Business Statistics
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Lecture – 13
Probability- Part 1

Hello friends, I welcome you all in this session, as you are aware in previous session we discussed, some basic concepts related to probability. We have seen, what is experiment, what is event, what is joint probability, what is the use of Venn diagram, and so on. We have seen, what is mutually exclusive event, what is the meaning of collectively exhaustive as well. In today session, we are going to work out a couple of exercises and will see different types of probabilities.

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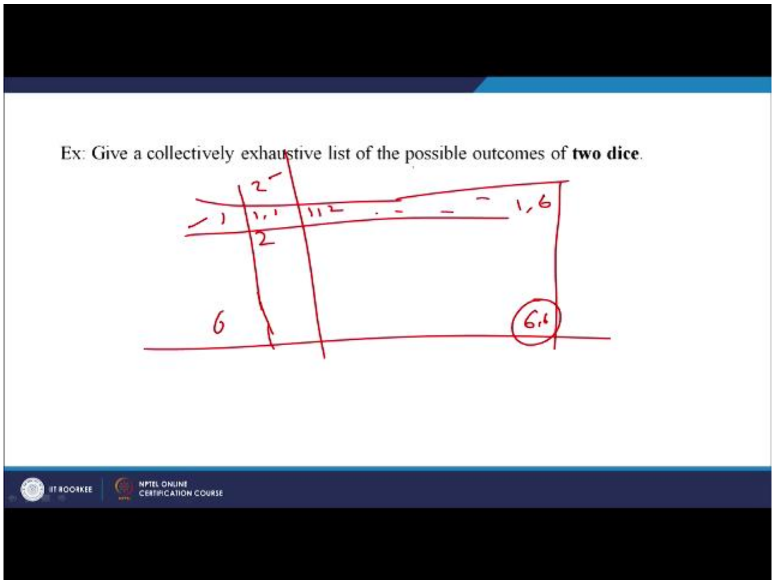

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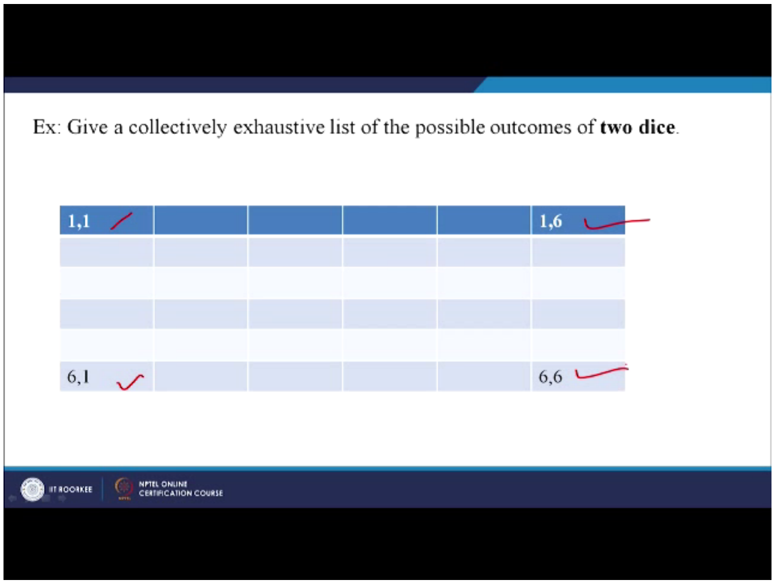

So let us look at this question, you have to collectively exhaustive list of the possible outcomes of two dice. Just think for a while and try to answer this question. So you have to swing two dice and find out what are the possible outcomes, ok. So the possible outcome let us say, it would be on first dice and let series second dice, right? So first die, second die, so the outputs are let us say on first you are getting one, and then on second also one, then on first one second two and so on, right. So on first one and on second 6, isn't it? Let us say 2 122 and so on like 26 so finally what you will have 66 isn't it? So you will have a list of 6 rows and 6 columns, where in you would be writing down all possible outcomes. This how you can prepare a complete table like, so 11216 61266 ok.

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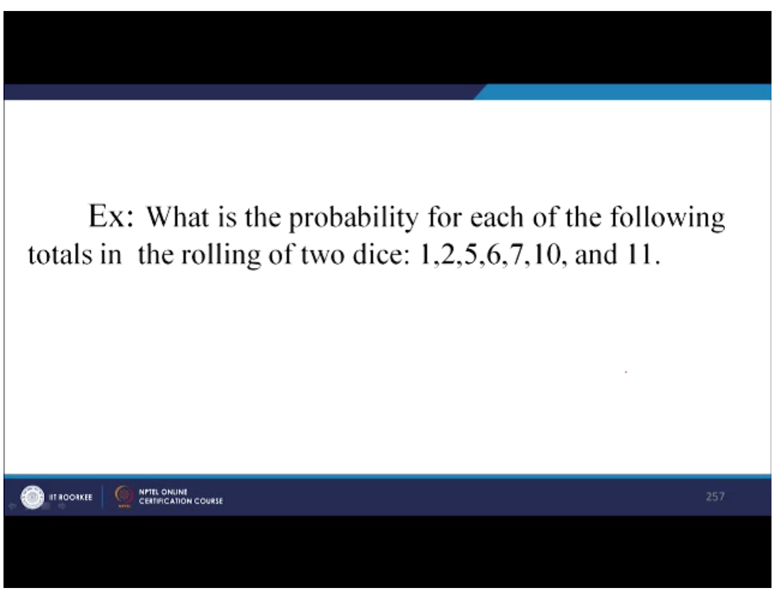

Now what is the probability for each of the following totals in the rolling of two dice, so again you are rolling two dice simultaneously, now what is the possibility that the sum, sum of the dots appearing on those two dice is one? Is it possible to have sum of dots on those two dice equal to one, you can't have a situation like this, isn't it? So this would be answer would be zero by 36 isn't it? What about two? What would be the probability?

