Business Statistics

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Lecture – 15
Probability- Part 3

Good afternoon friends I welcome you are in which session as you are aware in previous session we discussed probability and different types of probabilities. We have seen Probability under statistical Independence and Probability under statistical dependence. So, far as Probability under statistical dependence are concerned we have got conditional probability, we have got joint Probability and we have got marginal probability. So, we know that the conditional Probability and statistical Independence is this.

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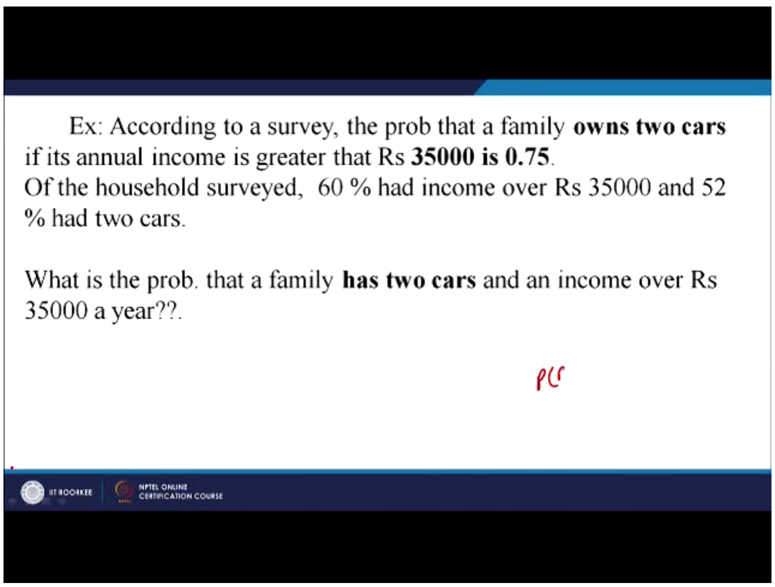

What is the probability of event be given that the event has already happened so this is probability of BA divided by probability of A, right? So this is conditional Probability under Statistical dependence. If you want to calculate the joint probability under statistical dependence is you just multiply this two term you will get this right and what is marginal Probability under statistical dependence it is the sum of all joint probability having single event ok.

And we have also worked out couple of examples on this ok. Today we will see in couple of more examples on statistical dependence that is conditional Probability under statistical dependence. So, let us look at this example according to survey the property that a family owns two cars if its annual income is greater than 35000 rupees is 75%. So, probability of having two cars is 75% having income greater than 35000.

Out of the household survey 60% had income over 35000 and 52% had two cars. Now what is the probability that the family has two cars given that income is over 35000 so this is the case of statistical conditional probability under statistical dependence?

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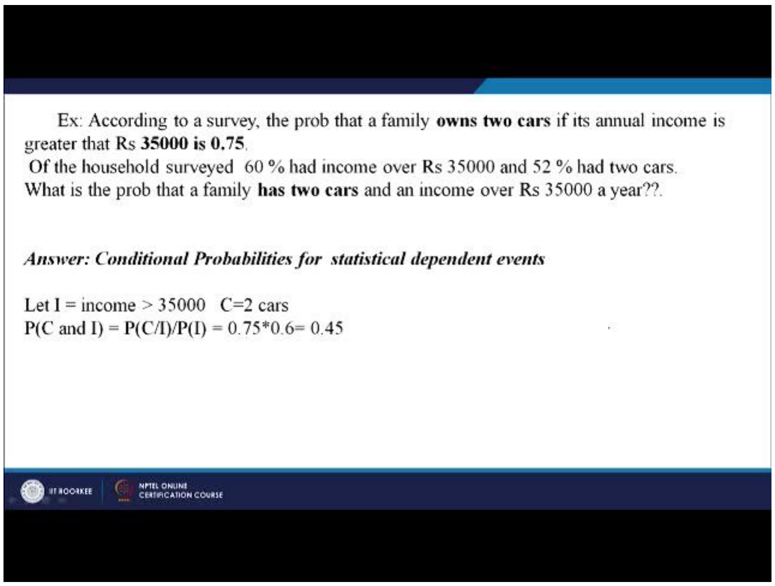

Let us look at this let us say more than 35000 C stands for 2 cars and I stands for income so probability that the family has 2 cars and income more than 35000 is this. So, given that income is more than 35000 and probability of car is C right. So, this would be you just multiply this two values right. So, this is how you should get answer to question like this right.

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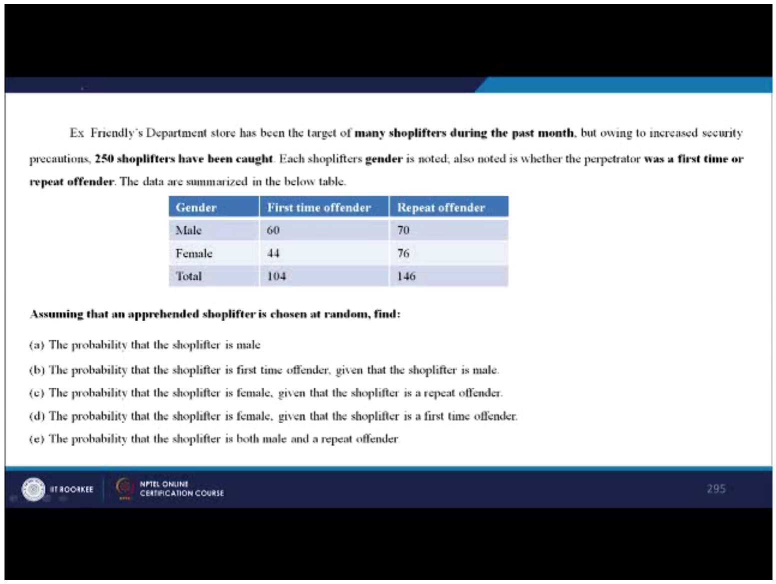

Let us look at one more question. Friendlys Departmental Store has been the target of shoplifters during past month owing to increase security precautions, 250 shoplifters having caught because of having security in place. Each shoplifter's gender was noted and it was also noted that whether the shoplifter was first time of offender or repeat offender. So, this is the summary. So if you look at this total genders total males 130, total females this is; it is 120 and this is total 250 right. So, 120 130 and this is total 250 right.

