

Marketing Research and Analysis-II (Application Oriented)
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Lecture – 28
Non-Parametric Test - IV

Welcome friends to the session of Marketing Research and Analysis. In this session, we will be continuing with the nonparametric test. Earlier we had done with some of the nonparametric tests like the Kruskal-Wallis test which is a similar kind of test as like the ANOVA in parametric test and then we did the Mann-Whitney U test which is similar to the independent sample t-test of a parametric test. Then also in the last lecture, we did the runs test.

The runs test is a test in which we try to check the number of occurrence of an event and whether it is random or nonrandom. So today we will continue with some of the more nonparametric tests and we will see how they can be useful for any researcher and they can be even quoted in refer to in some of kind of journals, publications, and in academic work. So a runs test just to brief, I have just kept this one slide.

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RUNSTEST & SIGNTEST

- Runs test is the test of randomness for a dichotomous variable.
- This test is conducted by determining whether the order or sequence in which observations are obtained is random.
- It can be effectively used in quality control situations. It helps in detecting whether the variation in quality is systematic or random and accordingly take action.
- Sign test is a test for examining differences in the location of two populations, based on paired observations, that compares only the signs of the differences between pairs of variables without taking into account the ranks.

So the runs test and the sign test which we did was a measure to test the randomness of a dichotomous variable. So in the runs test, we tested what is the randomness. So if it is random or if it is nonrandom, accordingly we can decide what actions needs to be taken and this is

especially true or more useful in case of a quality control check. So this test is conducted by determining will order of sequence in observations are obtained is random or nonrandom.

As I said this is used in quality control situations and helps to detecting whether the variation in quality is systemic or random and accordingly take necessary action. On the other hand, we said sign test which I had also said at that time I have been repeating is not a very powerful test but is a test for examining the differences in the location of two populations based on the paired observations that compares only the sign of the differences between the pairs of variables without taking we count the ranks.

So what is the rank that is immaterial to us, rather we would only say whether it is bigger or smaller, greater or lesser that is all and according to the number of signs, the number of greater value or the number of lesser value, we will decide whether the hypothesis is true or not true. So these two tests we had done. What we can do is, I am thinking of showing you how to do one of these tests in the SPSS. So this is the runs test which I wanted to show you.

(Video Starts: 03:17) So this if you can see this is the data in which if you can go to the variable view, so the quality of item is being measured. So if it is the quality of item if you go, if it is one, we say it is a good item and if it is zero, let us say it is a defective item. So there has been about 40 such samples have been taken and the first item was a good one second, third was defective. One is the defective and no defect is zero actually. So 2, no defects, then again a defective, then 5 no defects, then 3 defective, then again good, then defective, then good, then defective series, then good, it goes on.

So now we want to see whether this you say what is the hypothesis. There is no, it is a random occurrence, so the occurrence of the defective or the non-defective item is random in nature. So let us run the test on the SPSS. So if you go to analyze, go to nonparametric test and here you can see one sample, independent sample, related samples and you go to legacy dialogue, here you will see the different again examples of chi-square, binomial, the runs test and now we want this runs test. So we will take this variable into here.

As you know this is a nominal variable, so it is 1 or 0, so defective or not defective. Now this mean, median, mode if you see now if you remember in the case of a nonparametric test the value that we use is often the median, but in if the data or the value if the test is a parametric

test or a normally distributed, the reference is always the mean, but here we will take the custom point as something which we want as a cutoff value. Now what is the cutoff value. A value in between 1 and 0 here in this case.

So what I am doing is I am taking a value of let us say which is 50% between, so 0.5. So I will take this and what are the options. I will take just the descriptive to see what is happening, so okay. Now if you look at the descriptive statistics, the quality of item N 0, the mean is 0.43, but although it makes hardly any sense, deviation is 0.501, minimum and maximum is this much. Now look at the runs test, the table, now what is it saying?