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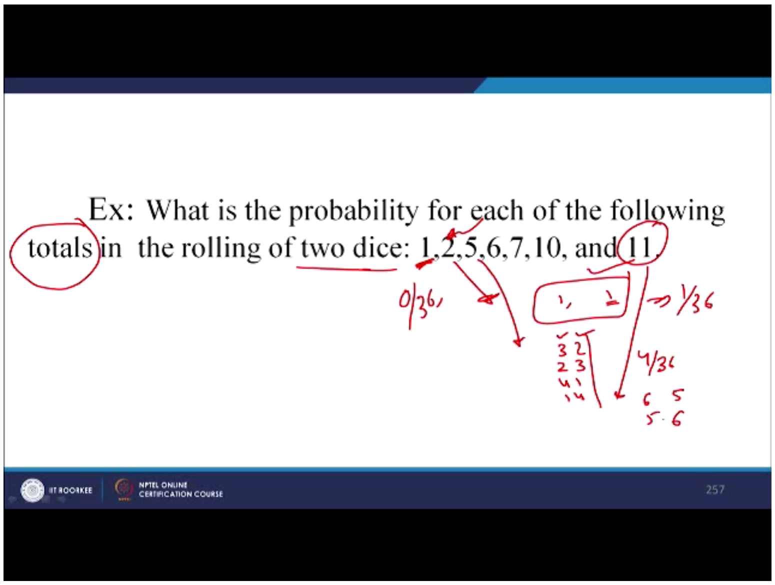

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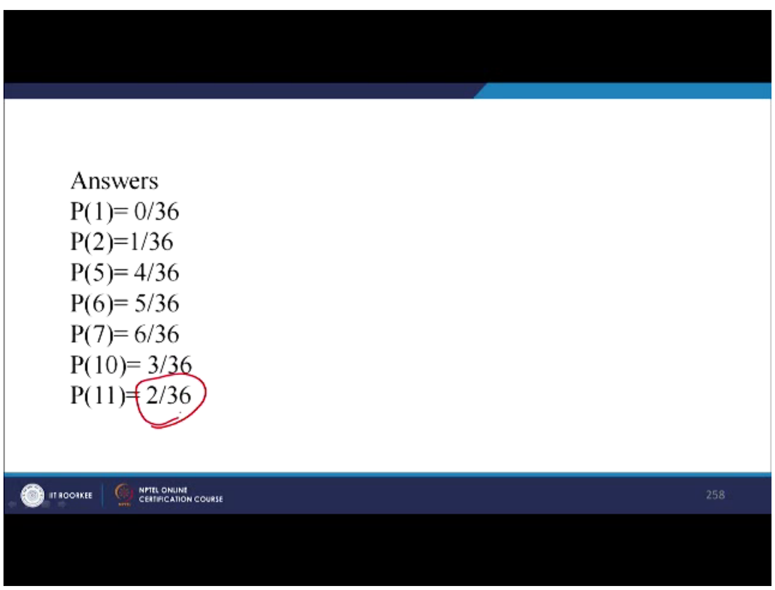

So you are getting one on 1st die and again one on 2nd die, right? So, there is only one way in which you can obtain, sum as two, right? So it is 1 by 36, what about 5? So let us say you are getting 32 on first die 3 and 2 on second die, then 2 3? Is there any other possibility for getting sum is 5 41 and 14 so 4 by 36. What about 11? Is it possible? So, you can have 6 and 5, 5 and 6, right? So it is 2 by 36. So this is, these are remaining answers, right?

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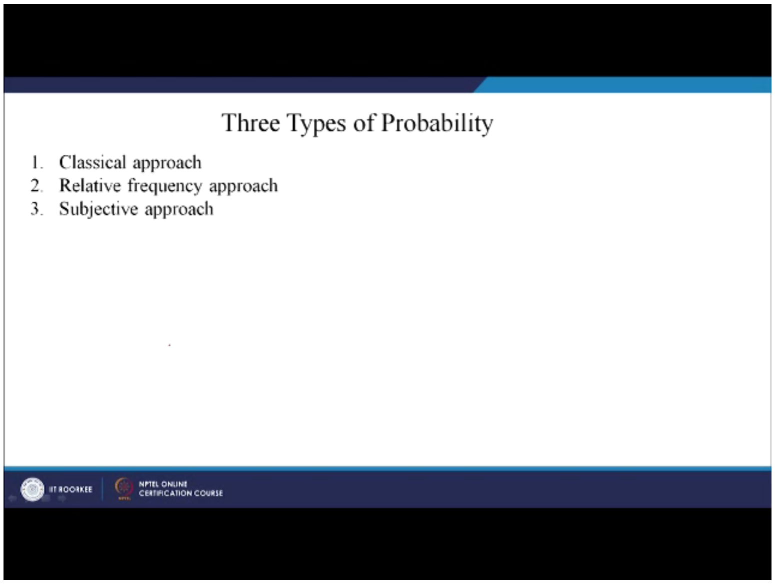

Now let us look at different types of probabilities, basically there are, three types of probabilities, the first one is, Classical probability or Classical approach to the probability then Relative frequency approach and the third one is Subjective approach. We will see each one of these in detail.

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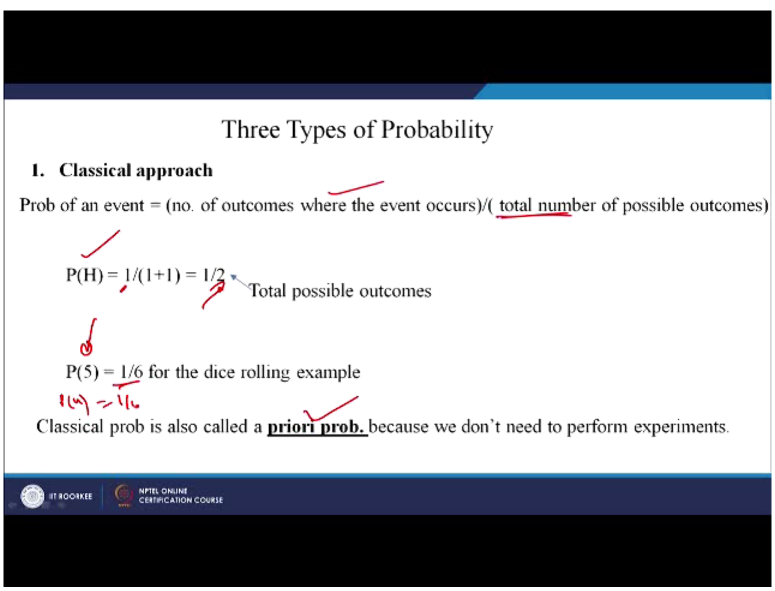

