Marketing Research and Analysis -II (Application Oriented)
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Lecture – 55
Confirmatory Factor Analysis in SPSS - II

Welcome everyone to the class of marketing research and analysis. So in the last lecture, we had started discussing about factor analysis, right. So we explain, we understood that there are 2 types of basically factor analysis. One, the exploratory factor analysis and the other is the confirmatory factor analysis, okay. the confirmatory factor analysis in fact comes under the SEM group, right.

So what is this factor analysis. We had learnt that factor analysis is a data reduction technique, okay which helps in data summarization and data reduction. And further, those data that has or the factors that are generated or developed, they further can be utilized for any regression kind of analysis, okay. So what if you learn that in the exploratory factor analysis, right, we had no theoretical background as such, right.

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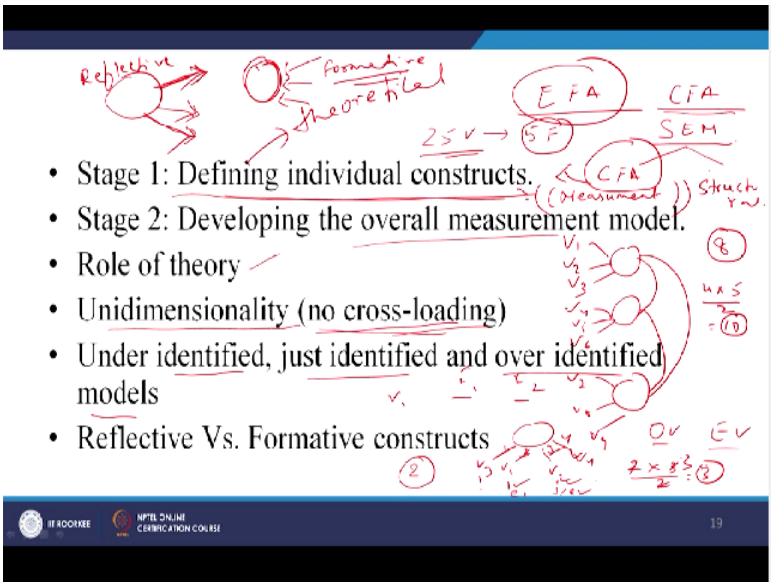

So we develop, we explored and we tried to find out what are the possible factors that may emerge after reducing the large number of variables that one has got. That means suppose somebody has got 25 variables, right. So out of these 25 variables, he draws, let us say, 5 factors.

So this 5 factors are nothing but representation of the 25 variables. And how do we select these 5 factors?

There are several ways which you can select. One of them could be like, for example, the amount of variance explained. It could be by the eigenvalue, right. Se several ways you can do that. May be with the help of a scree plot also you can find out graphically. So this is what we understood. But on the other hand, there are times when you do not explore but you try to confirm an existing theory, right.

So this method is called the confirmatory factor analysis. So confirmatory factor analysis actually is a part of structural equation modeling, right, which is, now it is very highly utilized in almost all research, by all researchers for writing research papers. And one of the 2 is the CFA, right. Or we say it is a measurement model, okay. And the other is, we say the structural model. So you can understand this that the factor analysis is either you go through SEM and try to understand.

So it is all the same. So there is one exploratory factor analysis and the other is called the confirmatory factor analysis which is nothing but we say it is a kind of checking whether a model, what is a model? A model is a theoretical representation of certain variables, okay. So when you are saying that there certain variables or constructs are required to explain a complete theory, in such a condition, we develop this measurement model and try to check it.

So that is what we are going to do it here. So we said while doing a confirmatory factor analysis, you have to define the individual constructs. So as you have understood that a construct is nothing but which consists of a large number of variables. So minimum we said there could be 2 to 3 variables or even more than that. So if you have less than 3 variables per construct, it could be an under identified model which is not good.

So let us see. So first you define the individual constructs. And you define it through certain theoretical inputs, right. So what theoretical knowledge you have got, accordingly you define the individual constructs. This constructs can also be taken from certain earlier literature. That

means somebody has done or developed a scale and suppose I have developed a scale, you can use my scale for your study purpose if it is a similar kind of a study and you can use it, so right.

So that is a theoretical backup. Second thing is developing the overall measurement model. Now how do you do, that is where the role of theory comes. Now we are saying suppose there are 3 constructs, right. So in the measurement model, each has got, let us say, several variables, V1, V2, V3, let us say V4, V5, V6 and this is again V7, V8 and V9. Now what we do in the measurement model is?

It is a covariance structure. So we try to see, right. So what kind of relationships exist. And first we will try to, in the measurement model, we will try to see whether the overall model is a fit model or not. Now what is this meaning of this fit, model is fit or not. So just imagine there are 2 things. One, if you have understood the chi square test for example. You must have seen in chi square that there are 2 things we were talking about.

One is the observed and the other is the expected value, right. So the observed value and the expected value is what we were checking. So if the difference between the observed value and the expected value is less, then we say the model is a very highly fit model. Similarly, the same concept holds true here. So in this case, what we are trying to do is? There is an observed matrix and the expected matrix.

So the difference between the chi square value between the 2, right, should be minimal, right. So this is what we do in the measurement model. Then we talked about unidimensionality. What we did here? In the exploratory factor analysis, we have seen that the factors, the variables can load into several factors. That means what? That means a variable may load into factor 1 and you can at the same time factor 2, right.