The test value is 0.5 which we had kept as the cutoff mark, the total number of cases is 40, the number of runs is 16, now how did this come, you have already done it, just to remind. So 1, then here 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, so you can go add and it will be 16, but suppose let us say if you see the output again, now what is it saying the Z is -1.328 and the significance value is 0.184. So since we have a value of generally we take the 5% level or a 10% maximum, it is a much higher value than 0.05 or 0.1 right.

So in such a condition, we will say that since this value the significance value is much higher than that, so the null hypothesis is to be not rejected and rather it should be accepted. So that means what, what is the null hypothesis, that the occurrence is random in nature, but that is not what the researcher is interested. You can just change the data a bit and see by suppose, what I will do is I will reduce the number of the occurrences. So what did I say if the number of occurrences is too few or the number of occurrences is too large, then the systematic error becomes larger in role than the randomness.

So let me show you an example. Let us take this case, what we will do is instead of this 1, I will make it 0, so this is 0, so I will put in more number zeros a bit. So let us say this is again a 0, this is a 0, let me see if we take little more, this is 0, this is so I can put it 0 right. So now let us rerun this data and I will go to the same nonparametric test and go to the runs test and I am keeping everything intact and just trying to check. Now if you see now what has been done here, what is the difference between the two?

If you look at the 2 values here, what did I do, I converted some of the 1s to 0s, that means what the 1 was the case of a defective item and the 0 was a good item. So what I did by doing

a large stretch of values, I took it without any break, so the number of occurrences I have reduced now, the number of occurrences of the groups I have reduced. So how many groups are there now; 1, 2, 3, 4, then you have let us say 4, then 5, 6, 7, 8, 9, 10. So earlier you had something 16, now I have brought it to 10, so 10 runs I have.

So when the number of runs has been reduced (**Video Ends: 09:29**), automatically the chance of getting a significant value is becoming higher. So too large, too many runs or too few runs, both create a chance of accepting the alternate hypothesis. So this is what is the runs test.

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Rank Correlation

So now we will do one more test which is the rank correlation test. So this test is basically a test of correlation between 2 variables which are measured in an ordinal scale in a ranked way. So what is this rank correlation? What is correlation by the way? Correlation talks about the strength of 2 values, the strength of the relationship of 2 variables. So if the 2 variables have a very strong relationship, so we say very strong positive relationship or it could be a very strong negative relationship or it could be a very poor relationship also sometimes if it is not very strongly related.

So let us say sometimes we say the height of a person and the weight of person, it has a very strong correlation. If he is more tall, then naturally there is a chance that the weight of the person also might be higher. Similarly one interesting observation has been seen that the people who wear let us say watch on the right hand are more artistic in nature. So the degree of being artistic in nature and the wearing of the watch in the right hand are highly correlated.

So such kinds of events are very important in life to understand which 2 variables are correlated. To do this if your data is a normal data, then there is no issue we use a pure sense correlation, but if the data is not in a continuous way, so we in that case is following a nonparametric distribution or a distribution free data, then we will say it is a rank correlation or we say the Spearman's correlation.

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Rank Correlation

The Pearson product moment correlation coefficient is a measure of the linear association between two variables using quantitative data. In this section, we provide a correlation measure of association between two variables when ordinal or rank ordered data are available. The **Spearman rank-correlation coefficient** has been developed for this purpose. ✓

SPEARMAN RANK-CORRELATION COEFFICIENT

$$r_s = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 + 1)}$$

where

- n = the number of observations in the sample
- x_i = the rank of observation i with respect to the first variable
- y_i = the rank of observation i with respect to the second variable
- $d_i = x_i - y_i$ ✓

So let us see the Pearson product-moment correlation coefficient is a measure of the liner association between 2 variables using quantitative data, but in this section, we will provide a correlation measure of association between 2 variables when ordinal or rank ordered data are available. So when the data is available in an ordinal or a rank ordered manner, in such a condition, we will use the rank ordered correlation or the Spearman correlation. So the spearman rank-correlation coefficient has been developed for this purpose.