Now assume that apprehended shoplifter is chosen at random find the probability that the shoplifter is male. So, is the probability that the shoplifter is a male, you have got total how many males 130 out of all the shoplifters right so it is 130 by 250. Now second question is the what is the probability that the shoplifter is first time offender given that he is a male or the shoplifter is male. What is the probability that the shoplifter is female given that shoplifter is a repeat offender and probability that the shoplifter is female given that shoplifter is first time offender and shoplifter is both male and repeat offender. So, let us look at solution to this question.

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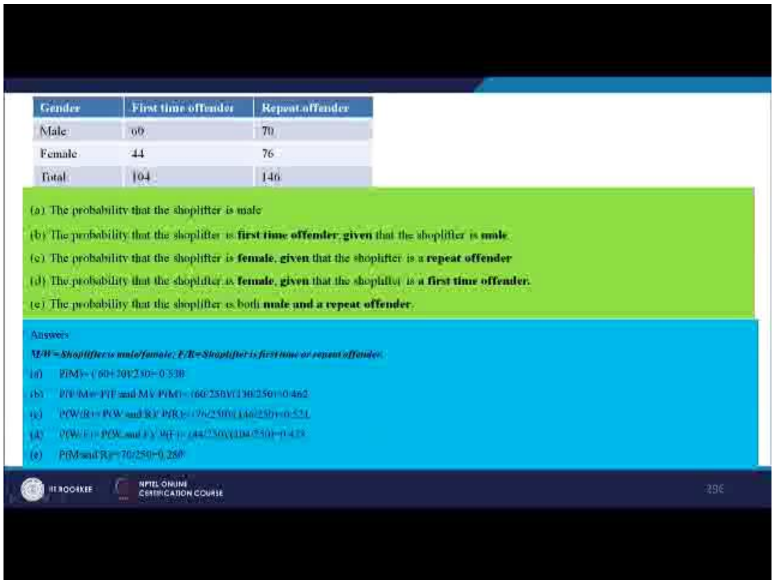

So, first part we already calculated it the probability that shoplifter is first time offender and male. So, these are the notations so shoplifter male, female right. First time shoplifter and repeat offenders this is how we done this symbols. So, probability that this shoplifter is male and first time offender right this is what we have to calculate. So, this is probability that the shoplifter is first time and male given that he was male right. So, this is the answer to this question, 60 by 250, 60 by total 250 total of these two divided by 130 all the males right with the first time offender or repeat offender all male.

Shoplifter is female given repeat offender so female you have got 76 repeat offender and 146 these are total repeat offenders. Ok the next one is shoplifter is female given that the shoplifter is first time offender, female and first time offender. So, this is how you should calculate. First time offender you have got 104 first time offenders and females are 44. Then shoplifter is both male female 70 by 250 this is how you should answer question like this.

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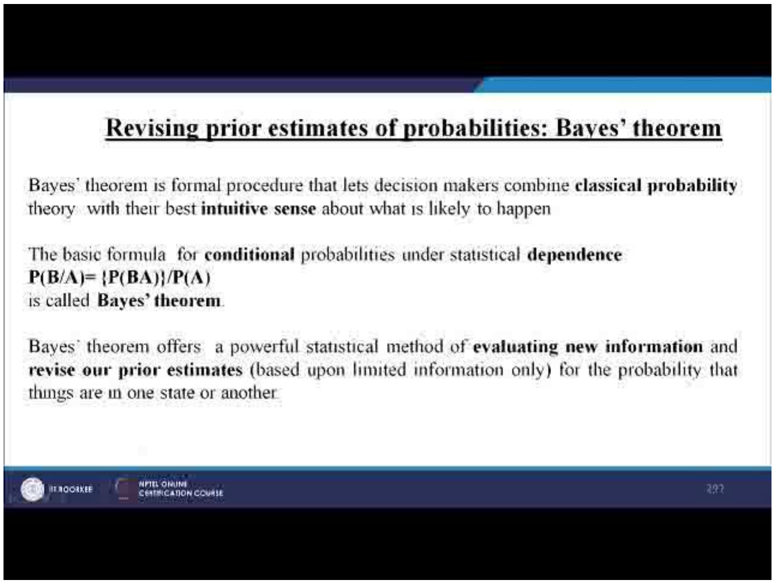

Now look at Bayes Theorem it is quite an important theorem so far as decision making process is concerned because what happens in real life situation whenever you make a decision you make the decision on the basis of information and in real life what happens initially what happens all the necessary information. So, whatever is information is available to you make decisions as per that. And over a period of time you keep getting some more information and then you would like to revise your probabilities of happening something or not happening. So, that is the application of Bayes theorem; it is also known as post priory probability right.

