Business Statistics
Prof. M. K. Barua
Department of Management Studies
Indian Institute of Technology – Roorkee

Lecture – 14
Probability- Part 2

Good afternoon friends, I welcome you all in this session. As you are aware in previous session we discussed about basic concepts of probability, types of probability, we have also seen the addition rule for mutual, mutually exclusive events and not mutually exclusive events. We did carry out couple of examples, today we will work out some more examples.

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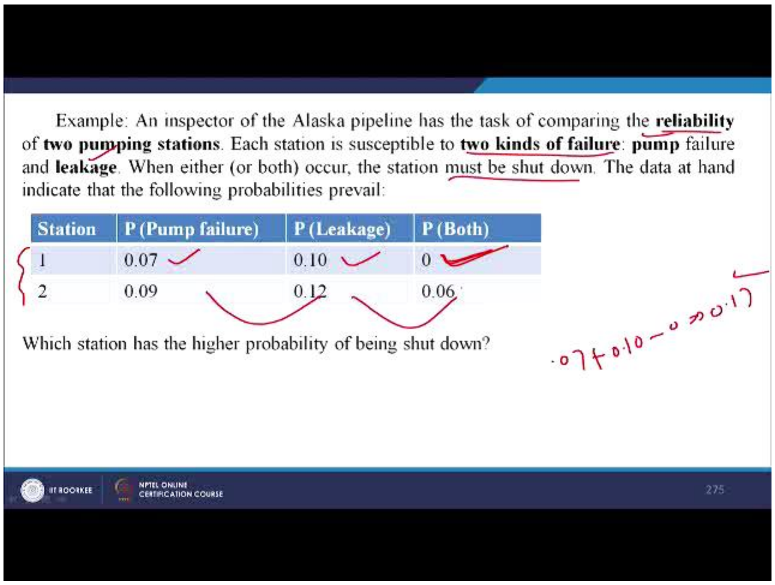

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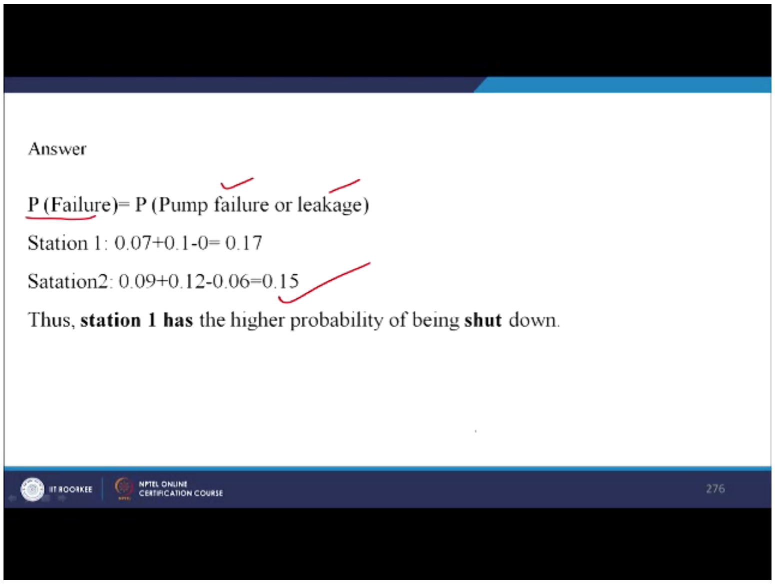

Let us look at this example, where in you have been given two stations, and an inspector of the pipeline has the task of comparing the reliability of these two pumping stations, which station is susceptible to two, two kinds of failure, failure and leakage. So there are two ways in which the functioning of these two stations can be or may be stopped. When either occurs, means either this failure occurs, or this failure occurs, the station must be shut down. Now you have to find out the probability of shutting down of a particular station, which has got more probability of shutting down. So this is a case, wherein this is, this something like, not mutually exclusive events.

So let us work out solution to this question, so probability of failure is equal to either, pump failure or leakage failure. So, probability of pump failure plus leakage failure minus both because the failures are you will have to stop the pump because of these two failures as well. Not only because of one of them. So this plus this minus, 0.07 + 0.10 - 0, so this is 0.17 and what about this, this is 0.21 minus this, so this is 0.15. So which has the more probability of getting down, Station One. So this value is higher than this value ok.

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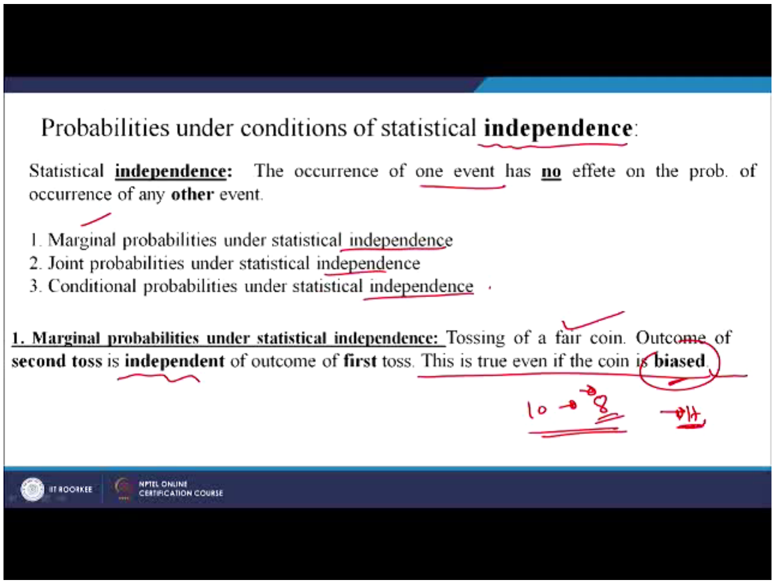