Now this is the formula. So as you know correlation is always given by the small $r = 1 - \frac{6 \sum d^2}{n(n^2 + 1)}$ summation of d square where d is nothing but a difference in the ranks divided by $n \times n$ square + 1 where n being the number of observations in the sample. So let us see what is x_i is the rank of the observation i with respect of the first variable, y is the rank of the observation same i with respect to the second variable and d is $x_i - y_i$. Let us solve a problem, so you will understand.

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Rank Correlation

Example

A company wants to determine whether individuals who had a greater potential at the time of employment turn out to have higher sales records. To investigate, the personnel director reviewed the original job interview reports, academic records, and letters of recommendation for 10 current members of the sales force. After the review, the director ranked the 10 individuals in terms of their potential for success at the time of employment and assigned the individual who had the most potential the rank of 1. Data were then collected on the actual sales for each individual during their first two years of employment. On the basis of the actual sales records, a second ranking of the 10 individuals based on sales performance was obtained. **The next Table** provides the ranks based on potential as well as the ranks based on the actual performance.

So this is the problem. A company wants to determine whether individuals who had a greater potential at the time of employment turn out to have higher sales records, would you understand? What does it mean? When we recruit somebody or a company recruit somebody during the interview, they think that this person has a large potential or a huge potential and he will do a better job in comparison to may be some of the others. So this thought is it actually going well or it is not.

So they want to determine whether individuals got a greater potential during the interview or the employment do they actually turn out to have a higher sales record? To investigate this, the personal director reviewed the original job interview reports, academic records, and letters of recommendation for 10 current members of the sales force. After the review, the director ranked the 10 individuals in terms of their potential success at the time of employment assigned to the individual and who had the most potential the rank of 1.

So the best performer as per that time during the interview or the employment while he was selected was given 1 and obviously the rank goes down till 10 if there are 10 people. Then these 10 people's data was collected on the actual sales that they have made, the sales people's sales record has been collected for the first 2 years of employment. On the basis of the actual sales record, a second ranking has been done on the basis of the sales performance. So let us see the next table.

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Rank Correlation

| Salesperson | Ranking of Potential | Two-Year Sales (units) | Ranking According to Two-Year Sales |
|-------------|----------------------|------------------------|-------------------------------------|
| A | 2 | 400 | 1 |
| B | 4 | 360 | 3 |
| C | 7 | 300 | 5 |
| D | 1 | 295 | 6 |
| E | 6 | 280 | 7 |
| F | 3 | 350 | 4 |
| G | 10 | 200 | 10 |
| H | 9 | 260 | 8 |
| I | 8 | 220 | 9 |
| J | 5 | 385 | 2 |

So what is it saying? These are sales people A, B, C, D, E, F, G, H, I, J so 10 people are there and the ranking as per their potential is. So D was given the first rank, second rank was given to A, third rank was given to F, fourth rank was given to B, fifth rank was given to J, and so on and till you get the last rank was given to G okay. After 2 years, the sales data was taken and then there was again a ranking done. So this time, the first rank has been given to A who earlier as per potential had got the second rank.

The second rank has been given to J who had earlier got as per potential the fifth rank. So see this also tells you whether the recruiters were also correct in the thinking process or not when they recruited the sales person. The third was the one who is B and who had got a fourth rank, so it goes on. Now who is the tenth, the tenth is the person who was earlier also tenth in terms of potential. So this is the ranking in terms of 2 year sales and this is ranking in terms of the potential which was assigned during the joining.