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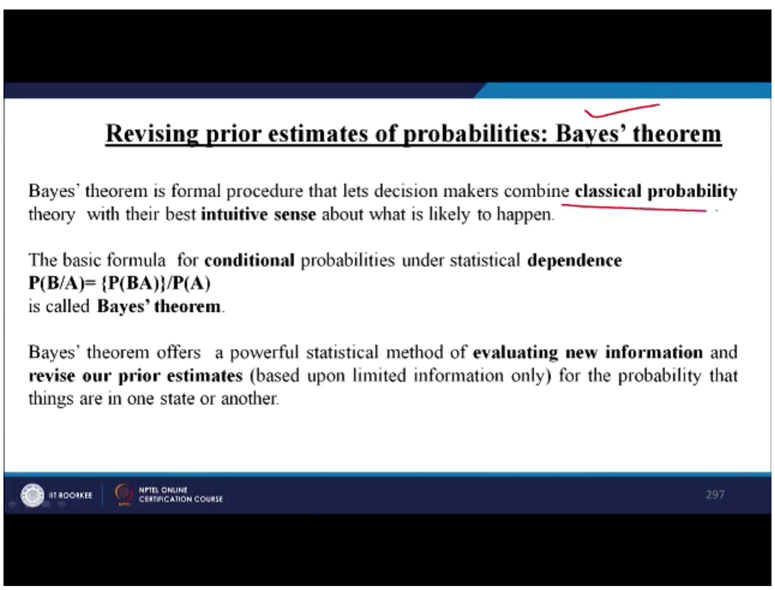

So, it is a perfect case of where we combine classical probability theory with subjective probability right with intuitive science and with some classical probability and with some information we make certain decision. We know that the conditional Probability under statistical dependencies this and this itself is called Bayes Theorem. So, what is Bayes theorem is the conditional probability under statistical independence. So, it is a powerful statistical tool for evaluating new information and revising our prior estimates of probabilities.

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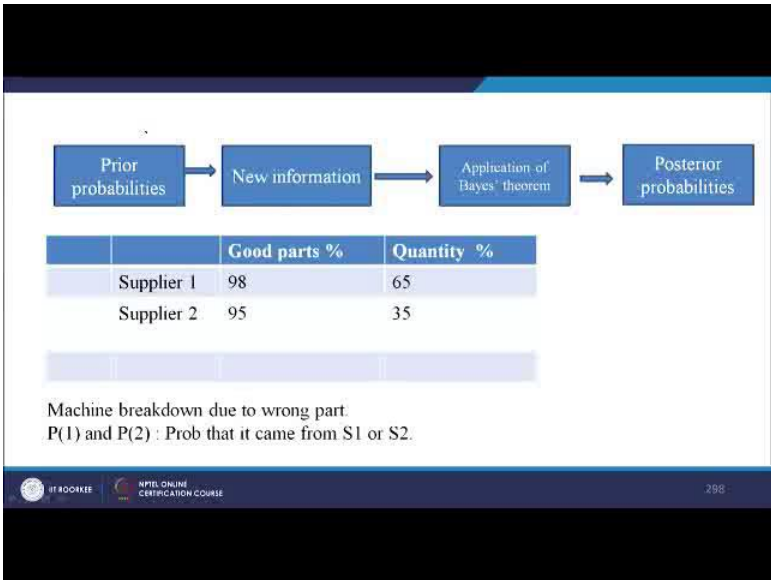

	Good parts %	Quantity %
Supplier 1	98	65
Supplier 2	95	35

So, how it works so initially you have got prior probability you have your calculator prior probabilities; probability of some events and then you get some more information then you apply Bayes Theorem and then you get output, posterior probability. So, let us take an example let us say you decided that the GDP in next financial year would be 8% DGP would be 8% or 8 point. Now after sometime of this estimate you have you have got some more information let your rupee has strengthened, oil prices have come down, your inflation has come down, you have good ties with your neighbouring countries.

Let us say you have got the public sector company starting doing well and so on. So, once you get all this information you try to revise this estimate and then you will say that the next years GDP would be 9 not 8. So, the growth rate of GDP would be 9 not the 8, so this how you can apply Bayes Theorem. So, will take up an example let us say you are manufacturing a part and for manufacturing that part of that product you have got 2 suppliers one was supplying you materials right.

So, there is a supplier supplying 65% of the quantity and the very supplier to supplier 35% of the quantity and we know from the past experience that supplier 1 supplies 98% good parts it means 2% parts are not good and in this case 5% parts are not good right. So, let us say you have got material from these two suppliers and you have started working on those parts and when you start manufacturing then you found that machine broke down due to some wrong part.

The wrong material which you might have received from one of them right, so we have to find out what is the probability that the quality part belongs to supplier 1 or supplier 2 we need to find out problems that the part came from supplier 1 or supply 2.

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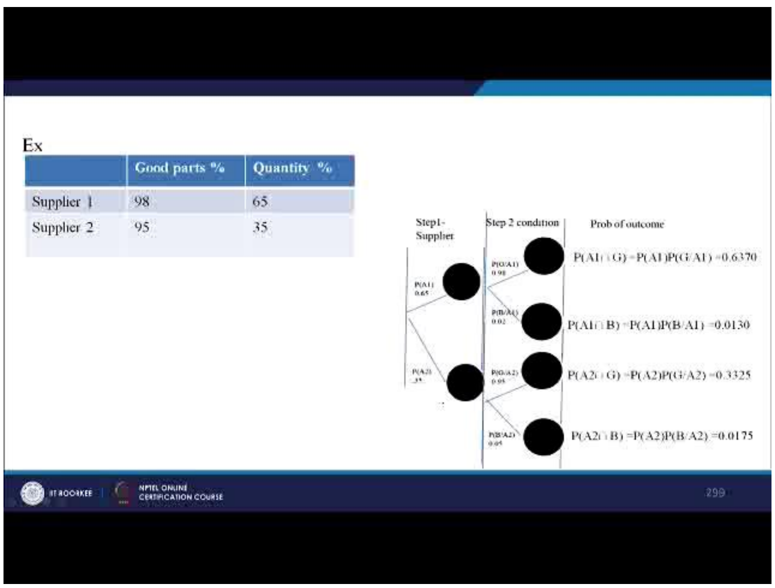

So let us look at this tree diagram, so you are getting probability of quantity is 65% you are getting from supplier 1 so P(A1) nothing but supplier 1 from supplier 2 your getting 35% quantity right now from supplier one good parts are 98% so .98 so probability that part is good and from supplier 1 is 98% just opposite to this would be what probability of bad part from supplier 1 just 2%. Let us look at second supplier.