Let us look at the probabilities under conditions of statistical independence, you can have two types of situations, where the events are independent of each other, or they are dependent on each other, when I said dependent means, the outcome of second event depends on the outcome of first event. So there are two broad cases probabilities under condition of statistical independence. When I say Independence means, the occurrence of one event has no effect on the occurrence of other event.

They are completely separated from each other, they are not related to each other, and they are not dependent on each other. So there are three types of these probabilities, marginal, joint and conditional. So, marginal probability under statistical independence, similarly joint and similarly conditional probabilities. Similarly marginal probability under statistical independence means tossing of a fair coin.

So when I say independence means, as I said the next event does not depend on the previous event. So, tossing of a fair coin. Let us see if you toss a fair coin and you want to know what is the probability of heads, so it would be 0.5, if you toss, toss it second time, then what is the probability of heads, does it depend on the outcome of first toss, no it is not like that. So the probability of heads during second toss is also 0.5. So they are called independent.

And this is true even in the case of biased coin. So when I say fair coin means, it has a probability of having heads and tails 0.5. Both 0.5. Now let us say, if you come up with a, with a coin, which is unfair coin or biased coin. Let us say in that biased coin, whenever you toss it, let us say 10 times then if you get let us say 8 times heads, so you are saying it biased coin, right, so you are getting more and more favour towards heads. So that would be a biased coin, but let us say, even if you take a biased coin, if you toss it once, and if you are getting heads, then second time again whatever the outcome would be that would not be dependent on the first outcome. That is known as independent marginal probabilities.

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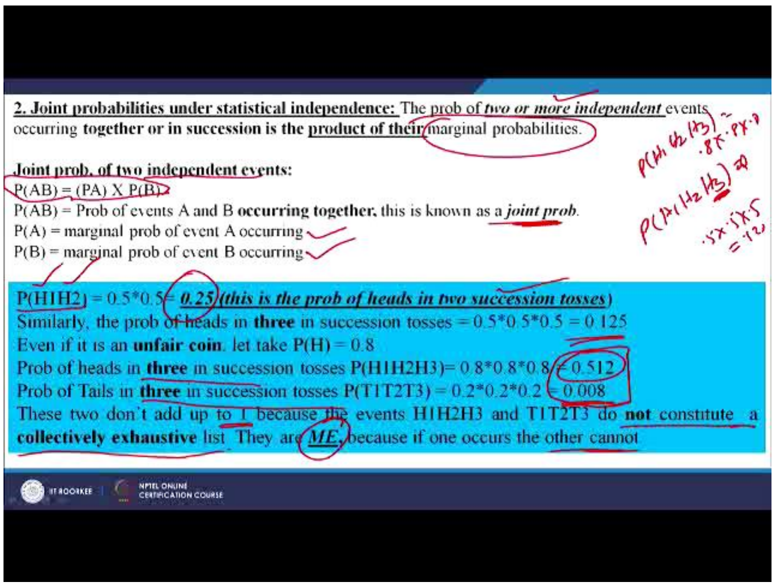

Let us look at joint probabilities. Joint probabilities mean two events can happen simultaneously, either the two or more independent events can occur together and it is the product of their marginal probabilities. What is marginal probability, we already seen, probability of A or probability of any other event let us say B. So there is nothing but marginal probability ok.

So, joint probability of two independent events, again these events are independent of one another. So P(AB) is equal to product of P (A) and P (B). So this nothing but, P(AB) is nothing but joint probability. This is marginal probability. Probability of event A and marginal probability of event B so you need to remember this.

Now let us have the probability of heads, when you toss a coin, and what is the probability of heads if you toss a coin second time. So this probability would be 0.25, so this is known as and this is the probability of heads in two succession process. If I ask you what would be the value if it is ph1 ph2 ph3, what was the probability that the third time in succession you will get point, you will get heads. So it would be 0.5 * 0.5* 0.5 is 0.125.

Even if you take unfair coin or unbiased coin, then also probability of heads would be, in case of unbiased coin it will be again same. H1, H2, H3. So it would be 0.8 * 0.8 * 0.8, so this is the probability of heads. Probability of tails is 0.008. If you look at, that the addition of these 2 is not equal to 1, why? Because they do not constitute a collectively exhaustive list they are actually mutually exclusive events. So, because of one occurs, the other cannot. That is the definition of mutually exclusive events. So this is joint probability is under statistical independence.