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Rank Correlation

Let us compute the Spearman rank-correlation coefficient for the data in above Table. The computations are summarized in the Table presented here. We first compute the difference between the two ranks for each salesperson, d_i , as shown in column 4. The sum of the d_i^2 in column 5 is 44. This value and the sample size $n = 10$ are used to compute the rank-r correlation coefficient $r_s = .733$ shown in the Table.

| Salesperson | $x_i =$ Ranking of Potential | $y_i =$ Ranking of Sales Performance | $d_i = x_i - y_i$ | d_i^2 |
|-------------|------------------------------|--------------------------------------|---------------------|---------|
| A | 2 | 1 | 1 | 1 |
| B | 4 | 3 | 1 | 1 |
| C | 7 | 5 | 2 | 4 |
| D | 1 | 6 | -5 | 25 |
| E | 6 | 7 | -1 | 1 |
| F | 3 | 4 | -1 | 1 |
| G | 10 | 10 | 0 | 0 |
| H | 9 | 8 | 1 | 1 |
| I | 8 | 9 | -1 | 1 |
| J | 5 | 2 | 3 | 9 |
| | | | $\Sigma d_i^2 = 44$ | |

$$r_s = 1 - \frac{6 \Sigma d_i^2}{n(n^2 + 1)} = 1 - \frac{6(44)}{10(100 + 1)} = .733$$

So how do you compute a spearman correlation for the data. Look at this, we first compute the difference. From here you if see, we first compute the difference between the 2 ranks. So this time, the rank of the potential was, go back let us see, so this was second rank and the same person gets a 1 rank, here 4 and 3 let us say, so 1 and 1. So d_i is $x_i - y_i$ right. So this is 1, 2-1; 4-3, 1; 7-5, 2; 1-6, -5; 6-7 goes on. So if you look at it you got a d_i , so this is the difference in the rankings as per the potential and as per the actual sales.

Now we take the d_i square, so this is 1, 1, 2 is 4, this is 25, this is 1, 1, 0, 1, 1, 9 and summation of the d_i square is 44. So r_s , the spearman correlation, this s is spearman correlation. So $r_s = 1 - 6 \text{ summation } d_i \text{ square} / n \times n \text{ square} + 1$. So this is equal to $1 - 6 \times 44 / \text{divided by } n \text{ is } 10$, so this is 10, 10 square is obviously 100 - 1, so this gives us a value, so the spearman correlation coefficient value for this case is 0.733, which is strong right.

So the more it is close to one, we can say it is strong correlation. So it ranges between -1 to +1, so -1 is extremely strong but negative, +1 is extremely strong but positive, and 0 is hardly any relationship.

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Rank Correlation

The Spearman rank-correlation coefficient ranges from -1.0 to $+1.0$ and its interpretation is similar to the Pearson product moment correlation coefficient for quantitative data. A rank-correlation coefficient near $+1.0$ indicates a strong positive association between the ranks for the two variables, while a rank-correlation coefficient near -1.0 indicates a strong negative association between the ranks for the two variables. A rank-correlation coefficient of 0 indicates no association between the ranks for the two variables. In the example, $r_s = .733$ indicates a positive correlation between the ranks based on potential and the ranks based on sales performance. Individuals who ranked higher in potential at the time of employment tended to rank higher in two-year sales performance.

So the Spearman rank-correlation coefficient ranges from -1 to $+1$ like any correlation coefficient and its interpretation is similar to the Pearson product-moment correlation. A rank correlation coefficient near 1 , which is in this case, indicates a strong positive association between the ranks for the 2 variables, while a rank correlation coefficient near -1 indicates a strong negative association. So in this example, $r = 0.733$ which indicates positive correlation between the ranks based on potential and the ranks based on sales performance.

So the now the director can understand whether there is a relationship between the potential that was envisaged or the gauged during the time of the selection and actually the way they are performing is there a strong correlation, yes, there is a strong correlation and this strong correlation is almost 73% . So **(0) (18:39)** individuals who ranked higher in potential at the time of employment tended to rank higher in the 2 -year sales performance. Let us go to have a visual look also.

So this was given 2 and the sales rank was 1 , so this is close; $4, 3$ close; $7, 5$ slight difference; $1, 6$, there is an absolute change in this case, so he was thought to be 1 , but he turned out to be the sixth person in his sales performance; $6, 7$, again close; $3, 4$, close; 10 , there is no difference at all; $9, 8$; $8, 9$; $5, 2$ this is again difference. So this is the highest difference you can see and this is the second highest. Otherwise more or less, the other 8 , they are very close to what they were expected to do.