But in the case of CFA, it is not possible, right. In CFA, because it is evolving from theory, we are saying that if a variable can load into 1 and only 1 factor, it cannot go into the other factor, it is restricted, okay. So no cross loading is possible, okay. The next thing is, we talked about the under identified, just identified and over identified models. So this is simple. You can

understand.

It depends upon the number of parameters to be calculated. For example, let us say you have a model with less number of variables. Let us say one construct with, let us say, 2 variables. So we said now if you want to check this model, this could be an under identified model. Why? Because if, let us say, this is the construct and these are the 2 variables, V1 and V2, right. So what we do is, and there are 2 error terms, E1 and E2, right.

So how many parameters are going to be measured? 1, 1, 2, 3, 4, right. And if you calculate the number of estimated parameters, it will be, the formula is
= 2 * (n+1), so 3/2. So that
= [n*(n+1)]/2. So that is equal to 3. So you see now you have 3 to be calculated and estimated and here the model has got 4. So this is an under identified model, okay. Suppose you would have got, let us say, 4 variables, 4 different variables, V3 and V4.

So how many parameters you can calculate. Now if there are 4 suppose then we would have each 1, 2, 3, 4, +4 error terms. So 5, 6, 7 and 8. So 8 you have an estimated parameters or the data points which I was saying is (4*5)/2. So that is equal to 10. So this says that we have more number of estimated parameters, so data points. So this is an over identified model, right. So over identified model is always preferable, okay.

Then we talked about reflective versus formative constructs. And mostly we have seen that we talk about the reflective construct that is what? One Latin construct explaining the variable, right. So Latin construct explains the variables and this is a condition which is known as the reflective construct. Had the, on the opposite side, had it be like this that the variables are explaining the construct.

Then this is called formative. This is called formative. And then the latent variable is explaining the individual items or the variables. Then that means arrow is moving towards the variables. It is a case of reflective, okay. And then the arrow is moving from the variables to the construct, then it is a case of a formative construct, okay. So this is what we had understood in the last

class. Now we will move into designing a study to produce empirical.

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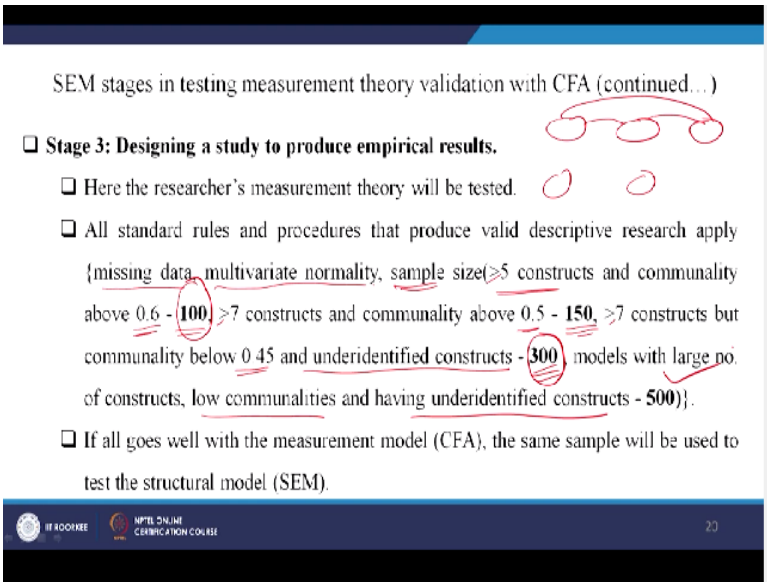

So stage 3 in the CFA, confirmatory factor is to design a study to produce some empirical results. So here the researchers measurement theory will be tested. Now researcher has got several some measurement theory. Now what it is? Now it says that there is certain relationship among the constructs. So there are 3 constructs, let us say. So there is a relationship, okay. So there is a relationship among them, right.

And this has to be tested. All standard rules and procedures that produce valid descriptive research apply to here also, apply here. Now what are these? For example, you have to check for missing data. When you are doing SEM or CFA, your missing data should be minimally there. It should be, if it is, if you can avoid it, nothing like it. If it is, there is no missing data, it is better, right.

So it is preferable to have a data set with no missing data, right. So what you can do is, you know I have already explained you in the earlier lectures where you can replace the missing data with some new value, okay. Multivariate normality is also a problem but that can get corrected with an increase in sample size. Because most of the models are sensitive to sample size, okay. Now I have given you this understanding.

You can see, when you have less than 5 constructs, so 1, 2, 3, let us say, 2 more, 5 constructs, let us say, are there. Then and the communality is above 0.6. Now communality, I had already explained, right. Communality is nothing but when the connection or the loadings or the relationships, the correlation between the variable and the factors. So the square of this value, the sum of the square of this loadings is called the communality.

So when this is above 0.6, you need to have a 100, around 100 sample size. But if the number of constructs is 7, this is equal to or less than, right. Is equal to or less than 7, and the communality if above 0.5, that is moderate, right. Then you need 150 sample size. When the communality, sorry, when the number of constructs is greater than or equal to 7, but communality is less now.

The communality is less, is 0.45 and below, then and you have under identified constructs. That means a construct with less than 3 number of variables, right, explaining it. There you require higher number of sample size, around 300, okay. When your models with large number of constructs, may be 8, 9, 10 constructs, right and the communalities are low, right and you have under identified constructs, a sample size of almost 500 is required, right.