So, classical probability is also known as a priori probability, it is the simplest of all these 3, so, the number of outcomes where the event occurs to the total possible outcomes. So let us see if you toss a fair coin, so the possible outcomes are only two and the probability of heads it is, 1 by 2 right? Or the probability of tails is again one by two? Or let us say if you throw a die then what is the probability that 5 dots would appear on surface, it would be just 1/6 or P4, what it would be again, 1/6, isn't it? So this is known as Classical probability, the number of outcomes to the total number of possible outcomes.

Now, we call this as I said, a Priori probability, why we call it a priori probability, because to get the answers, you need not perform any experiment. You need not actually throw the coin, to know what is the probability of getting heads. So, that is why, this one is called, a priori probability.

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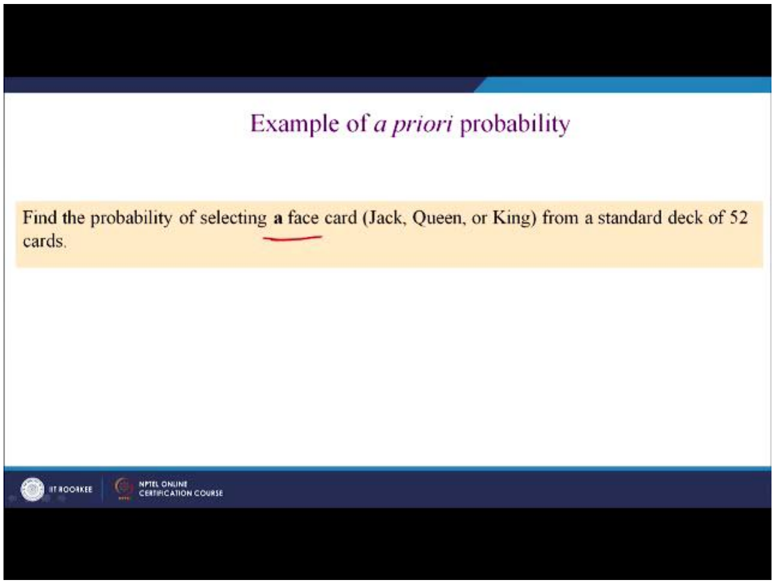

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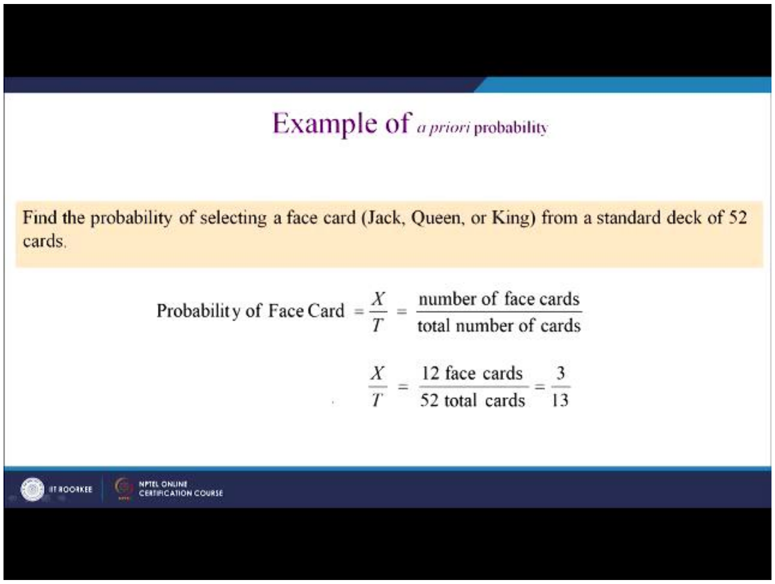

Let us look at some more examples to find the probability of selecting a face card from a standard deck of 52 cards. So how many face cards are there? So you have got three types of face cards Jack, Queen and King, right? So you will have four jacks, four queens and four kings, so it’s 12/ 52? So, this again, you need not actually, you know, select a face card from deck of card, you can just get the answer, using that particular formula, right? So this is 12/52, 3 /13.

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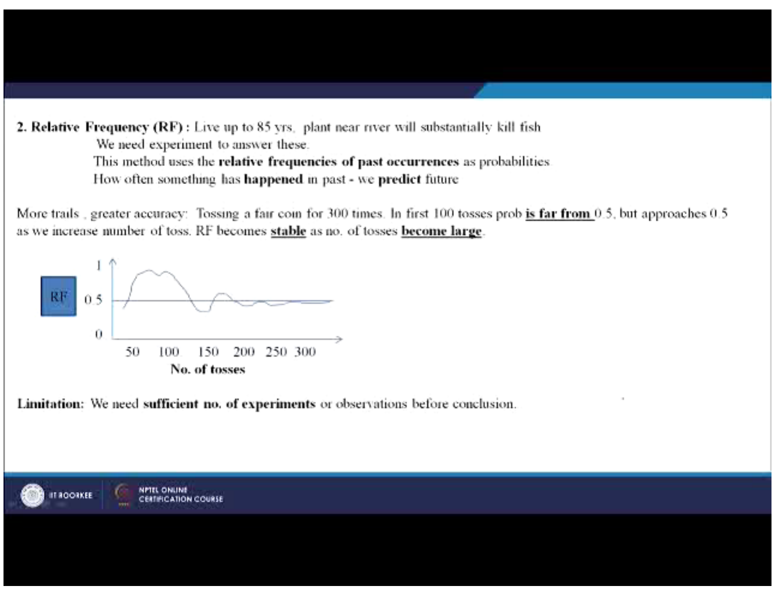

Let us look at the second type of probability; this is relative frequency approach to the probability. Now let me ask a question, what is the probability that you will leave up to the age of 85 years or 105 years? or let us say what is the probability if you, let us say, establish a plant near a river, and then how many fish would be killed from the pollutants which would be coming out from that plant, can you find out that probability or let us say, what is the probability that the speaker of the TV would blow off, when you turn its volume to its full level.