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Rank Correlation

At this point, we may want to use the sample rank correlation r_s to make an inference about the population rank correlation coefficient ρ_s . To do this, we test the following hypotheses:

$H_0: \rho_s = 0$ ✓

$H_a: \rho_s \neq 0$

Under the assumption that the null hypothesis is true and the population rank-correlation coefficient is 0, the following sampling distribution of r_s can be used to conduct the test.

SAMPLING DISTRIBUTION OF r_s

Mean: $\mu_{r_s} = 0$ ✓

Standard deviation: $\sigma_{r_s} = \sqrt{\frac{1}{n-1}}$

Distribution form: Approximately normal provided $n \geq 10$

$$\sqrt{\frac{1}{10-1}} = \sqrt{\frac{1}{9}} = \frac{1}{3} = 0.33$$

So rank correlation at this point we may want to use the sample rank correlation this value r_s to make an inference about the population rank-correlation coefficient. So do this, what is the inference you want to draw that there is a there is no difference or there is a strong correlation. So ρ_s is 0 and my alternate hypothesis is that the rank-correlation population correlation coefficient is not equal to 0. Under this assumption, the null hypothesis is true and the population rank-correlation coefficient is 0.

What is it saying, please understand. Under the assumption that the null hypothesis is true and the population rank-correlation coefficient 0, the following sampling distribution is used. Now what is it saying in this case the mean stands at 0 and the standard deviation is calculated by root over of $1/n-1$, so n is let us say greater than equal to 10, so let us take a 10 at this time, so $1/10-1$, so that is equal to $1/9$, is equal to $1/3$, so that is equal to 0.33 okay.

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Rank Correlation

The sample rank-correlation coefficient for sales potential and sales performance is $r_s = .733$. Using above equation, we have $\mu_{r_s} = 0$, and $\sigma_{r_s} = .333$. With the sampling distribution of r_s approximated by a normal distribution, the standard normal random variable z becomes the test statistic with

$$z = \frac{r_s - \mu_{r_s}}{\sigma_{r_s}} = \frac{.733 - 0}{.333} = 2.20$$

Using the standard normal probability table and $z = 2.20$, we find the two-tailed p-value = $2(1 - .9861) = .0278$. With a .05 level of significance, p-value $\leq \alpha$. Thus, we reject the null hypothesis that the population rank-correlation coefficient is zero. **The test result shows that there is a significant rank correlation between potential at the time of employment and actual sales performance.**

The sample rank correlation coefficient for sales potential and sales performance is 0.733 we have calculated. Using the above equations, we have $\mu = 0$ and standard deviation is equal to point 0.333 which we calculated. So now the z is equal to $r - \mu$, so μ is if you go back to the hypothesis what is it saying the null hypothesis $\rho_s = 0$ right, so this is what we are talking about 0 and standard deviation, so the z value is coming to 2.2.

So now using the standard normal probability table and $z = 2.2$, we find that two-tailed p-value is 1 minus one point this is the area under the curve 0.98, is this much right. So with a 5% level of significance the p-value is less than or equal to alpha. Thus so in this case it is less than right. So this is 0.02. We reject the null hypothesis that the population rank-correlation coefficient is zero. The test results show that there is a significant relationship rank correlation between potential at the time of employment and actual sales performance.

Had it been zero, that means you would have said there is no relationship, it is same, what it was assumed to be it is more or less the same, but in this case just there is a difference we say the test results show the significant rank correlation between potential at the time of employment and the actual performance, there is a significant difference. So this is the spearman correlation coefficient which we have understood and how important it is we have seen that.

Let us let me show you how to conduct this **(Video Starts: 22:36)** let us say I have created a data set for you. Now let us say this is the data of students, how many students are there, 30 students. See please understand I am creating a small data set, but if you increase the data set,

sometimes what looks insignificant to you will become significant. The sample size should be as high as possible. Now I want to see whether students who brought the marks in English and the same students brought some marks in Mathematics, is there any relationship between the marks that they have received in English and the marks that they have received in Maths.