So, from second supplier probability of good part is 95% right and bad part the probability is .05 or 5%. Let us look at probability of outcome, so what is the probability that the part would be good and coming from supplier 1 you just multiply these 2, .65 and .98, .6370 what is the probability that the part is bad and coming from supplier 1 and you just multiply this 2 .65 and

98 will get .6370 what is the probability that the path is bad and coming from supplier 1 so just multiply this and this so .65 * 0.2 hours .10308 probability that the path is good coming from second suppliers is .3325 and bad parties bad coming from supplied 2 is .1075 right.

So this is how you can calculate good part, bad part coming from supplier A and supplier B or supplier 1 and supplier 2.

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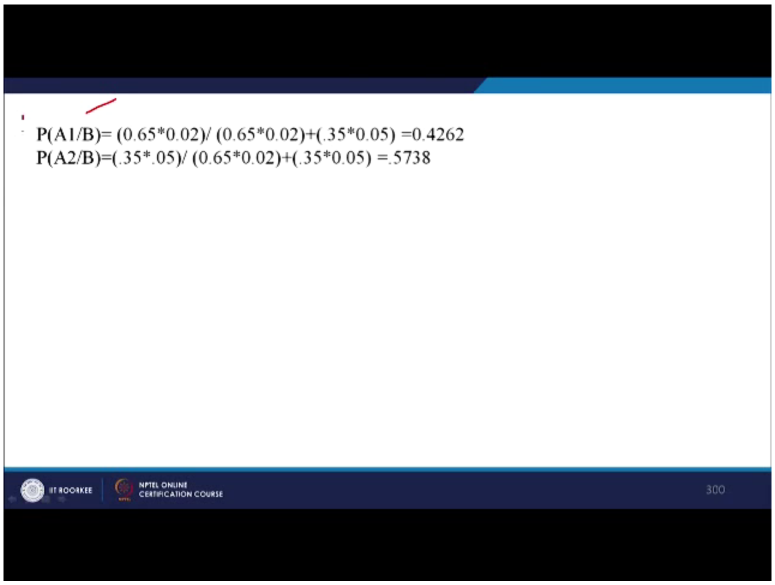

So, what is the probability that the part that this equation what is the probability is that the part which has come is bad from supplier one for this just to multiply this two divided by this summation of this one. So, from A1 you are getting how many parts from, A1 you are getting 65% of the parts right, please check this from A1 65% and how many bad parts 2% from supplier 1. From as far as supplier 2 is concerned.

What is the probability that the part is bad and coming from supplier 2, so, either you call it S2 or A2, one and the same thing. So, you are getting total 35% of the parts and 5% are bad divided by total bad parts getting from supplier and total bad part getting from supplier B this is what we have done in first case. So, let total parts received multiplied by bad parts and divided by the total parts and bad Probability from supplier A and total parts from supplier B and probability of bad parts so this how you answer the question.

So, what is the final answer the probability that the part is bad and which has come from supplies A is 42.62% and 57.38%.

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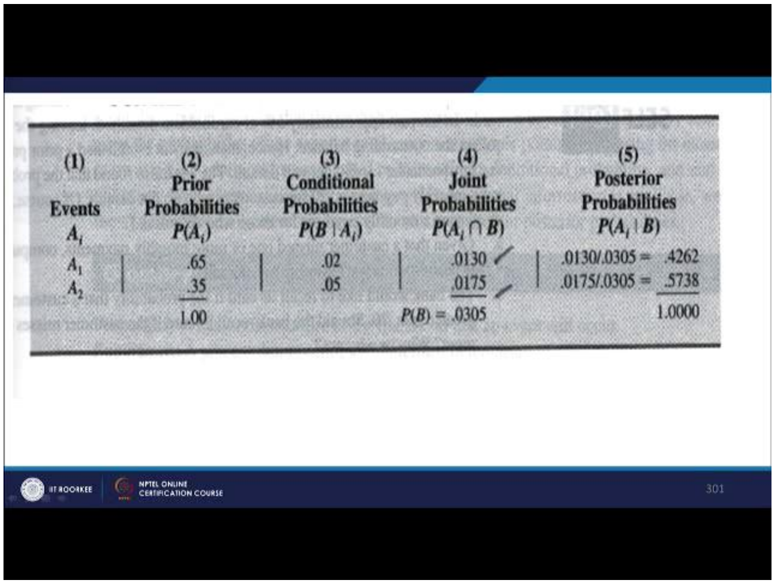

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B \mid A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5) Posterior Probabilities $P(A_i \mid B)$
A_1	.65	.02	.0130	.0130/.0305 = .4262
A_2	.35	.05	.0175	.0175/.0305 = .5738
	1.00		$P(B)$ = .0305	1.0000

Same answer can be found using tabular approach so we have got prior probability is 65% from supplier 1, 35% from supplier 2 so you are getting total 100% parts right. Bad parts conditional probability So, this the probability of bad part coming from supplier A is 2% from supplier B is 5% so you just multiply these two the bad probabilities summation of these 2 is .305 divide this value by this number and their this value by this number do you get 42.62 and 57.38 percentage.