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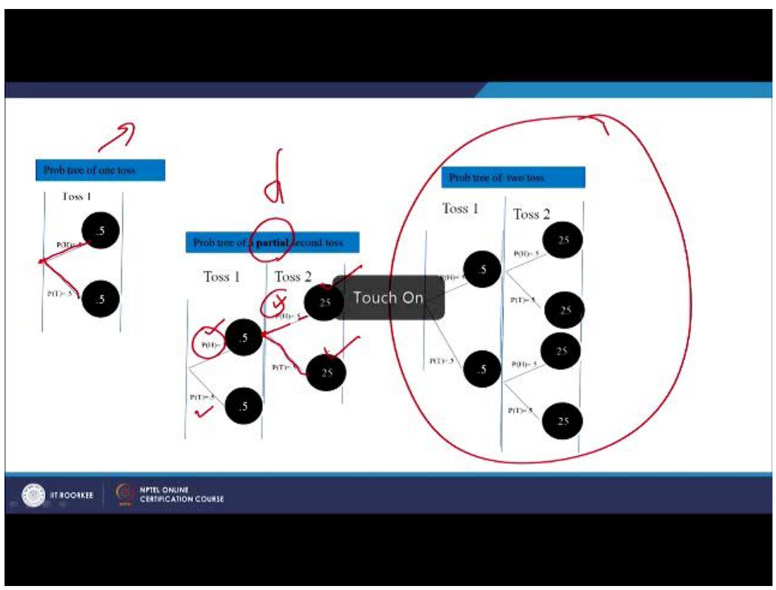

Let us look at look at conditional probability under statistical independence. The third one is conditional. So, before moving onto that let us explain this particular example using tree diagram. So let us say you are tossing a coin probability of heads 0.5, probability of tails 0.5. This is first toss, right. Now if you toss the coin second time then you will get this outcome but before this nothing but partial output or probability of partial 2nd toss. So, this is heads 0.5, heads 0.5. Now let's say you are getting head. Now when you toss the same coin, second time probability of heads is point five, probability of tails is point five. So this is 0.5 into 0.5, this becomes 0.25, this is what we were getting in previous slide also.

0.215, so this you can see in, this tree, probability tree as well. Now this the probability tree of two tosses. So you are getting heads here, tails once you get heads let us say probability of getting heads against 0.5 is again 0.5. Let us see if you get tails, then again problem of getting heads point 5 and point 5. So the total probability is 0.25. So if you look at the summation of these two, the summation of these two probabilities 1.0, similarly summation here is 1.08, summation has to be one, why this not one, because this is partial output or outcome.

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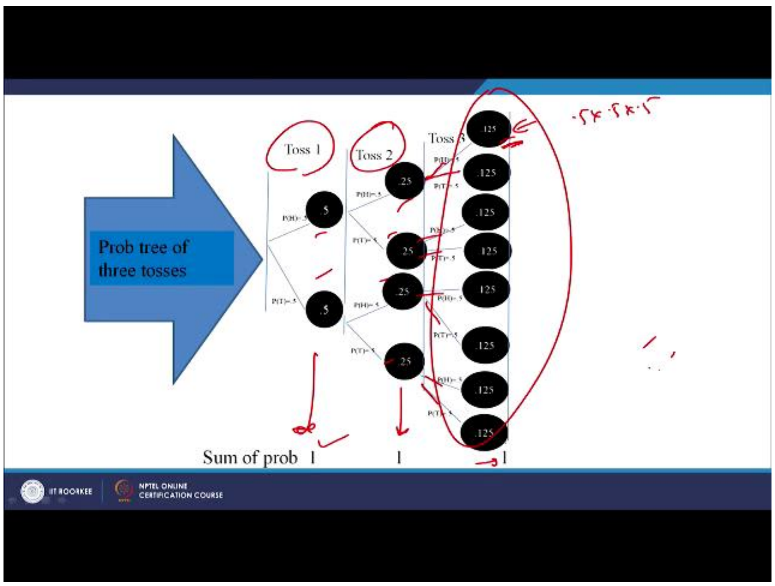

Let us look at the case where, you have three tosses in succession. So, this is first toss, these two are the output, second toss these four are the outputs. If you look at here again in 3rd toss probability of heads 0.5 tails 0.5. Similarly heads, tail, heads, tail. So this is how did you get 0.5 into 0.5 into 0.5, 0.125, which is there in previous slide as well. 0.125. So again here you can see that the sum of the probabilities is one, one here also one and here also one, sum of all these 8 values.

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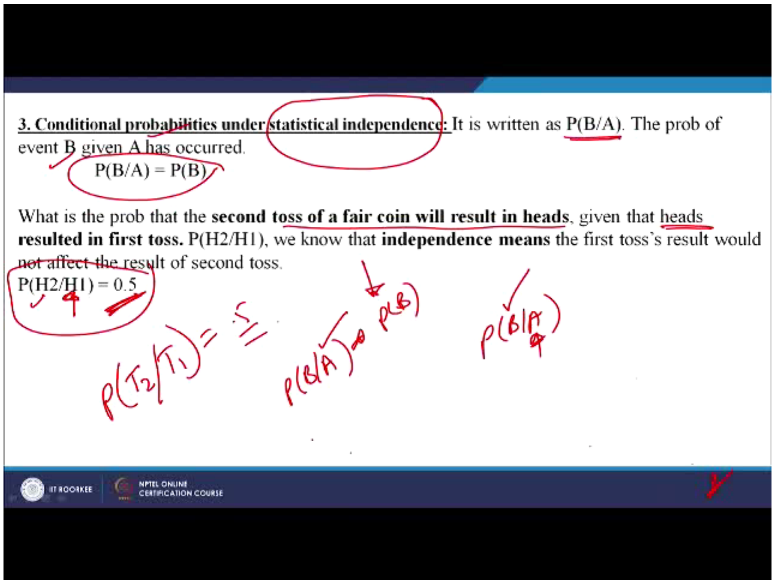