So you can understand the relationship, okay. If all goes well with the measurement model, then the sample will be used to test the structural model. So the SEM part. So as I said, so you can understand that SEM has 2 parts. First part is the measurement part. The second part is the structural part where we explain the relationship. So the measurement part is what we are talking in the CFA.

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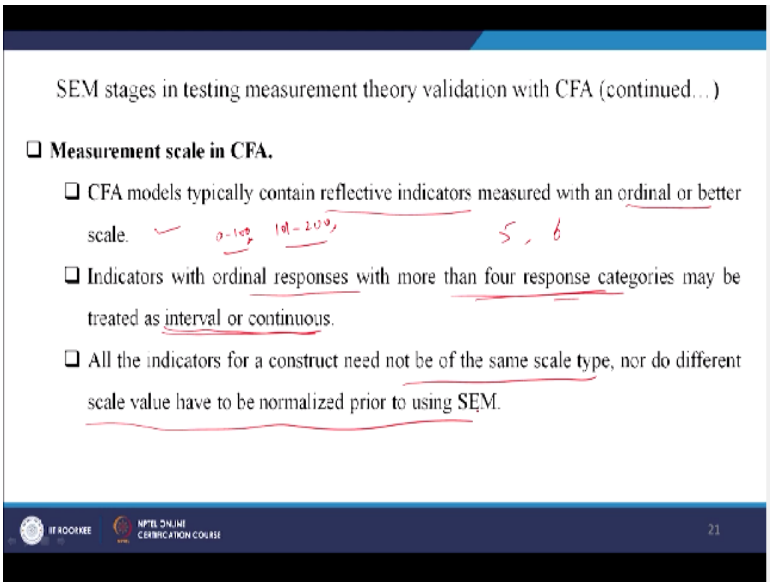

You see the CFA models typically contain reflective indicators, which we have understood, measured with an ordinal or a better scale. Better scale means? Ordinal or ratio, interval or ratio scale, higher scale. Indicators with ordinal responses with more than 4 response categories, that means 5 or 6 or 7 or whatever, may be treated as an interval or continuous, okay. Can you understand that?

It means that whenever you have collected the data in an original response, that means in a like a rank order form. And you have more than 4 response categories. Categories are for example, let us say, 0 to 100, 101 to 200, these are categories, right, class intervals. So when you have more than 4 response categories, you may be, it may be treated as an interval or continuous, right. All the indicators for a construct need not be of the same scale type.

This is very important because people are confused. Should it be of the same scale type or not. It is not necessary, right. Nor do different scale value have to be normalized prior to using SEM. So here you are not, you should not be worried about the type of the scale, right. But it is better if you have the same scale. There is no doubt about it.

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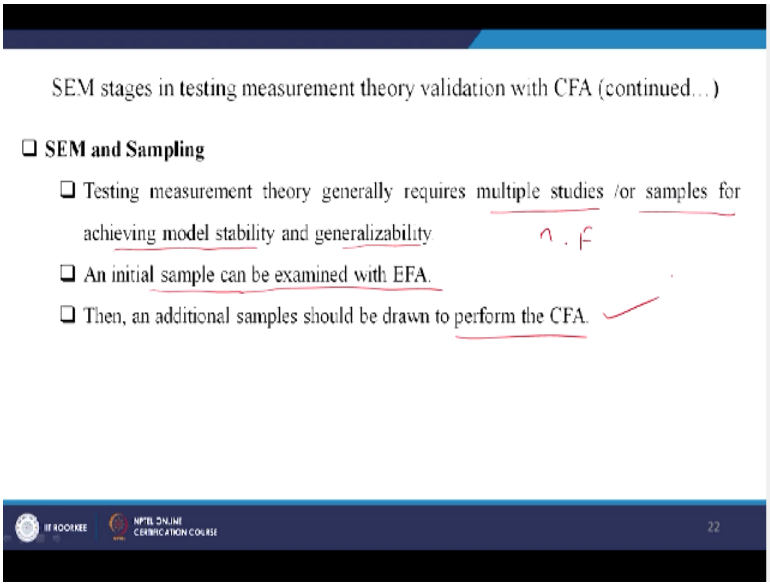

Testing measurement theory generally requires multiple studies. Obviously you have to validate, right, or samples for achieving model stability and generalizability. An initial sample can be examined with EFA, exploratory factor analysis. So you have a sample, you do a exploratory factor analysis and you, let us say, generated n constructs, okay, n factors or n constructs, right.

Then the additional samples should be drawn to perform the, some more samples, that is more data sets sample respondent should be selected and data should be collected to perform the confirmatory. So first you delete the EFA. Please this is very important. You can listen to it carefully.

First you should do the EFA and whatever factors have come, you may also check that through the CFA. Or when you have, this is only possible when you do not have any earlier idea. But suppose you have some earlier built up scales already there, some researcher has already developed a scale, then you can straight away test this through the CFA, right.