So in this case, we will say the $P_s = 0$, my null hypothesis is equal to 0 or the population correlation is 0, that means there is no correlation, but actually heart of heart we wants that there should be some strong correlation, negative or positive that is immaterial that because that is a rather a good thing to know whether there is a negative relationship or a positive relationship. So let us see how to conduct this test. So go to analyze, go to b correlation and then bivariate.

So let us take these 2 and now our data we are saying is a non-normal data so since it is a non-normal data, by default it is given a Pearson, but we do not want a Pearson because our data we want to check in a ranked form. So let us take the Spearman correlation. So when you take the spearman correlation, it automatically takes in for a ranking and thus then test the subject. So let us go to the output. Now in this case if you see the nonparametric correlation what is it saying, English mark and your Maths mark, is there a strong correlation ship, now yes there is.

English and Maths the relationship is 0.589, which the correlation is significant and is found to be significant at the 0.01 level and it is a two-tailed test. We said it is a two-tailed because -1, +1, so can go on both sides, so we do not know which side it will be going. So anyway, here found that the Spearman correlation coefficient in this case is giving us a clear indication that there is some strong correlation between the marks scored among the students for their English and Maths (**Video Ends: 25:12**).

Now what kind of a relationship that can be checked, but then we are saying is a significant relationship. So this is what we can understand, that means what if somebody is scoring good in let us say English, he can be scoring good in Maths also right, why because he is a good student or it could be possible that the student was good in English, his one side, his language processing part in the brain is better than the mathematical part, so if he is doing good in English, he should not be doing good in Maths.

So there are 2 logics right, so now which logic is coming to that can be checked, but here we are sure that there exist a strong correlation between the two values. So this is what we have done in the rank correlation.

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Wilcoxon test

Next, I will tell you about one more test called the Wilcoxon test.

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Wilcoxon test

- The **Wilcoxon signed-rank test** is a nonparametric procedure for analyzing data from a matched-sample experiment.
- The test uses quantitative data but does not require the assumption that the differences between the paired observations are normally distributed.
- The Wilcoxon signed rank test is used for that are at least ordinal in scaling.

Sign. test → Not a powerful.
Signed test → powerful.

So this test is a similar test which is again a nonparametric test and is used for analyzing data from a matched sample experiment. So there are 2, let us say the Wilcoxon test is a powerful test. This test uses quantitative data but does not require the assumption that the differences between the paired observations are normally distributed. The Wilcoxon sign rank test as I had earlier said to you, this is the difference the sign test which we had done earlier I had said it is not a very not a powerful test.

On the other hand, this Wilcoxon sign rank test is a very powerful test, is used of that are at least ordinal in scaling, so that means you measure the data in a ordinal scale.

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Wilcoxon test

- When a researcher wants to analyze two sets of data obtained from the same individuals, the appropriate test to apply is the related t test. → *paired 't' test.*
- However, when there is an extreme violation of the normality assumption, the Wilcoxon signed rank test can be used. ✓

Assumptions

- The scale of measurement within each pair must be at least ordinal in nature. ✓
- The differences in scores must also constitute an ordinal scale

So when a researcher wants to analyze 2 sets of data obtained from the same individuals, the appropriate test to apply is a related t test. So we say it is like a paired sample, paired t test. So for example, the case of blood pressure, when you check the blood pressure, or you check the efficacy of a food habit for people who are exercising if they take more protein what happens to their stamina or something. However, when there is extreme violation of the normality assumption, the Wilcoxon sign rank test can be used, obviously.

So it is used against the paired test which uses a normally distributed data. The assumption is the scale of measurement within each pair must be at least ordinal in nature, the differences in scores must also constitute an ordinal scale.