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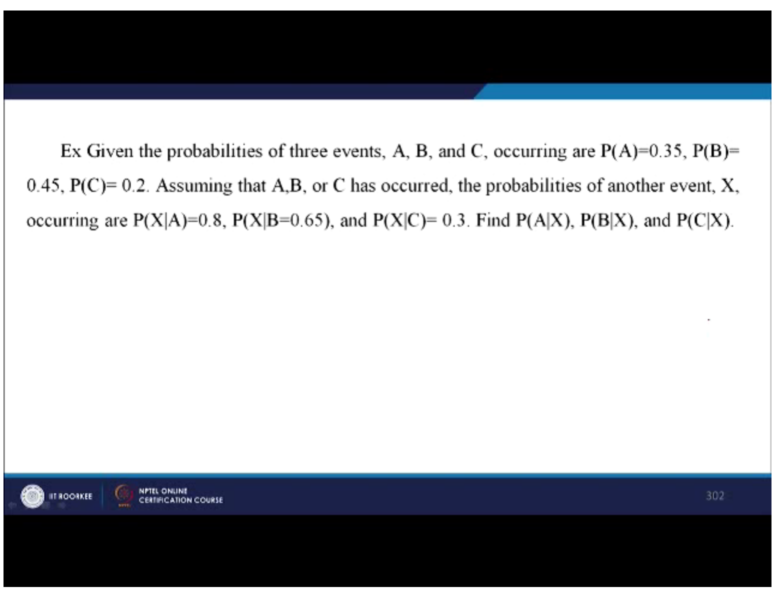

Let us look at one more examples given the probabilities of three events A, B and C occurring are 35%, 45%, 20%. So, this total is 100% right assuming that A, B and C are has occurred the probability of another event X occurring are so probability of event X occurring given that A has already occurred is 8% this is 65% and this is 3%. So, find probability of A given axis happened B given axis happened probability of C given axis happened. So, we have got probability of X even a happened but what we have to find out we have to find out what is the probability of A given X has happened. So this is how you can solve it in tabular form of event A B and C probability of X given A B and C happened right.

So, probability of X happened is just multiply these values. These two is this multiplication of these two is this now take the summation of all the three values now this value divided by this total this value divided by .6325 and so on right. So, this is how you can get the answer. Now after working out so many examples on conditional Probability under statistical dependence after going through several other points related to probability and types of probability. Let us workout couple of examples.

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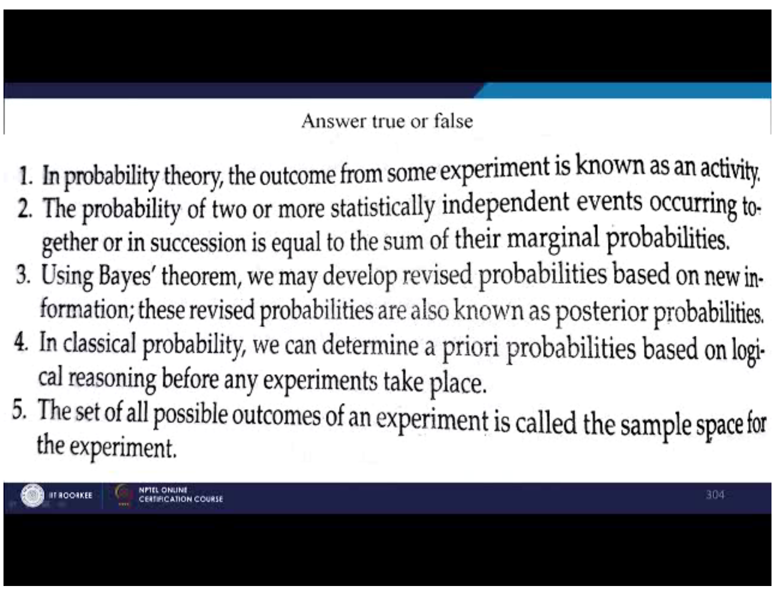

So, this is our exercise, so these are couple of statements you have to identify which one is true which one is false right. Let us look at first in probability theory outcome from some experiment is known as an activity is false not known as activity but it is known as either you call it sample space or outcome right. The probability of two or more statistically independent events occurring together in succession is equal to the sum of their marginal probability, is it? No this is false ok. Using Bayes theorem which we have just seen in this session using Bayes Theorem we may developed revise probabilities based on new information these revised probabilities are also known as posterior probability, yes this is true.

So, prior probabilities and posterior probability in classical probability we can determine prior probabilities based on logical reasoning before any experiment takes place so this is true yes. There is no need of any experiment in case of classical probability. The set of all possible outcomes of an experiment is called sample space yes or no, yes it is true. Let us look at this point under statistical dependence and marginal probability may be computed for some simple events by taking the product of probabilities of all join events in which the simple event occurs.

No this is false, this is not the product; it is a sum of the properties of all joint events in which simple event occur so this is false. When a list of events resulting from some experiment includes all possible outcomes the list is said to be collectively exhaustive, yes it is true, right. In conditional probability is known as marginal probability yes it is true, unconditional probability is also known as marginal probability that it is true.

A subjective probability maybe nothing more than an educated guess yes it is, so if you are a experience person you can estimate the probability of happening of an event. So, when the occurrence of some events has no effect on the probability occurrence of some other event then the two event are said to be statistical Independence, yes it is correct. There are statistically independent. When using the relative frequency approach of the probability, probability figures become less accurate for large number of object is it so, no it is not it becomes more accurate so this is false 11th is false.

Symbolically marginal probability no it not is denoted by P(AB) it is just typed either P(A) or P(B) right so this is false. If A and B are statistically dependent events probability A and B occurring is this 13 is false, no this not correct. Classical probability each of the possible outcomes yes it is true.