Now let us look at conditional probability under statistical independence. Again we are taking case of statistical independence. So we have seen two cases, marginal and joint, ok. Now in this case conditional probabilities, this how we write it. So the probability of event B given that event A has already occurred. So when I write this, it means what, what is the probability of event B given that event A has already happened.

So let us look at this example, what is the probability that second toss of a fair coin will result in heads, given that, the first time, it was also resulted in heads. Because they are independent second time also the probability of heads would be 0.5. So, this how you should remember this formula Probability of B given that A has already happened is equal to nothing but probability of B. So second time you are tossing a coin is again this P(B). So answer is .5 so what does it mean probability of getting heads second time given that the first time it was head ok. So you be, you can have it, like this, probability getting tails second time given that it was heads in first toss tails in first time because they are statistically independent.

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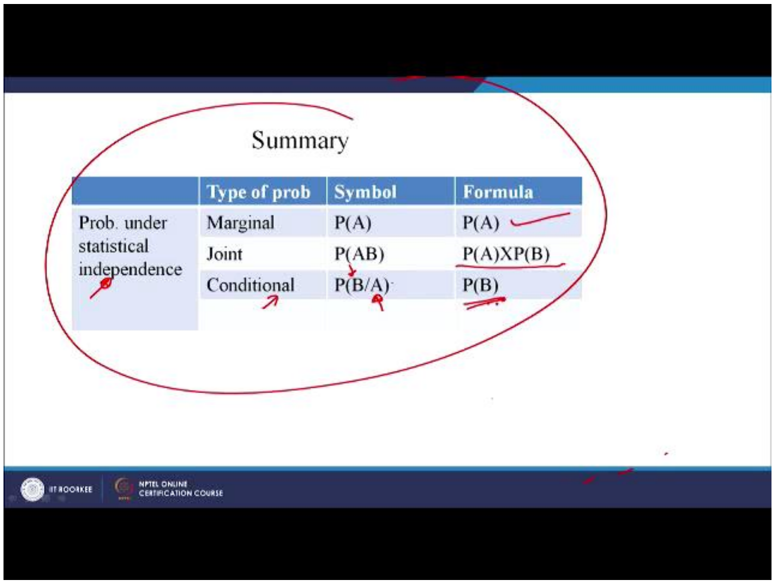

Let us summarize all these probabilities, so probabilities under statistical independence, marginal that is we should remember. This P(A), P(A) X P(B), P(B). This is conditional probability of obtaining B given that A already occurred.

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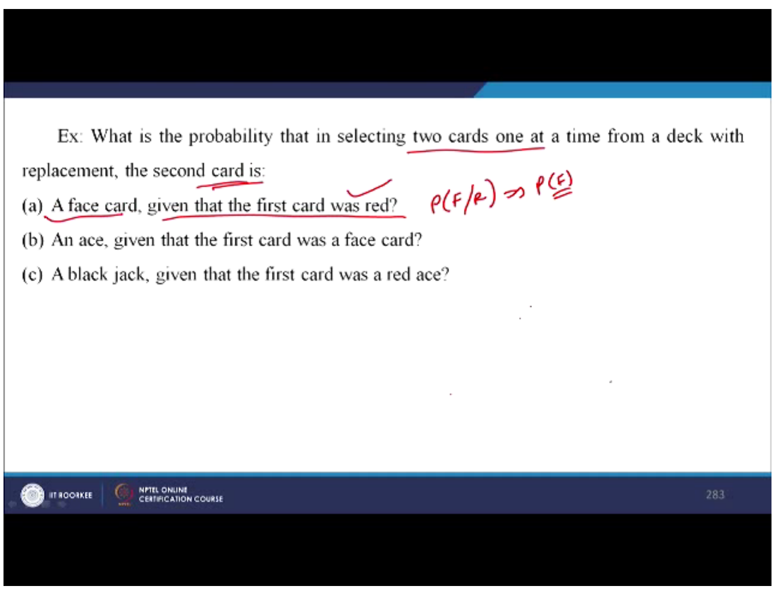

Now let us look at this example, what is the probability that in selecting two cards one at a time from a deck with replacement the second card is, the second card is a face card given that the first card was red. So, what you have to find out what is the probability of getting a face card given that the first card was red card. So this is a case of conditional probability under statistical independence. So what is this, let us call it like this, P face even it was already red, for simplicity you can write so it is P face your face card, so many face cards are there in a deck? 12.