So, for knowing answers to all these questions you need to perform some experiment then only you can find out answers. So this method uses relative frequencies, or you should have some past data. So, if you get past data you can predict future on the basis of past data. So how often something has happened in past, we predict, will predict, future on the basis of that. So relative frequency is absolutely based on past data or you need to perform experiment.

So let us say if you are finding, what is the probability of getting heads, so if you toss a coin, you may get tails? second time again you may get tails, but if you toss a fair coin let us say, for let us say, 300 times or 1000 times, then over a period of time, after sometime the probability of heads would be or probability of tails would be stable and that would become point 5.

If you look at let us say, first few 75 price, this is 75 was the probability here, that is near let us say point 8? But as you increase number of trials, then this curve becomes stable right? So, relative frequency becomes stable as number of tosses becomes large. But the limitation of this particular types of probability approach is that, you need to perform sufficient number of experiment, before getting answer. So this is relative frequency approach to the probability.

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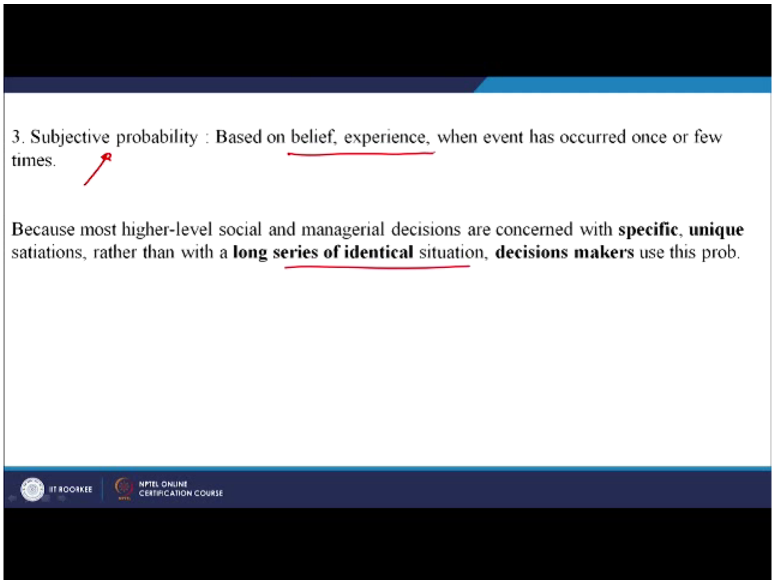

The third and the most important, because most of the people sitting at the top of decision makers, either in business or in any other sector, or in, government or army or whatever area you take they are faced with unique situations, most of the times, rather than facing a long series of identical situations. So they use their experience, their knowledge, their wisdom to make necessary decisions. So this type of probability is known as Subjective probability based on belief, experience and judgment.

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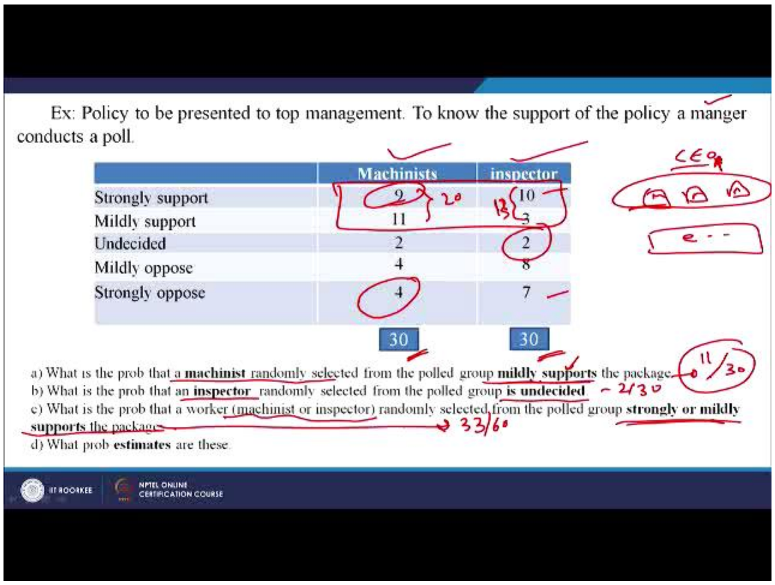

Let us look at couple of exercises. Let us say there is a CEO in a company and there are several managers working under this fellow, these are different managers and there are several employees' right. Now these managers have decided and they would come up with a retirement policy and policies to be presented to CEO, but before presenting the policy to CEO, in fact one of the managers decided to conduct a poll, amongst machinists and inspectors working in that organization.

So he contacted 30 machinists and 30 inspectors, out of 30, 9 of them strongly supported it and 4 of them strongly opposed, similarly for inspectors as well right, now the question is what is the probability that a machinist randomly selected from the polled group mildly supports the package. So first of all, how many machinists were contacted, 30 right, and mildly support right, 11 so 11 by 30 is the answer to this question, this part of the question.

What is the probability that an Inspector randomly selected from the poll group is undecided. Inspector and undecided is 2/30, isn't it? What is the probability that a worker either machinist or inspector randomly selected from the polled group strongly or mildly support the package or the policy, strongly or mildly, so strongly, mildly and all the workers, right? So this becomes 20 and this becomes 13, so total is 33, 33/ 60 isn't it? Because we have to look at all the workers, right?

