the conclusion at the end of the day see if you keep reorder point let us say 25 gallons.

So, whenever you reorder this particular oil you should reorder 25 gallons of it not the 21 otherwise there would be the probability of stock out higher than this. Now this is just 5% is not it earlier it was some 20 + percentage isn't it. So, this how you can convert Z value into X value, so what will say by raising the reorder point from 20 gallons to 25 gallons on hand the problem of stock out decrease from 20% to 5%.

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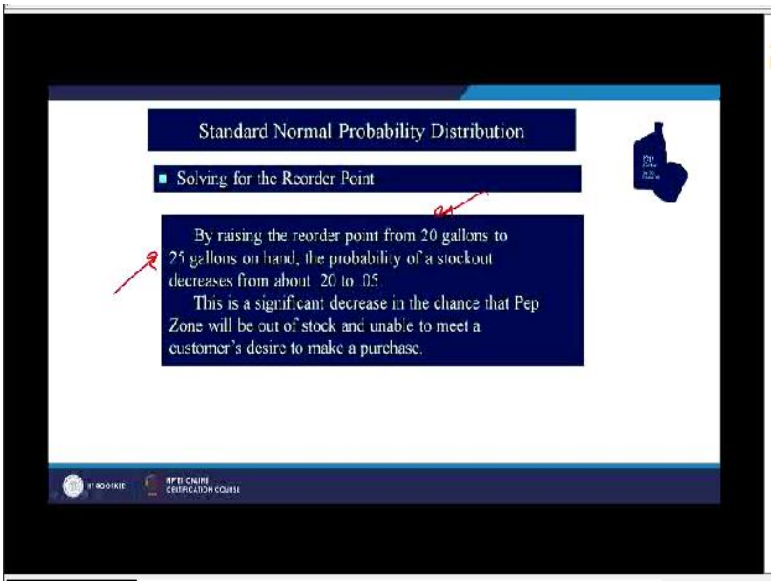

And this is significant decrease in the chance that pep zone will be out of stock and unable to meet customers demand during lead-time.

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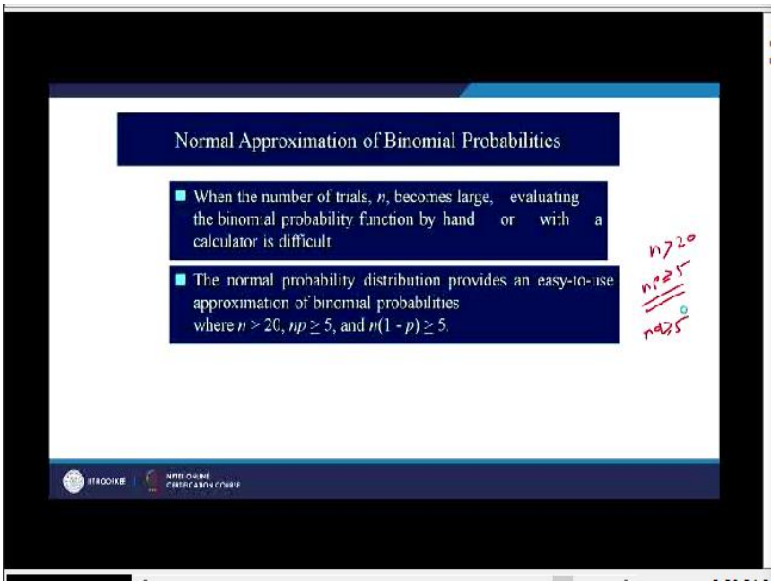

Now let us look at normal approximation of binomial probabilities, we have seen in case of discrete distribution we have used poisson distribution instead of binomial distribution why did we use poisson distribution. Because in Binomial distribution you got very difficult calculations are very tedious. So under certain conditions we could approximate percent distribution as binomial distribution and what are those conditions.

The n has to be large is equal to or more than 20 and P has to be very small right is less than or equal 0.05 right. So, similar to that we can also approximate binomial distribution with normal distribution, so rather than using binomial distribution you can use normal distribution. When the number of trials n becomes large evaluating binomial properties function by manually or with calculator is difficult the normal probability distribution provides an easy to use approximation of binomial probabilities under this condition.

So, when n is equal to more than 20 np is equal to or greater than 5 and np this is nq right. So np greater than or equal to 5, nq greater than equal to 5, and n greater than 20. In this situation you can always use normal distribution instead of binomial distribution, so let me summarize what we did in this session. In this session we have calculated different probabilities when whether the probabilities were under upper tail or lower tail or in between.

You need to keep in mind that you should convert the given X value properly by using the Z formula into Z value. And then you need to area under the curve from Z table carefully and you need to be very sure when you have to subtract area from which remaining area is not it. Sometimes you need to area in 50% of the area sometimes you need to subtract area from 50% of the area.

Sometimes you have a situation where you are finding area from left side to the 0 and area from right side to the 0. And then you are adding both the areas now in all these circumstances it is very necessary to add and subtract areas. So, with this let me come to the end of this particular session. In next class will see how normal approximation or how normal distribution can be used to solve those questions which we can solve using binomial distribution. But in binomial distribution you will have to perform lots of calculations, so thank you very much.