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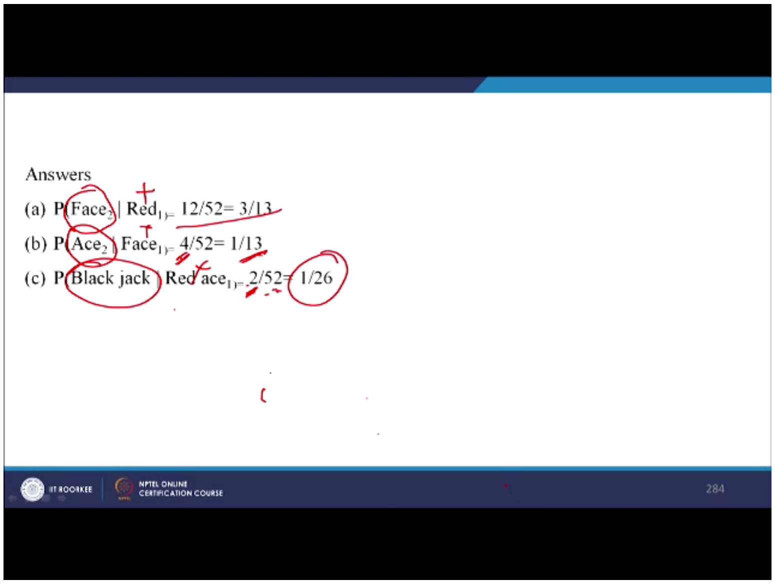

So that would be 12 by 52. So this is your answer. Let us look at probability of that it would be an Ace given that the first card was a face card. So again the probability would be of Ace. This is what you need to this would this is would be your answer. And so how many aces you have got? 4 aces. So this is the answer. And what about a Blackjack given that the first card was a red ace? So Blackjack how many cards you got 2 blackjacks. So this is 2/52. So this is the answer. So you need to just remember that the answer to conditional probability under statistical independence is this only. You don't have to bother about this part ok.

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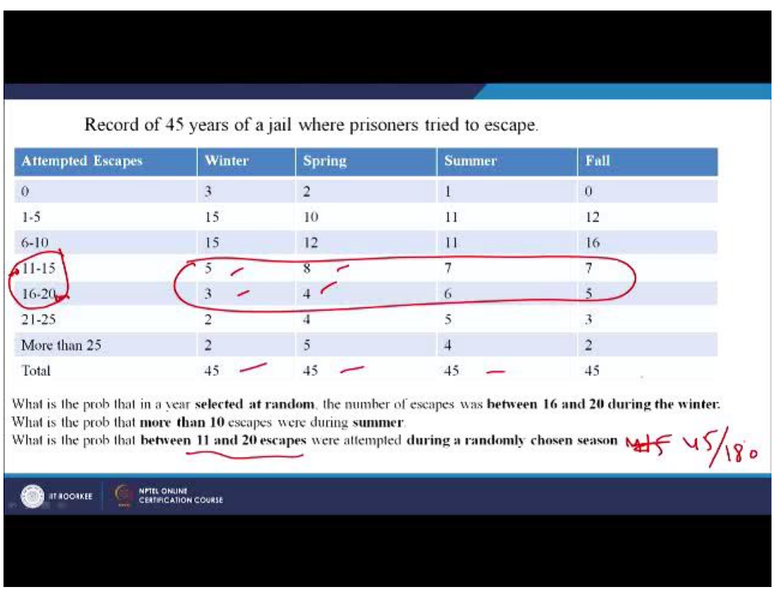

Let us look at this question, very interesting question, let us say a jailor has recorded the incidents of prisoners escaping jail and there is a record of let us say last 45 years. Ok so these are attempted escape and these are different seasons, winter, spring, summer and fall. So attempted escapes 0 between 1 and 5, so 15 attempts were made in winter, there were 1 to 5 attempts, in winter, ok, so this is nothing but your classes, isn't it?

So there were 15 cases in winter where the number of attempts were 1 to 5, ok, more than 25 two in winter, and let us say two in fall, so these are total attempts, in these different seasons. So what is the probability that in a year selected at random, the number of escaped, was between 16 to 20 during the winter, number of escaped was between 16 to 20 where it is 16 to 20 and in which season, winter, where is winter, this right. So what should be the answer, total attempts in winter, 45. So this would be 3 by 45. What is the probability that more than 10 escapes were during summer, more than 10, now more than 10 can be 11 or more than 25 or more than 100.

So all those attempts which were more than 10, So 11 onwards, so more than 10 means till this point, ok let me delete some of these, ok so what is our question? What is the probability that more than 10 escapes during summer, so first of all summer season, and more than 10, all these values these are 10 and less than 10. So summers, this is summers, so you just write these two, these values four values, this is 13 + 9, 22 / 45, is the answer.