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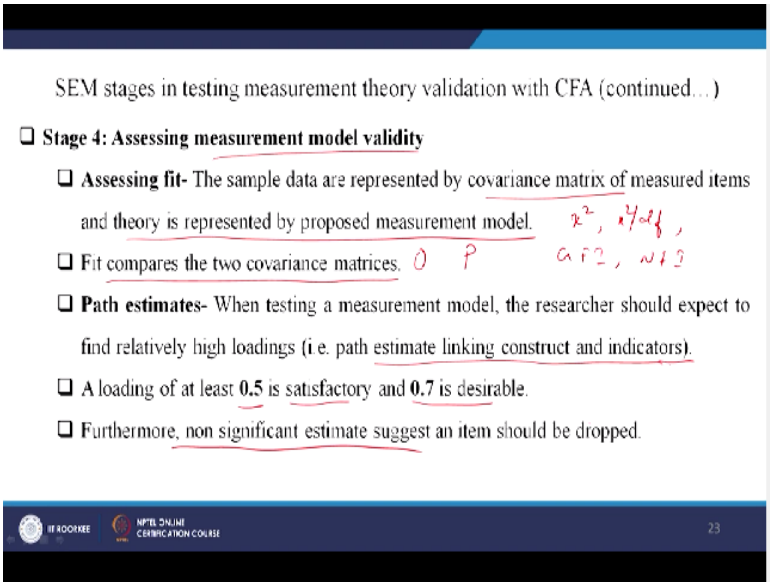

Next is, you have to assess the measurement model validity. Now here, we are saying that the model should be fit. Now how do you check? Now this is important. The sample data are represented by the covariance matrix or measured items and the theory is represented by the proposed measurement model, right. So what we talk about is the model fitness. So here, there are certain fitness indices which for example we say the chi square; the chi square degree of freedom, right; chi square/degrees of freedom, right.

And then we talk about the GFI, so goodness of fit indices. And a phi norm fit indices. So there are several fit indices. And we take, and this fit indices usually tell us about whether the model is a fit model or not, right. So this is the time or stage where we test the model, okay. So as you can see I had mentioned here, it compares the 2 covariance matrices. Which 2 covariance matrices I am talking about?

The observed and the predicted or the expected, okay. Path estimates, when testing a measurement model, the researcher should expect to find relatively high loadings, right. Now this is very important. That means what. Your loading should be very high. What is the loading? Loading is the relationship or the correlation between the variable and the construct or the factor, right.

That is path estimate linking the construct and the indicators. Indicators or the variables, right. A

loading of at least 0.5 is satisfactory and 0.7 is desirable. Even more than that is desirable. Further this is to be noted. Non-significant estimate suggest an item should be dropped. How do you check this? I will show you.

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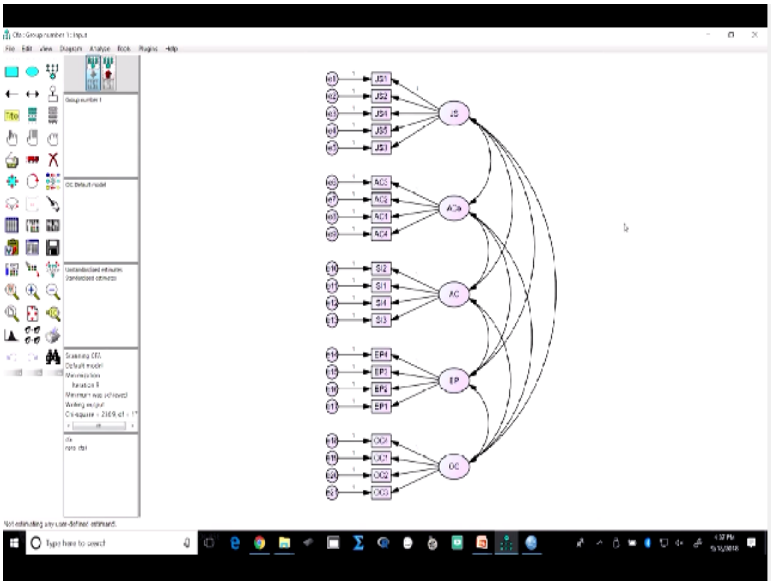

Now let us see, this is the data set I will be using later on. I have explained in my PPT also. Let us run this, okay.

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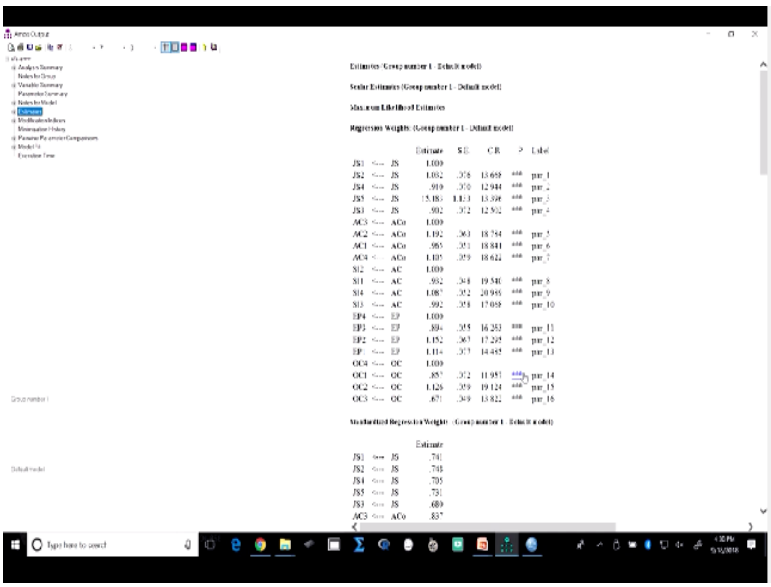

Now let us go to the data. This is the estimate I am talking about. Now if you see, this is the estimate, right. This is the covariance values, okay. This is the standard error, this is the critical ratio. So this critical ratio is nothing but this divided, estimate/standard error, okay. And this tells

you whether it is significant or not.