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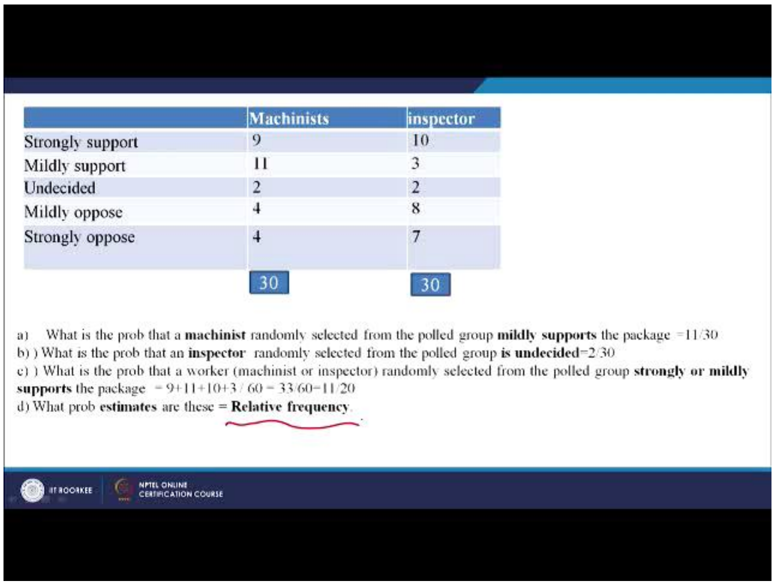

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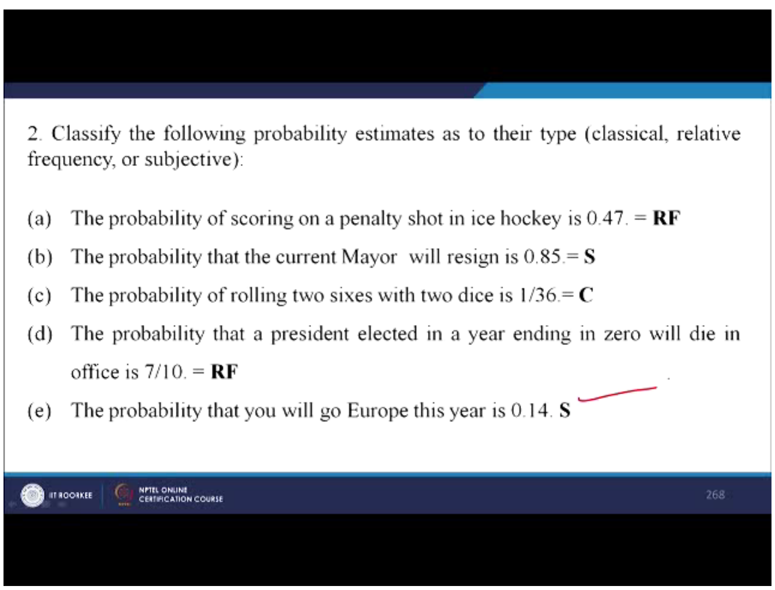

What is the probability, what probability estimates are these? These are relative frequency probability, right? Let us look at one more exercise, now classify the following probability estimates as to their type, so these are different questions and we have to identify which statement is of which type of probability right? Let us look at this; the probability of scoring on a penalty shot in ice hockey is 0.47, so what kind of this probability is this? The probability of scoring on a penalty shot in ice hockey is 0.47, so this is what this information from the relative frequency approach.

The probability that the current mayor will resign is 85% this is subjective, right? The probability of rolling two sixes, probability of rolling 2 sixes with 2 dice is 1/36, could it be relative frequency or subjective, no it is classical approach right? The probability that a president elected in a year ending in 0 will die in office is 7/10. This probability that a president elected in a year ending in 0 will die in office is point 7. This again relative frequency approach, the probability that you will go to Europe this year is what is 14% is again a subjective probability ok.

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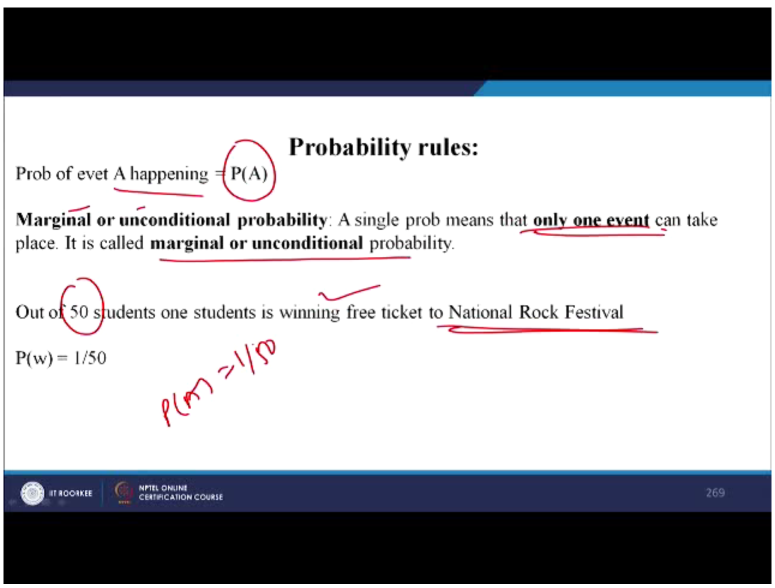

Now let us look at couple of probability rules some addition rules so first of all let us look at what is the probability of happening of an event is P(A), this how we denote it, right? so either we call it marginal or unconditional probability, that is basically a probability of only one event and this all this is called marginal or unconditional probability, right so let us see in your class there are 50 students was the probability that one of them is winning free tickets to National Rock festival or any other festival for that matter it would be just 1/50. So P(A) would be just 1/50.