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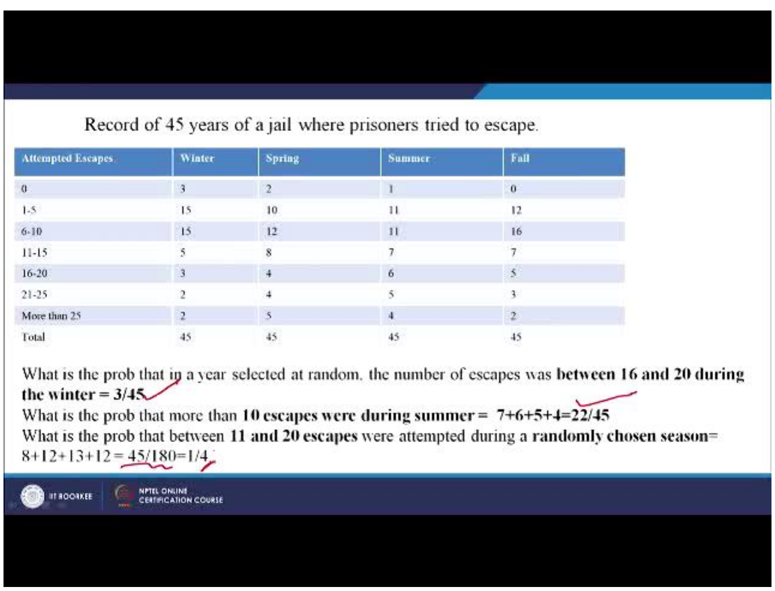

Now what is the probability that between 11 to 20 escapes were attempted during a randomly chosen season? So you have chosen seasons randomly, from all this and the number of attempts between 11 and 20. 11 and 20, between 11 and 20 means what? Starting from here and ending here. So these two classes is the reverse. During any randomly chosen season so you have got all these values. So, just add all this to this 8, this is 12, 20 then 13, 33 and 12, 45/180. So these are your answers 3/45, 22/45, and 45 by 180 that is 1/4 this is how you can solve a question like this.

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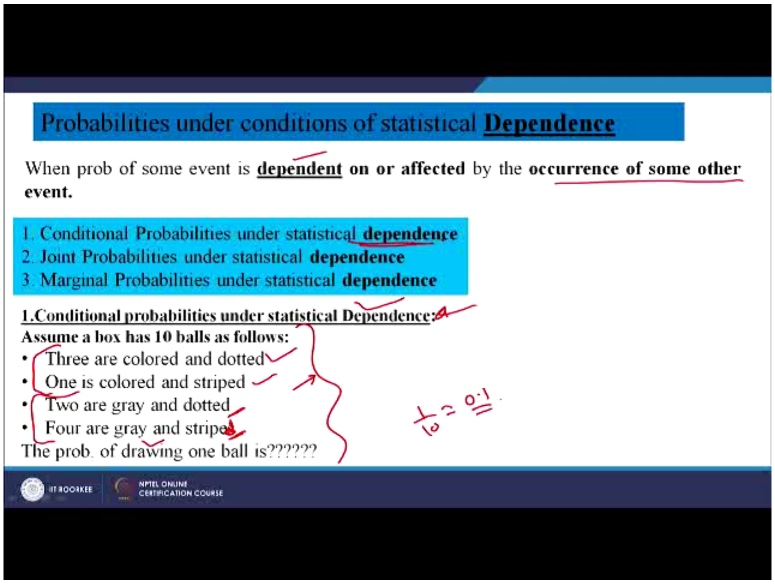

So far we have seen probabilities under condition of statistical independence. Now will see the probabilities under conditions of statistical dependence when I say dependence means the outcome of second toss would depend on outcome of first toss or the second event would depend on the first event. So probability of some event is dependent on, or affected by occurrence of some other events like this known as dependence. So again, there are three types, marginal, joint and conditional.

Now if you look at conditional probabilities under statistical dependence, now this is conditional probabilities under statistical dependence. So before starting this let us take one example and let me explain through an example. So there is a box which contains ten balls, in those ten balls there are 4 colored balls and there are 6 grey balls. Ok. So out of 4 colored balls 3 are dotted as well and 1 of them is stripped. Ok. Out of 6 grey balls 2 grey balls are dotted and 4 are stripped.

So if I take out a ball if I draw a ball out of this container, what is the probability of getting one ball selected? It is 1/10 ok.

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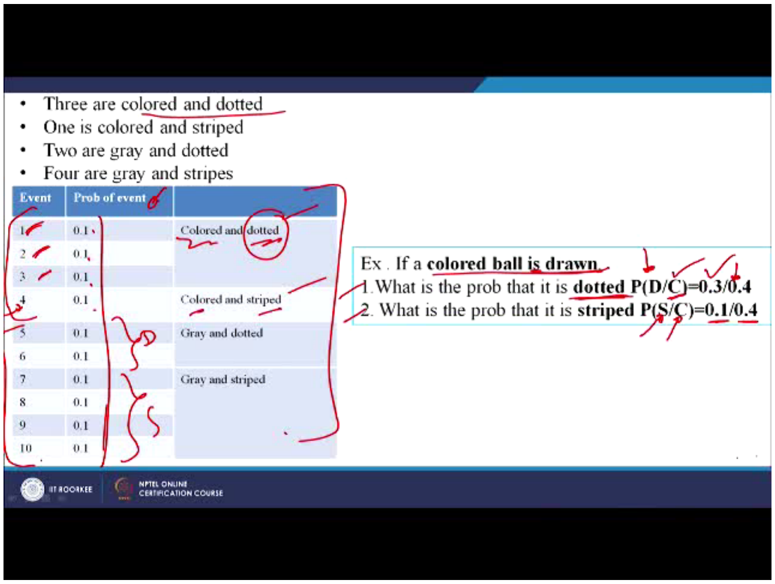

Now you can arrange the same information in tabular form. So three are colored and dotted, 1st 2nd 3rd, as we have already said that the probability of drawing up a ball is 0.1. So, all these are point one. So first, second, third are colored and dotted, the fourth is colored and stripped so these are four colored balls 3 dotted and one stripped, now these are 6 grey balls, 2 are dotted, and these 4 are stripped. Now if a colored ball is drawn what is the probability that it is dotted? So this is a case of conditional probability under statistical dependence.