So 3 stars means significant at the 0.001 level, right. So now suppose something would not have been significant, then it would have been more than 0.01 or 0.001 or 0.05, whatever significance level you have taken. So if it is more than that, then in that case, it is better to drop that item, okay.

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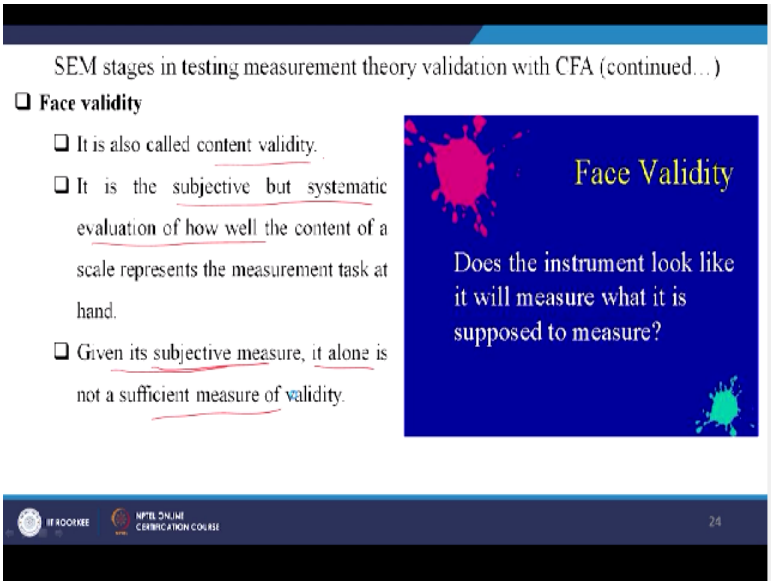

Now coming to certain validity issues. First of all when you are doing CFA, you have to check for validity. Now what are the different types of validity. One is face or content validity, right. Then we talk about convergent validity. Then we talk about discriminant validity. Then we talk about nomological validity. Now these are the 4 validities together which is called the construct validity, right.

Now we will see, and any scale that you develop, must go through these phases of validity and they should come successfully, they should be successfully tested and they prove their validity, okay. So what is this face validity. It is also called content validity. Is a subjective but systematic evaluation of how well the content of a scale represents the measurement tasks at hand. That means suppose you have done a questionnaire.

You have several indicators too as a part of your study. Now whether this question that you have

asked, do they actually represent the objective of your study, yes or no. Now how will you do that? To do that, may be you will have to take opinion of experts from that area and ask them and check whether what you are doing is correct or not correct, okay.

It is subjective. Given its subjective measure, it alone is not a sufficient measure of validity. So it is the first step but it is not. Individually it is not sufficient to measure validity. So there are other things which I just said. For example, convergent, discriminant, and other things and nomological.

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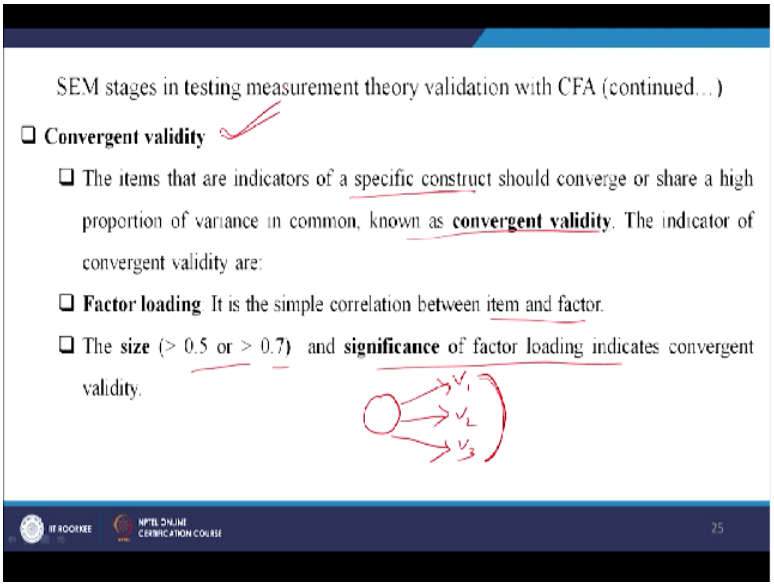

So let us see. So second is the convergent validity. The items that are indicators of a specific construct. Now as a construct as I said the construct can have, let us say, minimum I am taking 3 items, 3 indicators or items whatever you say, right. V1, V2, V3, okay. This is a reflective construct, okay. So what is it. The items that are indicators of a specific construct should converge or share a high proportion of variance in common, right, known as convergent validity.

Now this means what? When there is a high correlation among this 3 indicators. That means what? In a layman's term, you can understand that they all are coming from the same population, right. And if there is a high correlation, the unexplained variance would be quite low, right. As you have talked about in the same thing in regression, right. Now factor loading. It is a simple correlation between the item and the factor. Item or the variable or indicator, whatever you say,

right. So size should be 0.5 or 0.7 and significant, right. If it is there, then it is passing the test of convergent validity.