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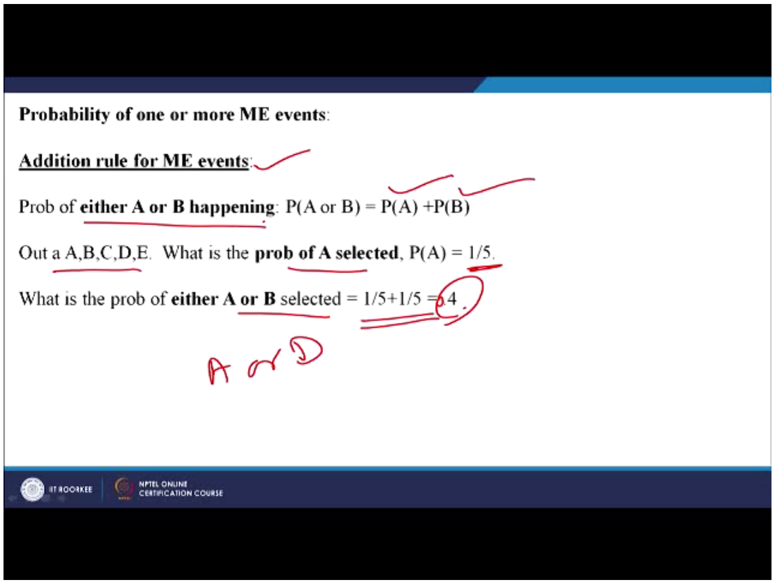

Probability of one or more mutually exclusive events, so what are, what are mutually exclusive events? Those events where in only if there are two then only one will occur at a time, right? so addition rule for mutually exclusive events, so probability of either A or B happening is probability of a + b. so let us say there are 5 students what's the probability of A is getting selected for placement is 1 by 5, A or B selected is just summation of these two or A and D selected again is .4 this is 1/2 and 1/2, 0.4.

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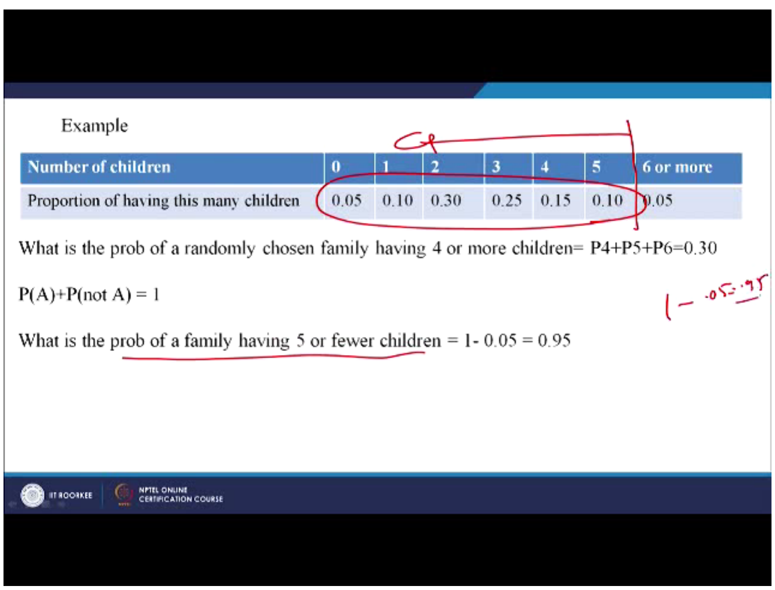

Let us look at this example, so in a survey you have found that the proportion of; in a city you are found the proportion of having these many children, 0, no child in a home and the probability is 0.05. Probability that in home there are five children, is 0.10 and 6 or more is again 0.05, so this information is given to you, you collected this data from a survey. So what is the probability of randomly chosen family having four or more children?

So 4 or more, it means, all these probabilities you need to sum up, so 0.15, 0.10, 0.05, so this becomes 0.30. And we also know that P(A) and P(not A) has to be equal to one, ok so let us have in this question we was we have asked is, the question is like this, what is the probability of randomly chosen family having four or more children if I ask a question what is the probability that the families have got, up to three children, then what it would be its 1 minus 0.3, so 0.7.

Let me delete all this, what is the probability of family having five or fewer children? So we have to look at 5 or fewer, right? Five or less than 5, right, just said all this probability or since you know the probability of 6 or more, you just subtract 1 - 0.05 so you will get 0.95. So this is how you can work out a question like this.

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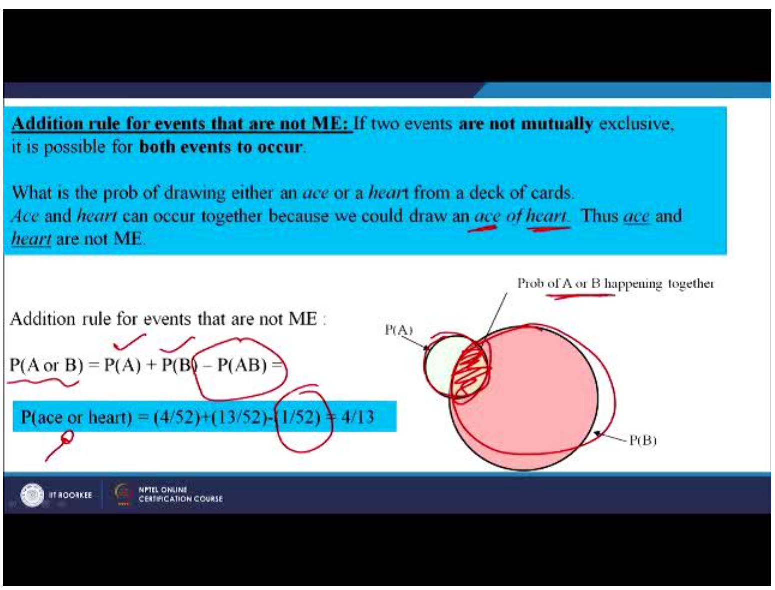

Now let us look at additional rule for those events which are not mutually exclusive, earlier you seen additional rule for events which were mutually exclusive, so not mutually exclusive means what? Both the events can occur simultaneously. So, let us look at this example what is the probability of drawing either an ace or a heart from a deck of cards. Ace and heart can occur together because we can have a situation where in an ace is of heart as well.