So what is the question, if a colored ball is drawn what is the probability that it is dotted, so if a colored ball is drawn, what is the probability that it is dotted. If a colored ball is drawn, what is the probability that it is stripped? So, this is very simple. So colored balls how many colored balls 4.1,2,3,4. So, 0.4 in denomination how many dotted out of his colored balls 3 are dotted so, 0.3. Similarly we know that colored balls 0.4 and stripped is just one, this one is colored and stripped its 0.1. So this how you can find out answer to these questions.

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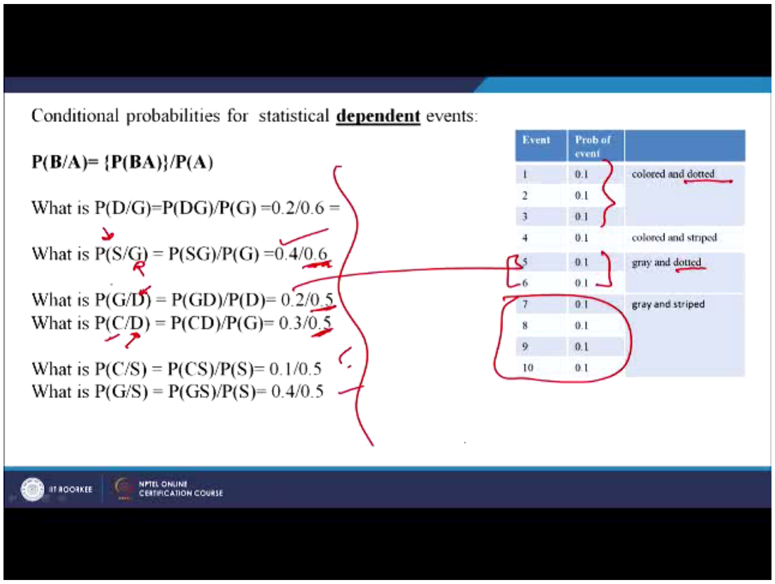

Now this we have formula for conditional probability under statistical dependence the probability of event B given that the probability given that event A has already taken place, is nothing but probability of B and a divided by probability of A. So we will work out all these questions and with the help of this table we can easily find out answers. So what is the probability that the ball is dotted given it is Grey.

What is the probability that the ball is dotted given it is grey, so how many grey balls, 6. So 0.6 and how many of them are dotted? Two, these two are dotted 0.2, 0.2 and 0.6, so this is 1 by 3. Similarly let us, let me remove all this points ok, now what is the probability that it would be stripped given that the ball was grey. You know that there was 6 grey balls, out of 6 grey balls, how many are stripped 4. This is 4 this 0.4. Similarly what is the probability of grey, given that the ball was dotted?

So dotted balls how many dotted 4 are here and 1 is here 0.5 and how many of them are grey, two, these 2. Probability that it is colored given that it was dotted one, so dotted of course we know. 5 in how many of them are colored these 3. So 0.3 and so on and so you can easily calculate remaining answers so, this nothing but statistical probability under conditional probability under statistical dependent case.

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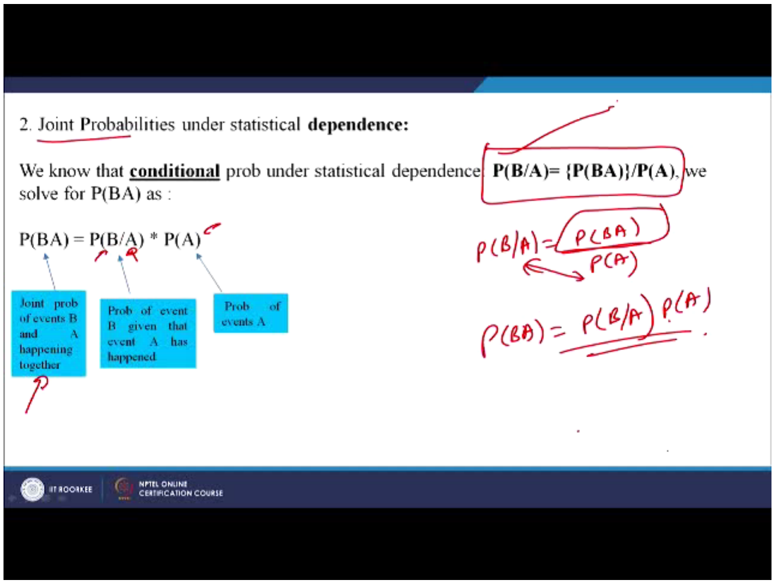

Let us look at joint probabilities under statistical dependence. So first using conditional probability let us look at joint probabilities and if you look at joint probabilities we know that it is the formula for conditional probability under statistical dependence. So joint probability can be easily calculated from here, so we know that P(BA) is equal to P (B/A) divided by P(A), so this is nothing but joint probability, so how will calculate, just multiply thee two. So P(BA) is equal to P(B/A). So this joint probability of events, B and A altogether, probability of event B even that A is already happened and probability of event A. So joint probability is very simple since you know the conditional probability under statistical dependence, can easily calculate joint probabilities.