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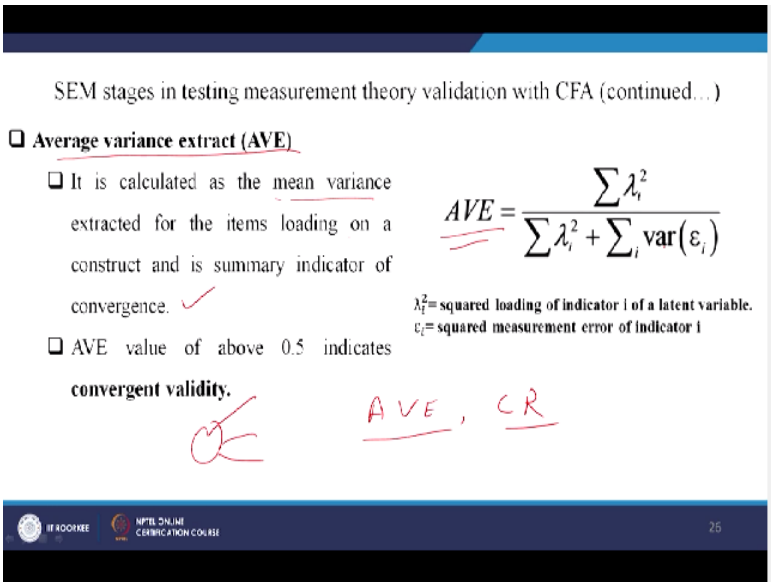

Next, you will talk about convergent validity, there is also a case called, we need to understand that in CFA, there is an extremely important to, in order to understand about the next case, is the discriminant validity. So what is this? Before that, we will understand what is this term average variance extract. Now what is this?

It is calculated as the mean variance extracted for the items loading on a construct and is a summary indicator of convergence. Now, so while checking for convergent and discriminant validity, there are 2 terms which are very important. One, the average variance extracted. This is the one.

And there is something called a composite reliability, right. So these 2 indicators are very important to be measured. And they help you in the convergent and discriminant validity. Now what is it and how it is helpful, let us see. So as it says average variance extracted is nothing but the mean variance extracted for the items loading on a construct. So there are 3 items in this; suppose in the case, there are 3 items, okay.

And this is summary indicator of convergence. Now let us see. Now this one.

AVE = lambda, lambda is nothing but my loading, summation of the loading square, right. There are 3 variables. So

= $\sum (\lambda_1^2 + \lambda_2^2 + \lambda_3^2) / (\sum \lambda^2 + \text{variance})$.

Now what is this variance. Variance is nothing but the (1- λ), you will see that in the next, right. An AVE value of above 0.5 indicates convergent validity, right.

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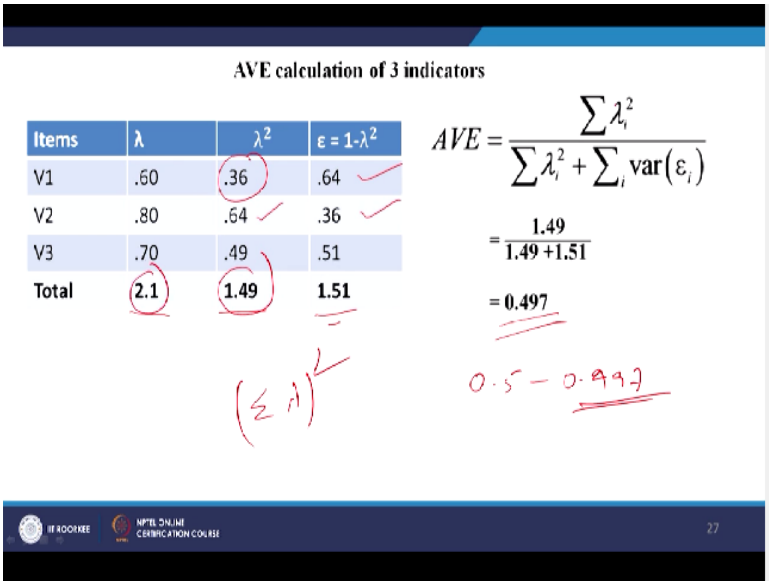

So you can see here. So what it says? Now E, this one, right, this part, variance, is equal to 1-lambda square. Now this is, suppose lambda, the loading, the correlation between the variable and the factor, is let us say 0.6. So what is my lambda square? 0.36. So what is my variance? Now 1-this much.

Now how much? Now 0.64. V2 0.8. Now this is 0.64 and this is 0.36. So if you say total my lambda is this much, my lambda square is this much and my variance is this much. Now using this in the formula, we can calculate. Similarly you can calculate for your own, in your own study. So

= 1.49/(1.49+1.51)

= 0.497.

Now you see, we are just missing the case of, case by 0.03, right, not even 0.03, 0.01, less than that even, right.

Because 0.5 is the cut-off value. So 0.497 is quite a small margin. You can, if you round off, it will be 0.5 only. But understand that it should be always, the variance extracted should be more. That means what? What does it mean in a, you have heard the word variance more often in regression, right. So in the method of least square.

So you say that if the variance extracted is more, that means what my unexplained variance is less, or my error term is less. So that is what we are trying to do here, okay. Similarly, composite reliability is another measure which is an indicator of, is also an indicator of the convergent validity. So it is computed from the squared sum of the factor loadings.