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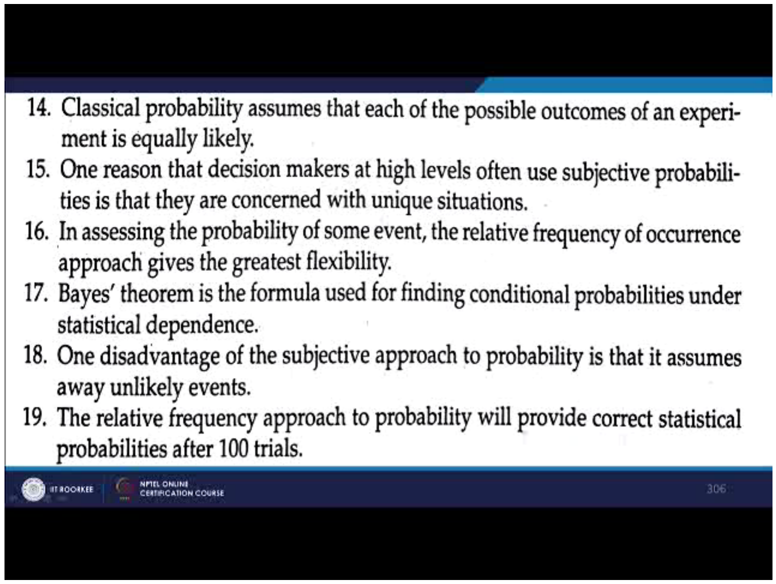

Classical probability assume that each of the possible outcomes of an experiment is equally likely as it is true. One reason that decision maker at high level of a new subjective probability is that they are concerned with unique situation, yes it is true. Because for management most of the times they will face unique situations that they do not have data of those kind of situations right past data are not available right so they go for subjective approach.

In assessing the probability of some events the relative frequency of occurrence approach using greater flexibility 16th number is false. In assessing the probability of an event the relative frequency of occurrence approach use greater flexibility, no this is false. Actually what happens when you repeat experiment for larger amount of time then accuracy increases? So, it does not give you greatest flexibility. Bayes theorem formula is used for finding conditional probability under statistical dependence, yes it is true.

One disadvantage of subjective approach to probability that it assumes away unlikely event so this is false, no in case of subjective approach a person can think of likely events in unlikely events this is false. This is an advantage of subjective probability is not a disadvantage right would had been advantage in this question then it would have been true statement right but it is unlikely and disadvantage then it is false right.

The relative frequency approach to Probability will provide correct statistical probability obtained, this is false ok.

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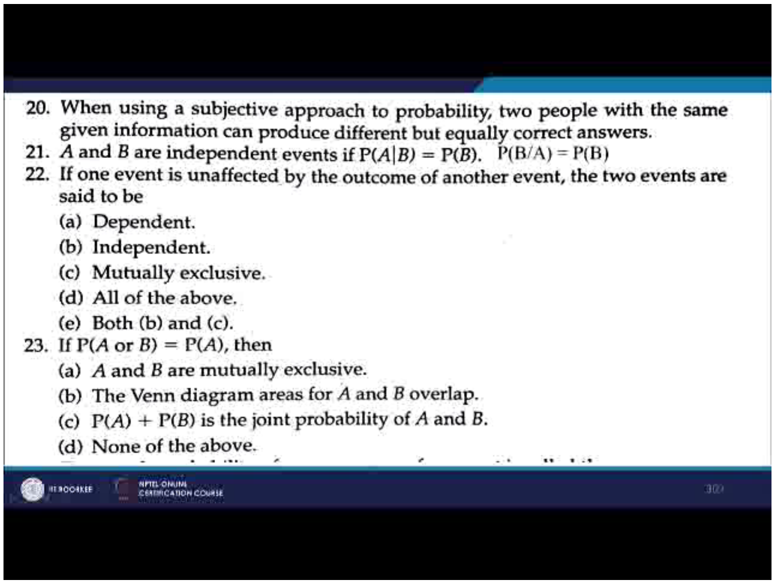

Let us look at next question when using a subjective approach to probability two people with same given information can produce different but equally correct answer yes it is possible, this is true. So, it depends on how much information both of them. So, A and B are independent events if probability of A given that B has already happened is this no this is not correct, so this is the correct answer. If one event is unaffected by the outcome of another event two events are said to be independent so b part is the correct one.

If probability of A or B is equal to probability of A then what is this is there any statement correct A and B are mutually exclusive? No the Venn diagram areas of A and B overlap, no. P(A) and P(B) are joint probability of A and B it is not like that so in fact this is none of the above for this.

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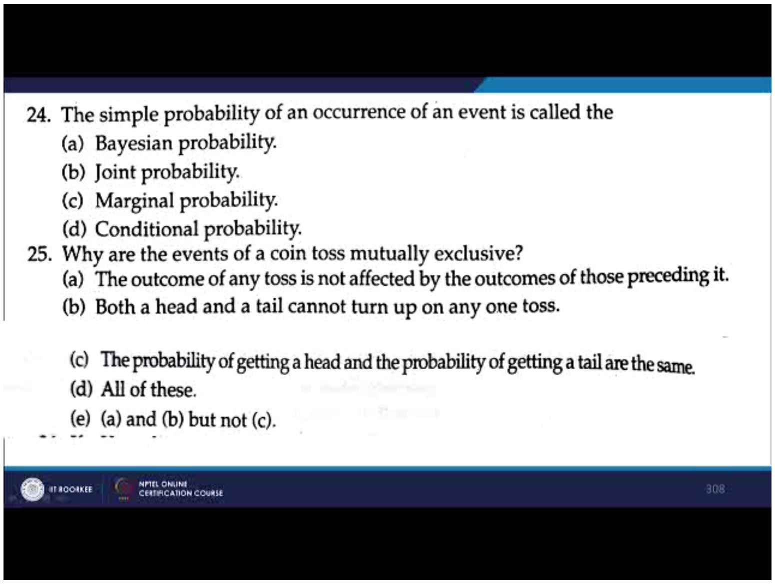

The simple probability of an occurrence of an event is called marginal probability right why are the events of coin toss mutually exclusive? Why the event of coin toss are mutually exclusive because the outcome of any toss is not affected by the outcome of those preceding it both head and tail cannot turn up once any one toss the probability of getting head and the probability of getting a tail are the same, all of this no, this is the correct one right because both of them cannot happen and the same time so that is why it is mutually exclusive.