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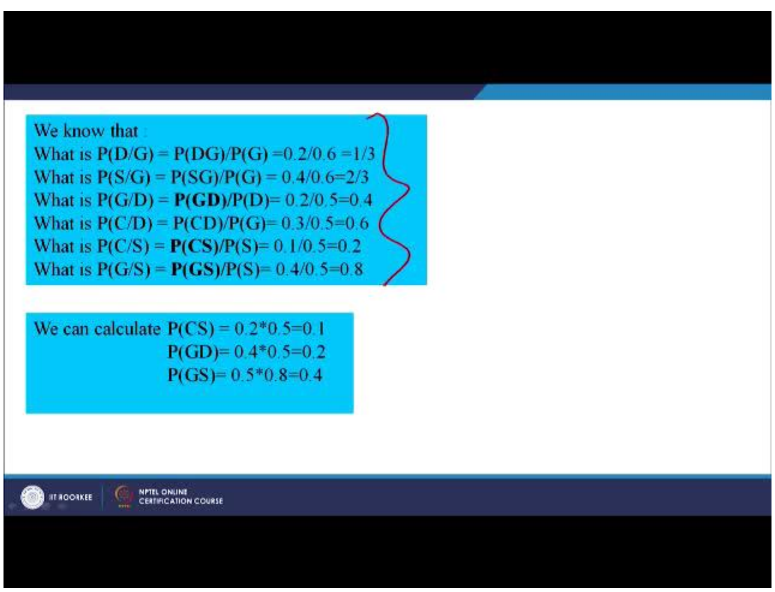

So we know these answers, in fact in previous to previous slide, these are different questions we have solved and we just write down all these questions and answers over here. So, I know all the answers. If I want to know the joint probability of CS what is the probability that the ball is colored and stripped so what is the; what that value would be colored and stripped here it is, color and stripped. So we know that P(C/S) is equal to P(CS) divided by P(S), what you want you want P(CS) we want P(CS), so what was this value, this is 0.2 is equal to 0.1 by 0.5, so we want PCS will multiply point two and point 5.

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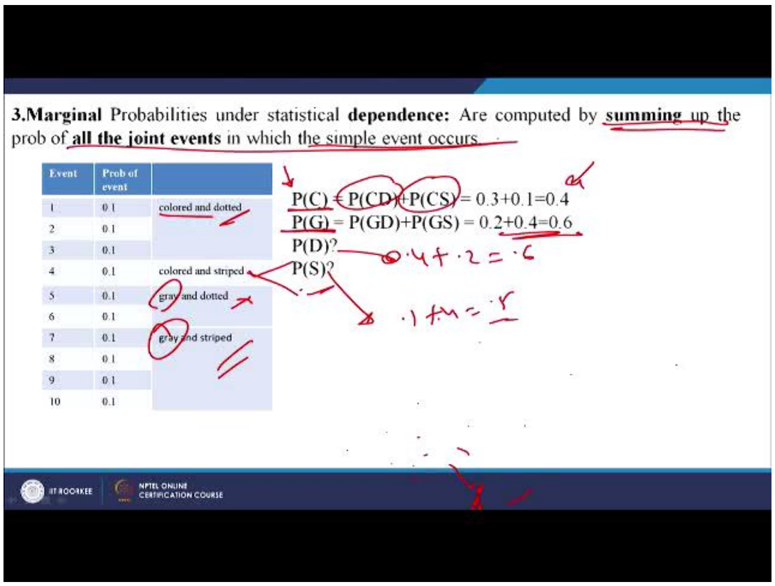

Similarly GD is here, so you just multiply 0.4 and .5 and .4, you will get probability that the ball is green and dotted. 0.2. Probability that it is not Green grey, so grand stripped Grand stripped is here, so just multiply .5 and .8 it is 0.4. So, third one is marginal probability under statistical dependence. Now these are computed by summing up the probabilities of all the joint events in which the simple event occurs, so if I want to know what is the probability of event that the ball is colored? So just write all those probabilities, where you are getting colored ball. So this is colored and dotted and colored and stripped so, .3 and .1, .4.

Probability that it would be grey, so grey here and here so .2 plus .4 what would be this probability that it would be dotted, just look at dotted, 0.4 plus any .2, .6 stripped, just look at this is .1 plus stripped, .4 .5. So this is how you can calculate marginal probabilities under statistical dependents. So in today's session we have seen different types of situations wherein, we have seen the, probabilities under statistical independence you have seen marginal, you have seen conditional and joint. Similarly probabilities under statistical dependence again marginal conditional and joint in next class we will work out couple of examples. Thank you.