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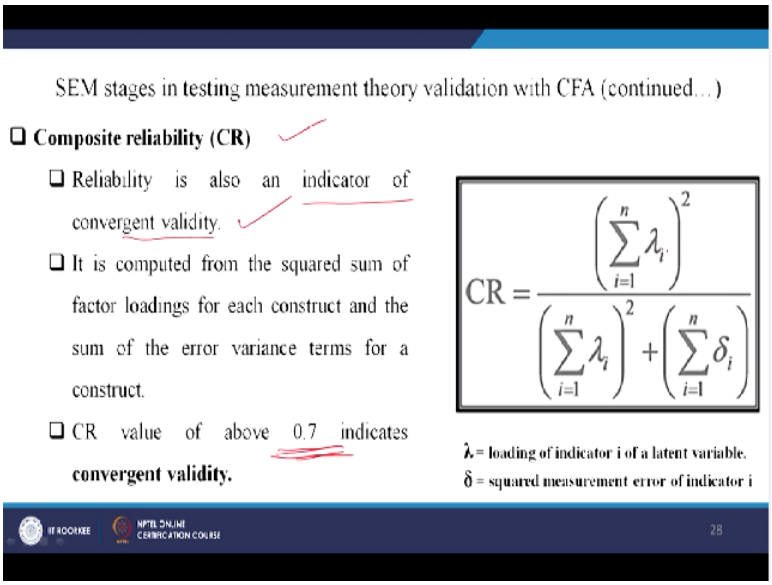

Now we will just directly go to the formula. So composite reliability

$$CR = \frac{\left(\sum_{i=1}^{n} \lambda_i\right)^2}{\left(\sum_{i=1}^{n} \lambda_i\right)^2 + \left(\sum_{i=1}^{n} \delta_i\right)}$$

Now composite reliability is the cut-off value, has been kept at 0.7, right. Now let us see.

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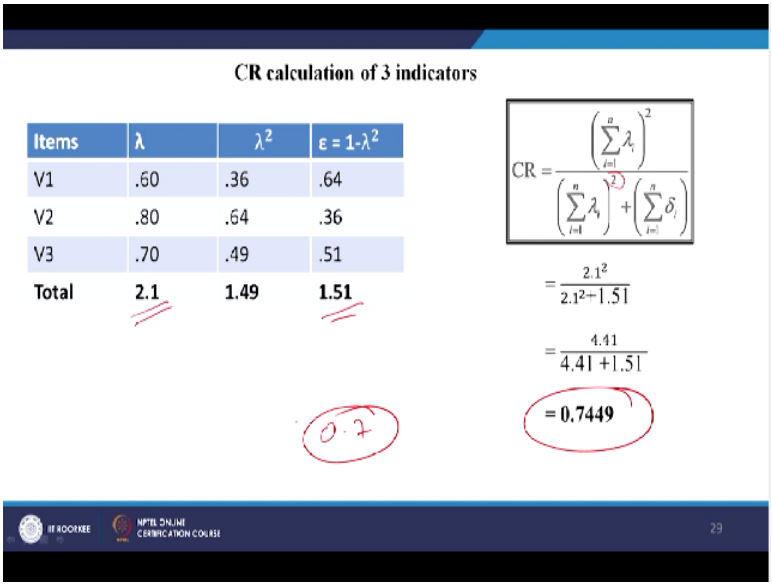

So what is happening here? 0.6, the same thing we have taken. So

$= (2.1)^2/[(2.1)^2 + 1.51]$

$= 0.7449$

If you can see the only difference here is that, there we were not squaring this part. Let us go back and check. So the next thing is the composite reliability. As you can see the reliability, this is also an indicator of the convergent validity. And how it is measured? It is much similar to the AVE.

The only difference here is, we are taking the summation of the lambda and then squaring it, right. So look at the difference here. So we were taking here as this value, right. So each time the lambda square summation of this. But this time we are taking this value, right. The summation of the lambda which is 2.1 and then squaring it here in this case. Now look at this. For example, in this case, we are calculating the composite reliability.

So this is equal to 2.1, this

$= (2.1)^2/[(2.1)^2 + 1.51]$

$= 0.7449$

Now this is coming to 0.7449. So we have set the cut-off value is 0.7. So our composite reliability is sufficiently okay, good or high, right. So these 2 measures are stringent measures, okay. Now coming to the next thing as I had said after convergent validity, there is something

called a discriminant validity. So the discriminant validity is the extent to which a construct is truly distinct from the other constructs, okay.

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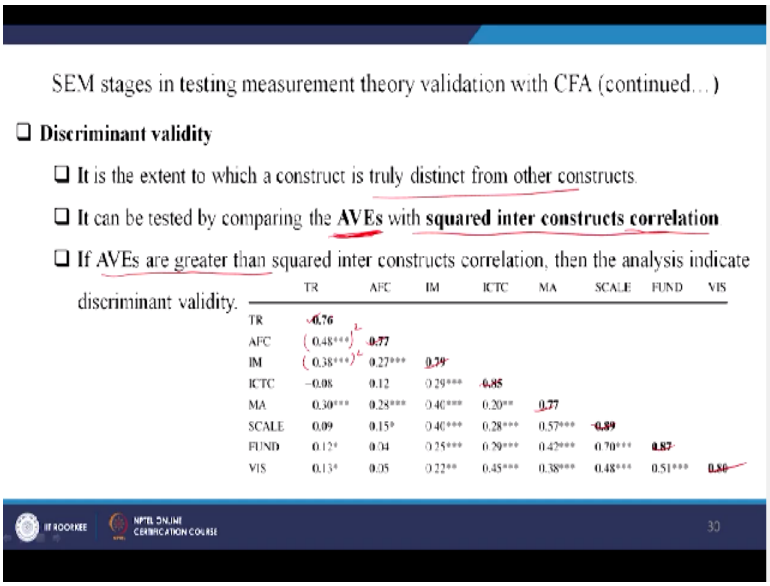

So it can be tested by comparing the AVE. How do you check it? Now you can do this by checking, comparing the AVE, the average variance extracted with the square inter constructs correlation. Now this is the table for example, let us say. What do you do is, all the AVE that has been calculated or has been kept in the diagonals, okay. So you have calculated the AVE for each construct.