So thus ace and heart are not mutually exclusive, they may happen together. So, let us look at this Venn diagram, probability of event A, probability of event B, and this is probability of A or B happening together. So how to work out this example, so probability of A or B is equal to P(A) + P(B) – P(AB). So this area is to be subtracted, otherwise there would be double counting of this area. So probability of ace or heart is what you got 4, number of aces are 4, right, you have got 13 cards of heart, right? And you have got this is 1 by 52. This is to be subtracted, so 4 by 52 this becomes, 17 by 52 - 1 by 15, so this is 16 by 52 which is 4 by 13.

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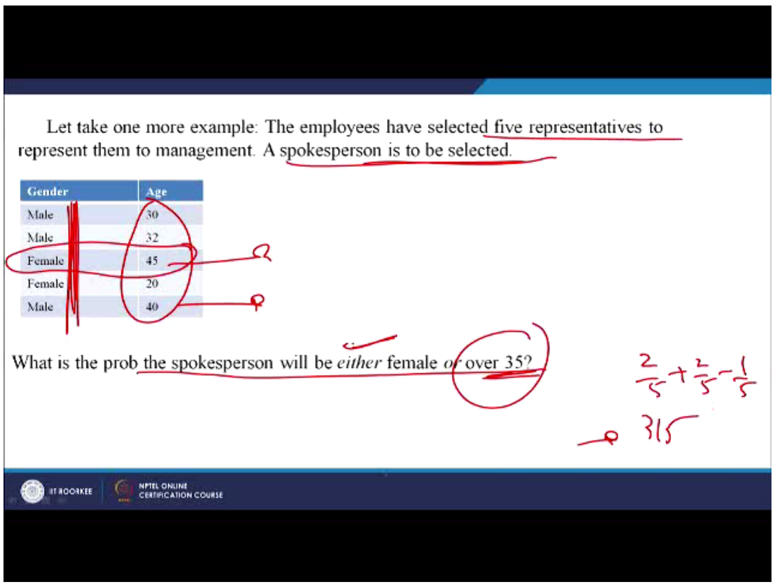

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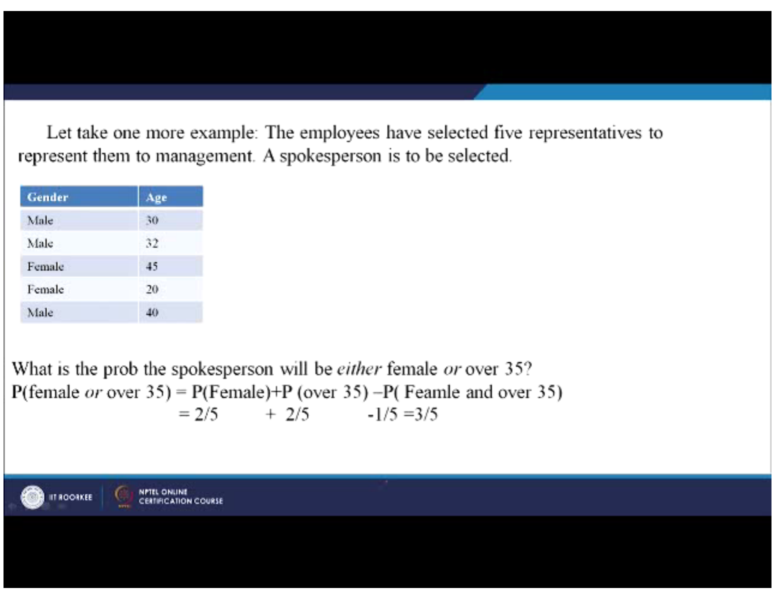

Let us look at this example the employees have selected five representatives to represent them to the management and out of these 5 by spokesperson is to be selected these are different ages and these are genders, so you have got male and females, what is the probability that a spokesperson will be either female or over 35? So, you look at both. You have to find out if probability of a spokesperson is female or more than 35, so how many total people over here, 5.

So, either female right, so how many females, 2 females' right plus over 35, how many people are of over 35, just see this one and this one right. Those employees who are over 35 so this is 2/5. Now what is the probability that the selected person is female as well as more than 35, only one, right, so minus 1/5. This becomes 3/5, ok, this is the answer 3/5.

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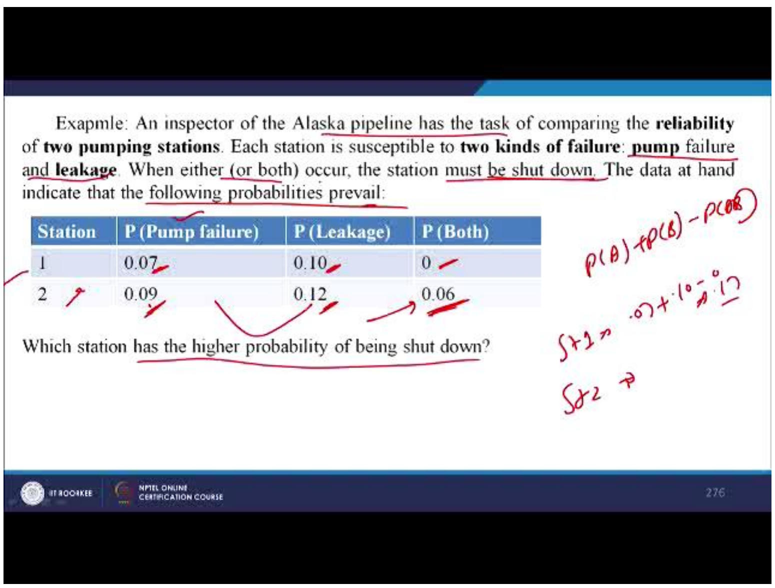