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Wilcoxon test

Example

Consider a manufacturing firm that is attempting to determine whether two production methods differ in terms of task completion time. Using a matched-samples experimental design, 11 randomly selected workers completed the production task two times, once using method A and once using method B. The production method that the worker used first was randomly selected. The completion times for the two methods and the differences between the completion times are shown in Table.

| Worker | Method | | Difference |
|--------|-----------------|-----------------|--------------|
| | A | B | |
| 1 | 10.2 | 9.5 | .7 |
| 2 | 9.6 | 9.8 | -.2 |
| 3 | 9.2 | 8.8 | .4 |
| 4 | 10.6 | 10.1 | .5 |
| 5 | 9.9 | 10.3 | -.4 |
| 6 | 10.2 | 9.3 | .9 |
| 7 | 10.6 | 10.5 | .1 |
| 8 | 10.0 | 10.0 | 0 |
| 9 | 11.2 | 10.6 | .6 |
| 10 | 10.7 | 10.2 | .5 |
| 11 | 10.6 | 9.8 | .8 |

Now let us take this. So there are 2 methods. So there are some workers who have been taken. Consider a manufacturing unit that is attempting to determine whether 2 production methods differ in terms of task completion. There are two production methods A, B. Whether these 2 methods, they take different time in completing the job. Using a matched sample experimental design, 11 randomly selected workers completed the production task 2 times for one with A method one with a B.

The production method the worker used first was randomly selected. The completion time for the 2 methods and the differences between the completion times are shown. So the first one took 10.2, in the B method 9.5; 9.6, 9.8; third 9.2 it goes on and the difference has been measured. If you can see the difference, how much difference of time has come when they are using between the two methods, so $A-B =$ this is what we have got, let us say.

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Wilcoxon test

A positive difference indicates that method A required more time, a negative difference indicates that method B required more time. Do the data indicate that the two production methods differ significantly in terms of completion times? If we assume that the differences have a symmetric distribution but not necessarily a normal distribution, the Wilcoxon signed-rank test applies.

In particular, we will use the Wilcoxon signed-rank test for the difference between the median completion times for the two production methods. The hypotheses are as follows:

H_0 : Median for method A - Median for method B = 0

H_a : Median for method A - Median for method B \neq 0

If H_0 cannot be rejected, we will not be able to conclude that the median completion times are different. However, if H_0 is rejected, we will conclude that the median completion times are different. We will use a .05 level of significance for the test.

A positive difference indicates that the method A required more time, a negative difference indicates that the method B required more time, obviously. Do the data indicate that the 2 production methods differ significantly in terms of completion times. If we assume that the differences have a symmetric distribution, but not necessarily a normal distribution, the Wilcoxon test applies. Now what it is saying is we will use the Wilcoxon sign rank test for the difference between the median completion times.

I had told you several times that in a nonparametric test, we are more interested in the median and not the mean. So the two production methods. The hypothesis are the median for method A minus the median for method B is equal to 0 or that means the median for method A is equal to the median for method B. In the alternate hypothesis, we say the median for method A minus the median for method B is not equal to 0, so this might be more, sometimes this might be less or this might be less, this might be more whichever.

If H_0 cannot be rejected, we will not be able to conclude that the median completion times are different, obviously that is how the null hypothesis to be understood. However if H_0 is rejected, we will conclude that the mediation completion times are different and this is what is of interest to a researcher. So the level of significance uses 0.5.

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Wilcoxon test

The first step in the Wilcoxon signed-rank test is to discard the difference of zero for worker 8 and then compute the absolute value of the differences for the remaining 10 workers as shown in column 3 of the next table. Next we rank these absolute differences from lowest to highest as shown in column 4. The smallest absolute difference of .1 for worker 7 is assigned the rank of 1. The second smallest absolute difference of .2 for worker 2 is assigned the rank of 2. This ranking of absolute differences continues with the largest absolute difference of .9 for worker 6 being assigned the rank of 10. The tied absolute differences of .4 for workers 3 and 5 are assigned the average rank of 3.5. Similarly, the tied absolute differences of .5 for workers 4 and 10 are assigned the average rank of 5.5.