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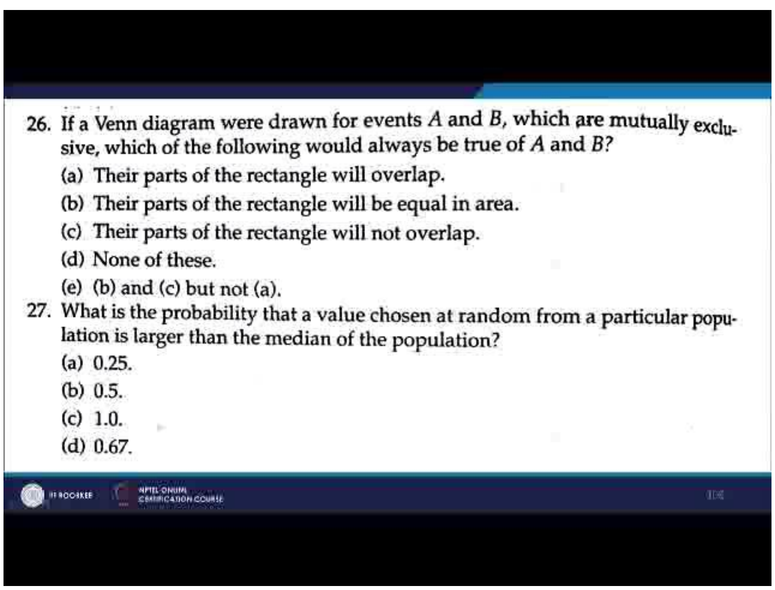

If a Venn diagram were drawn for events A and B which are mutually exclusive which of the following would always be true of A and B their parts of rectangles will overlap, their parts of rectangles will be equal in area, their parts of rectangles will not overlap none of this so answer to this question is this they will not overlap because they are mutually exclusive it will be like this A and B right what is the probability that ok, let us skip this one. But anyway let us try what is the probability value chosen at random from a particular population is larger than the median of the population so you have got lots of data with you and you chose one data points.

So what is the probability that data point is larger than median of the population so it is 50% chance, so this is correct.

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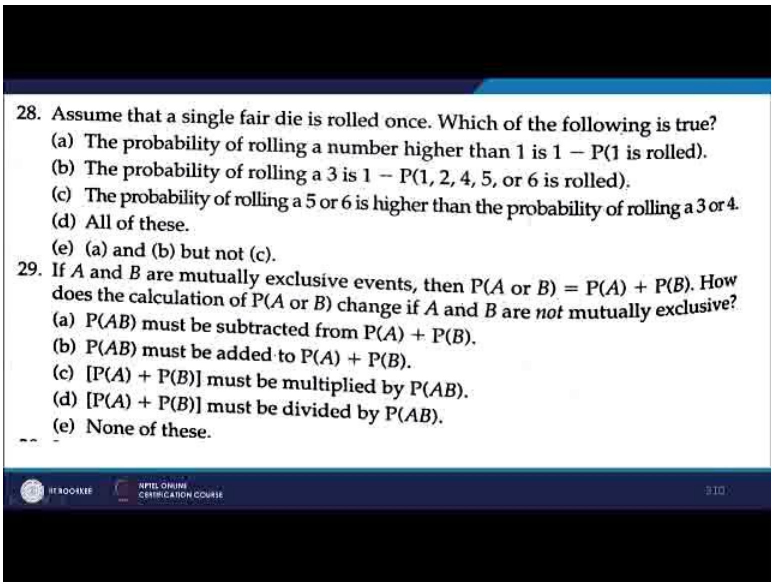

Assume that the single fair die is rolled once which of the following is true, assume that a single fair die is rolled once what is which of the following is true. The probability of rolling a number higher than one is 1 minus probability of one is rolled. The probability of Rolling a 3 is 1 minus probability of 1 2 4 5 6 is rolled, the probability of rolling of 5 or 6 is higher than probability of rolling 3 or 4, all of this so the correct answer to this question is this is e, a and b are correct and c is incorrect right.

Let us look at next one if A and B are mutually exclusive events then P(A) or P(B) is equal to this how does the calculation of PR be changed when we are not mutually exclusive. In fact if you look at this you will find that none of this is correct. In fact e is not the right answer, the answer is must be subtracted from yeah this is; you need to subtract A and B, Why? because this is a case like this right. So, Probability of A Probability of B, A and B so PA or B = P(A) + P(B) - P(AB) this common areas to be subtracted so A is correct in this case.

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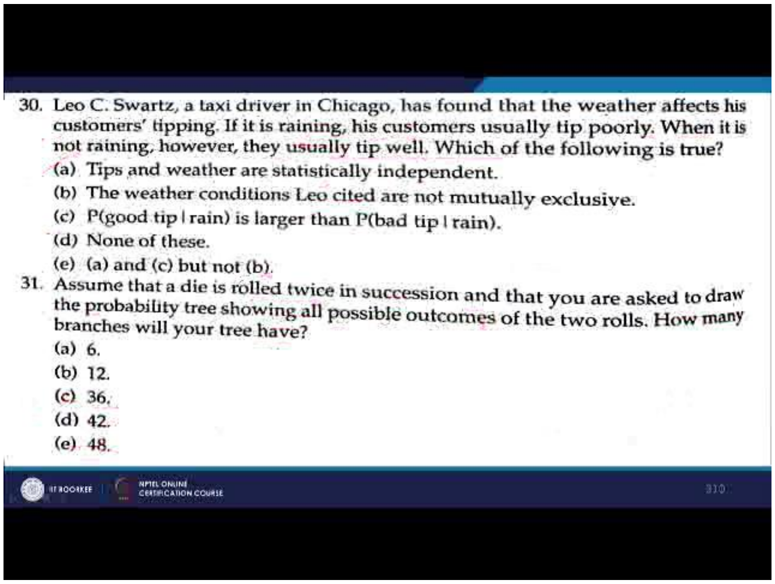

Let look at this Leo C. Swartz a taxi driver in city has found that the weather affect his customers tripping his customer tip poorly when it is not he get good tips right. However they usually tip well which of the following is true tips and weather are statistical independent, the weather condition Leo cited are not mutually exclusive, probability of good tip given that it as rent is larger than probability of bad tip even it is rent, none of this. So, for this none of this is the answer.