Now keep it in the diagonal. And all the correlations, for example, between the constructs, right, has been kept it here. Now what it says is the AVE should always be greater, should be greater than the squared inter constructs correlation. So suppose this square, so whatever value comes, right, squared, this one. So this value should always be lower than the AVE value, right.

So if it is true, then we can say there is a clear discriminant, this is a clear case of difference between the 2 constructs. That means the items are discriminant, there is a discriminant validity among the constructs, okay. The last is the nomological validity. Now although it is highly theoretical but it is very important.

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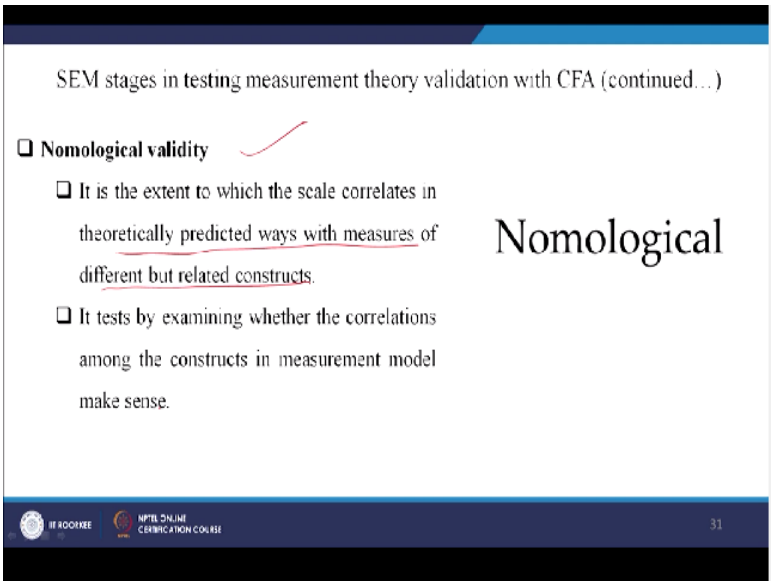

It says that the extent to which the scale correlates in theoretically predicted ways with measures of different but related constructs. Now what does it mean? Okay, let us read this, then we will go to. It tests by examining whether the correlations among the constructs in the measurement model make any sense or not. Now nomological validity says that because you have developed a model out of theory.

CFA has been developed out of theory, right. So whatever theory you had in mind and you have drawn a model out of it, now is this, and what you are getting, is this value that you are getting, is this falling in line with your theoretical understanding or theoretical inputs. If it is yes, then you have a nomological validity. If it is no, then you will say that it is not proving nomological validity.

Did you understand? That means what? Whatever we are assuming and while developing the model, it should fall in line with actually the theory that has been, that it has been based on, right. If it is yes, that means our observed, our calculated values falling in line, then it is fine, right. Otherwise, it is a no, okay.

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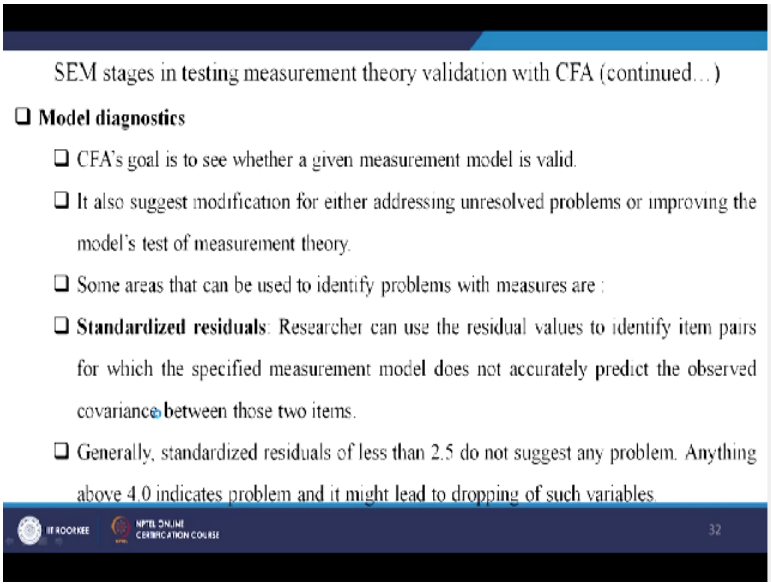

Then we talk about the model diagnostics, right. So the model, just I will take a break here. We will take it in the next lecture but let me explain you. Now once you have done the model, as I said there are several fit indices, right. And you have to understand this fit indices and check whether your model is actually meeting these indices or not, this, the cut-off value for this. If it is a yes, then it is good.

Sometimes, the model do not behave in the manner you want to. And in those cases, you have to improve it, right. And there are ways how you can improve the models, right. So by improving the model, you can get a better model, right. So what we will do is? In the next lecture, I will explain you about the same thing when a researcher has a problem in hand that the model is not a fitting model.

And now he is confused whether should you, what should he do? Should he try to manipulate, should he try to do something without any understanding. So there I will explain what should be the scientific approach to handle that model in that case, right. So we will continue the same in the next lecture. Thank you so much.