Once the ranks of the absolute differences have been determined, each rank is given the sign of the original difference for the worker. The negative signed ranks are placed in column 5 and the positive signed ranks are placed in column 6 (see next table).

So what you do is the first step in the Wilcoxon test is to discard the difference of 0 for worker 8, let look at the worker 8. Se there worker 8 has got a difference of 0 because the same, so we need to discard it and then compute the absolute value of the differences for the remaining 10 workers. So next, we rank this absolute differences from lowest to highest because you want a ordinal scale. The smallest absolute difference of 0.1 for worker 7 is rank 1.

So what is saying, so the worker 7 has a difference, that means both the methods are taking almost the same time. The second smallest 0.2 is 2 and it goes on till you get the highest difference and that is the tenth rank. The tied absolute differences of 0.4 for workers 3 and 5 are assigned the average rank of 3.5. This is how you do. So there are 2, let us see, so 3 and 5, 0.4 here and 0.4 here correct, so if you see the absolute value is so for 3 and 5. So the absolute difference of 0.4 for workers are assigned average rank of 3.5, yes, why because if you can see let us go to the front one.

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Wilcoxon test

For example, the difference for worker 1 was a positive .7 (see column 2) and the rank of the absolute difference was 8 (see column 4). Thus, the rank for worker 1 is shown as a positive signed rank in column 6. The difference for worker 2 was a negative .2 and the rank of the absolute difference was 2. Thus, the rank for worker 2 is shown as a negative signed rank of 2 in column 5. Continuing this process generates the negative and positive signed ranks as shown in the table.

| Worker | Difference | Absolute Difference | Rank | Signed Ranks | |
|--------|------------|---------------------|------|---|----------|
| | | | | Negative | Positive |
| 1 | .7 | .7 | 8 | | 8 |
| 2 | -.2 | .2 | 2 | -2 | |
| 3 | .4 | .4 | 3.5 | | 3.5 |
| 4 | .5 | .5 | 3.5 | | 3.5 |
| 5 | -.4 | .4 | 3.5 | -3.5 | |
| 6 | .9 | .9 | 10 | | 10 |
| 7 | .1 | .1 | 1 | | 1 |
| 8 | .0 | | | | |
| 9 | .6 | .6 | 7 | | 7 |
| 10 | .5 | .5 | 3.5 | | 3.5 |
| 11 | .8 | .8 | 9 | | 9 |
| | | | | Sum of Positive Signed Ranks $T^+ = 49.5$ | |

So the rank you see, 1, 2, so lowest, then 3, you do not 3, so you have two same. So what you have done is instead of 3 you have given a rank of 3.5, 3.5. Then 4 you do not have, so then we will go to 3.5, 3.5, then the next is you are going to fifth, where is 5, so 4 is also there is some similarity I believe, yes okay, this is also 0.5, this is also 0.5, the difference. So that means after 3.5, the next is 5.5, so 5.5 here 5.5. here. So the next is, after this is the seventh okay, then eighth, then ninth, then tenth. So this is how the rank is to be made.

So after these ranks are made, the negative and the positive values are taken, the sum of the positive sign ranks is $T = 49.5$. If you add up this total it becomes 49.5. So if you see what is it saying, the difference of worker 1 was positive 0.7 and the rank of the absolute difference was 8, this one right. The rank for worker 1 is shown as a positive rank in column 6, thus difference of worker 2 was a -0.2 , this one and the rank of the absolute difference was positive 2 and thus the rank of the worker 2 is shown as a negative sign of a rank 2.

So this is what it is saying is so this gets a negative sign rank 2, okay. This is difference is 0.2 absolute difference. So continuing this process generates negative and positive sign. So the scale of measurement within each pair must be ordinal in nature and the it basically uses a ordinal scale right. So this the example. So what we will do is I worked on this example, so we will take it this example in the next lecture.

We will work on it how the difference of two methods has been taken to understand the effectiveness of the workers and how the Wilcoxon rank sign rank test has been used for this purpose. We will stop it here. Thank you so much.