Let us look at 2 or 3 questions then we will wind up this session assume that a die is rolled twice in succession and that case let us skip this part 31.

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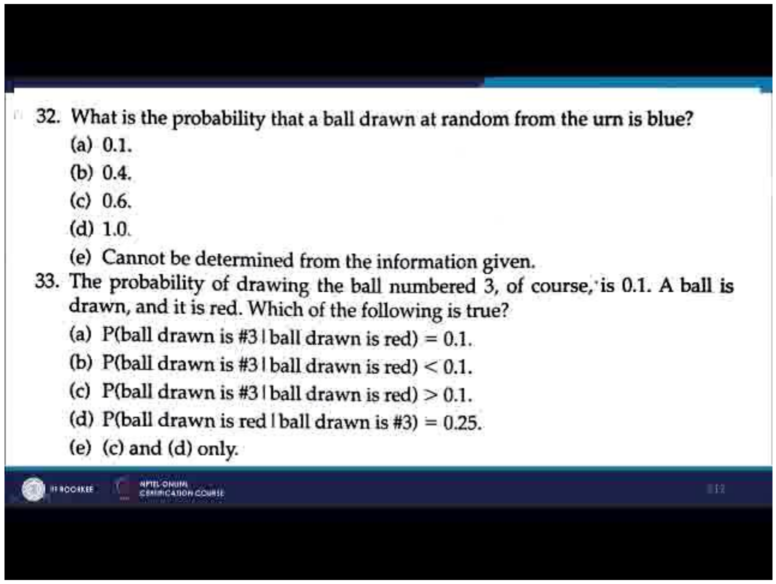

We will skip this one also 31 and 33 right. Let us look at these questions in fact you can skip this as well right ok. Let us look at symbolically the marginal probability is what? P(A) so none of this is right all these are not, right answer is this. If we sum all the properties of the conditional event in which the event A occurs while under statistical probability the result is none of this.

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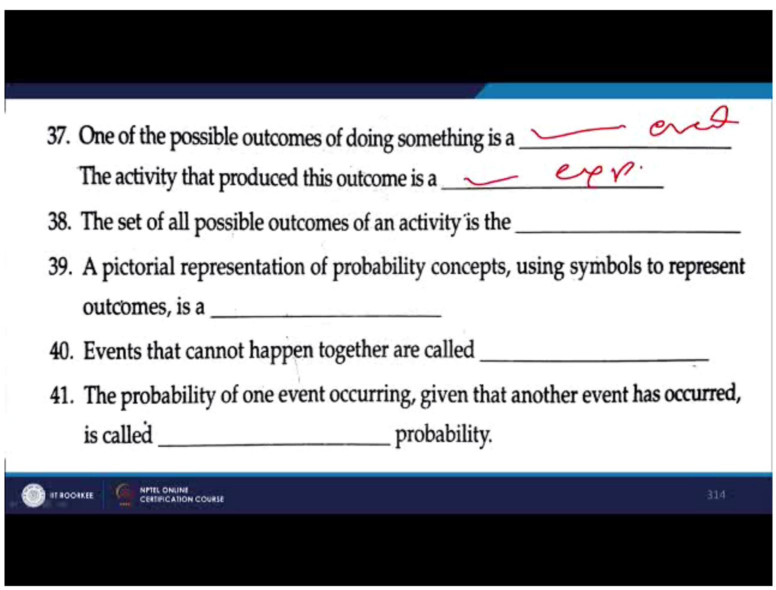

One of the possible outcomes of doing something is the activity that produce the outcome is maximum so this is 37 this event right and this is experiment. The set of all possible outcomes of an activity is what sample space, right. A pictorial representation of probability concept using symbols to represent outcome is Venn diagram. Events that cannot happen together are called mutually exclusive events the probability of one event occurring given that another event has occurred is called conditional probability right.

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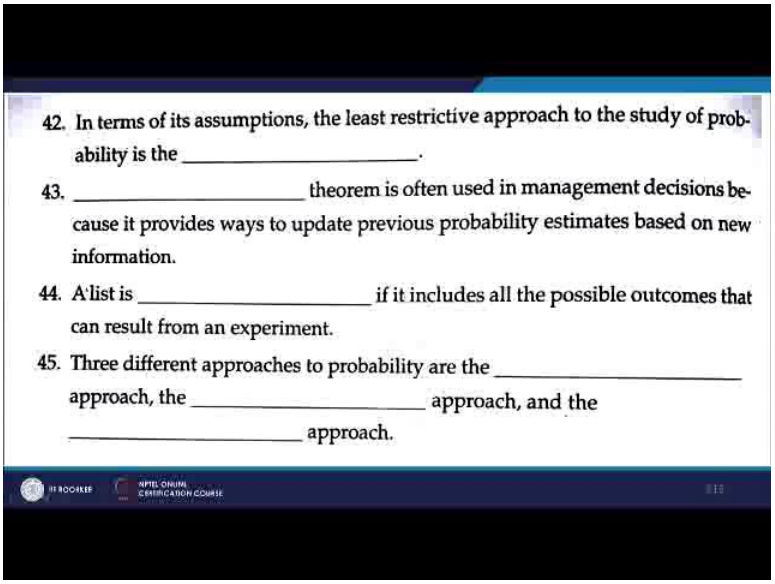

In terms of its assumptions the least restrictive approach to study of probability what least restrictive its subjective approach. Bayes Theorem is often reciprocity discussion theorem is often used in management decision because it provides way to update previous probability so it is Bayes Theorem. List is if it includes all the possible outcomes that can result from an experiment is collectively exhaustive right. And 3 different approaches classical, relative frequency and subjective right. So, with this I complete today's session and in next session we will start with probability distribution, thank you very much.