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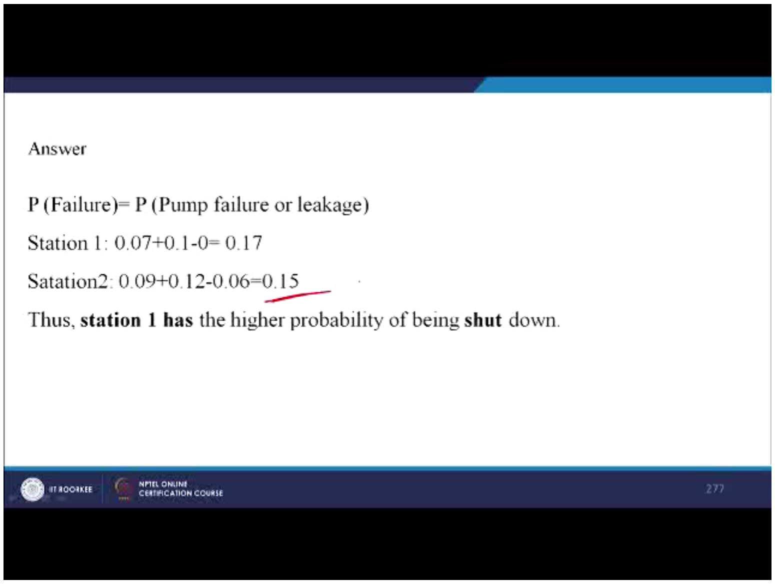

Now let us look at one more question, An inspector of the Alaska pipeline has the task of comparing the reliability of two pumping stations, so there are 2 pumping stations and they may fail in two different situations, there are two types of failures possible, Pump failure and leakage when either both, when either both of these failures are both occur, the station must be shut down. The data at hand indicate the following probabilities, so station 1, probability that there would be pump failure, 0.07, 0.10 and both happening together not possible, right? Probability zero station 2, 0.09 pump failure, 0.12 leakage.

And both because of pump may fail because of both these reasons is 0.06, right, so which station is the higher probability of being shut down, so either this may happen or and this may both may so again is P(A) plus P(B) minus P(AB) isn't it? So far station one, let us look at station 1, 0.07 + 0.1 - 0, becomes 0.17? Similarly you can calculate for station 2, just add these two and subtract this. So, that this 0.09 + 0.12 -0.06 is 0.15. So which station has got the higher probability of shutdown, station 1? It is probability is 17% compared to 15%.

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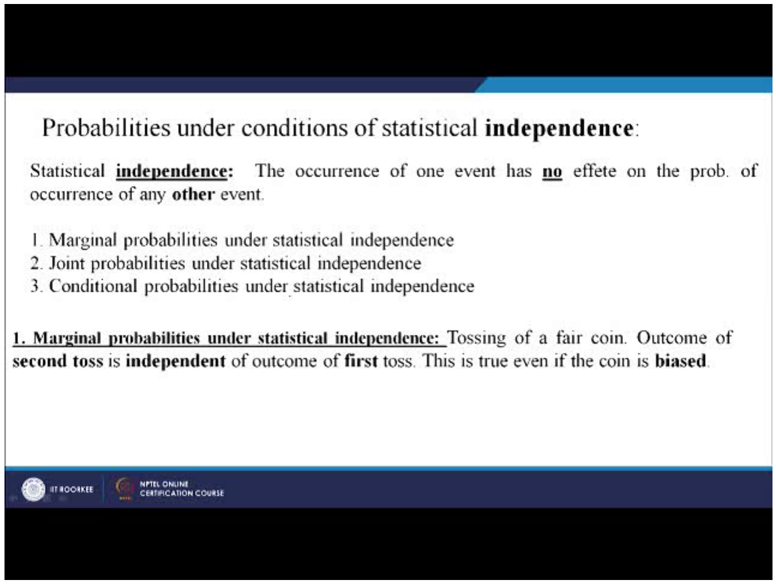

Now before going on to the probabilities under condition of statistical dependence and under statistical independence, let me summarise what we have done in today's session, we have looked at the several concepts related to probability, we have seen what is an experiment, experiment is basically a process which gives us different outcomes. So if you toss a coin, then it's an experiment and it would result in two outcomes. We have seen, what is mutually exclusive event, those events which cannot happen simultaneously. We have seen, what is collectively exhaustive list. So, collective exhaustive list is a list which gives you, all possible outcomes.

Apart from this we have seen different types of probabilities. We have seen classical probability, which is the simplest one amongst all those three probabilities, which we have seen. In classical probability which is also known as a priori probability, so you can find out answer to a question, without doing any experiment or without using your own knowledge. So you have got formula available with you, you just put your values in that formula, and you will get the answer. That's why it is also called a priori probability.

The second type of probability which we have seen is relative frequency approach. Now this type of probability is useful when we, we perform experiments, and when the questions are such that for which you cannot put them into any formula. So either you will have passed information, so on the basis of past information you can know what will happen in future. So we have seen couple of examples, one of the examples I gave was, what is the probability that the speaker of

TV would blow off due to high volume. Now to know the answer, you need to actually turn on TV to its full volume.

So you need to perform experiments and when you increase the number of experiments, you will get better answer, the answer close to the real one. The third probability which we have seen and which is being used widely by several people is subjective probability. In subjective probability, people make decisions on the basis of their experience, on the basis of their intuition, on the basis of their knowledge of that particular industry or that particular area, in which they have been working in. And this is a type of probability, which is being used most widely.

We have worked out couple of examples related to all those three, and I hope that you would have understood, the basics of probability and different types of probability. With this, let me complete today's session, will have the probability under statistical dependence and statistical independence in next few classes. Let me just tell you what we would be discussing in next few classes. So you can have statistical independence probability, where in the probability of occurrence of an event, depends on the probability of occurrence of, of previous event.

You may have a situation where the probabilities are statistically independent. So the probability of happening an event will not depend on the probability of event which is already happened. And under both these categories, will have marginal probability, joint probability and conditional probability. So these 3 probabilities under statistical dependence and statistical independence, we will see all these in next couple of lectures. Thank you very much.