

Marketing Research and Analysis-II (Application Oriented)
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Lecture – 26
Non-Parametric Test - II

Welcome friends to the class again. This course of Marketing Research and Analysis we had been doing for the last many sessions now and we have covered several important aspects of marketing research. So currently, we are doing the non-parametric test and why we should be doing it I had explained in the last lecture.

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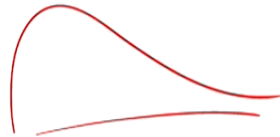


So a non-parametric test is a test as I had explained is one which does not follow a normal distribution, so the normal distribution is something like this like a bell's curve. Non parametric tests are those tests which violate the normal distribution conditions. So for example if the data is let us say skewed data for example let us take this, although I will be explaining today.

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KRUSKAL-WALLIS TEST

One
in
paired
ANOVA.



So if the data is a skewed data or there are lot of outliers in the data, so we say the analysis that we need to do is one which violates the normality assumptions and therefore the best method is the nonparametric test and under nonparametric test, I had also said several tests like for example the Mann-Whitney U test which we had started in the last lecture. so the Mann-Whitney U test is nothing but as I had said it is very simple, it is an analogy of the t-test, independent sample t-test.

So if you remember while doing the t-test, you must have heard of one sample t-test, then independent sample, paired sample. So when you have more than 2 samples, then you went for let us say the ANOVA, one-way ANOVA. So there are several tests that we are talking about and the Mann-Whitney U test is one of the first starting points where had done with like the independent sample t-test. Now I said I will be explaining you the Mann-Whitney U test at the end of the class in the last class through SPSS also okay.

So the software which we are using is SPSS. So how do I do it on the SPSS? Now if you can look at this table, the data sheet. **(Video Starts: 02:35)** now it talks about several variables, for example x1, x32. This first is the id number, then x1 which is the delivery speed, x2 price level, x3 the price flexibility, x4 manufacturer's intention, x5 service, similarly all are there, then up to x7 if you see the data is measured in a continuous scale, x8 is which is firm size is measured as a categorical variable, so small firm large firms.

So for example again going to if you see the x12, x12 is again a categorical variable, x13 is again a category variable which is firm type. So you have some categorical variables in this

case. The x14 is also a category variable with just 3 categories, new task, modified rebuy, straight rebuy, I think you if you have heard of it, in industrial buying process, we use this language. So there are several variables. Now how do I go for the nonparametric. First of all, you check whether the data is following a normality or not.

Now how do I check. Now first I will see let us say what I will do is I will go to the explore and I will take one of the data list, say I am taking anyone, let us say price flexibility. I want to see a plot and I want to seek whether it is following a normal distribution or not. So if you remember in the last class itself I had said to you, just you can do one thing you can check the skewness and kurtosis. Now luckily, the data is not skewed and it is a normal data, but suppose you find some data is not normal, so I do not have may be a data which is, we can do one thing, we can check for all otherwise.

So we will take from delivery speed, price level, manufacturer's image, service, this one and this one, usage level, and satisfaction level. So I am trying to see if these data, any of them follows a non-normal pattern or not. So let us see skewness, skewness, skewness, not this also, two four three, two four three, three six seven. So I think you remember how we do it, this one, let us say salesforce image, although it is very close to normal distribution, the skewness is not very high and just for an example I am trying to show you.

Just let us imagine salesforce image, if I if you remember I told you very clearly that if your data is slightly skewed, moderately skewed, even you can march ahead, if you have a decent sample size, you can march ahead with the normal parametric tests, but however, if your data is extremely skewed, then only you should go for a nonparametric test and even after correction, it is the transformation of the data, the data is not getting the normality, it is not showing normal distribution, then only.

So assume for a second this data is not showing a normal pattern. So what I am doing is I am just to show how do you do a nonparametric test in SPSS, so you can see this. So I go to the legacy box. There are few things here, chi-square test which is a nonparametric test, binomial test, runs test they are all nonparametric tests. Now here out of this, I am picking up this independent sample test. So what is this independent sample, if you remember the two different sample groups, sample 1 and sample 2 and we are trying to check, how, whether there is any similarity, whether the sample 1 and sample 2 are same or they are different.

So let us say salesforce image I am taking here and what I am checking against, what is the grouping variable. The grouping variable is let us say I am taking x8 which is the firm size. Now firm size when you take, you have to define the group. Now look at it. So I am using from the keyboard. What I will do is I will put a value of 0 and for here I will put a value of 1, now I do not require this, continue. Now you see x8 has 1 and 1 right so I have taken this. Now what do I need here, this exact if you see, this is what I require right.

So let us go to options, you may use the descriptive, continue, and if you see here automatically the Mann-Whitney U test is selected. Let us check this. Now if you see here, it is because you had asked for descriptive, it is telling you the firm size, what is the mean, the mean is this much, the standard deviation is given to you. The Mann-Whitney test values are given, the mean rank and the number of salesforce, n is given to you, the mean rank is given, the sum of ranks is given to you.

If you remember in the last class lecture where I had explained how to measure by calculating the U if you remember the U value. To measure the U value required the mean, the standard error, to calculate the Mann-Whitney U value. So after this, this is the one which is important to us, the test statistics. Now if you look at the Mann-Whitney U value for the salesforce image is 1189. What is Wilcoxon, this is another parameter always compared. So this is but not important to us. Let us go the Z, is -0.074 and the significance value if you see is 0.941.

So if you remember from the earlier classes when you have a significance value of 0.941, why it is two-tailed because the value can fall either to the left or to the right. So 0.941 means the null hypothesis which said that there is no difference between the salesforce image in small versus large firms, that means the salesforce image in small firms is same as the salesforce image in the large firms. So this null hypothesis is not rejected in this case because it is above point 0.05, suppose I take it at a 5% level of significance or even at a 1% level of significance.

So it is 0.941 which is almost saying they are same, so in that case, we are saying the salesforce image across the two firms sizes, small and large, they are not different, they are same. So this is what you do in the Mann-Whitney U test (**Video Ends: 09:48**). So today we are starting with another test which is the Kruskal-Wallis test. Now the Kruskal-Wallis test is

a test which is very similar to Mann-Whitney test only, but it has more than 2 samples now, so it is close to as good as it looks like one-way ANOVA right. So let us see this.

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KRUSKAL-WALLIS TEST

- Kruskal- Wallis Test is the non-parametric test.
- It is the extension of the Mann-Whitney test to situations where more than two populations are involved.
- This test too depends on the ranks of the sample observations. *Rank sums test*
- This test also fall under the family of rank sum tests.

It is a non-parametric test obviously. It is the extension of Mann-Whitney test to situations where more than 2 populations are involved. So here more than 2 populations are involved. This test too depends on the ranks of the sample, so it is also comes from the family of rank sums test, like the Mann-Whitney.

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PROBLEM

- The score of a sample of 20 student pilots on their Federal Aviation Agency written examination are arranged according to the method used in their training: video cassette, audio cassette, or class room training.
- FAA is interested in evaluating the effectiveness of theses training methods. Specifically, it wants to test at the 0.10 level of significance, the hypothesis that the mean written examination scores of the student pilots trained by each of theses three methods are equal.
- Because we have more than two population involved, the Kruskal-Wallis test is appropriate in this case. ✓
- To apply the Kruskal-Wallis test to this problem, the values in the following table is ranked from lower to higher.

$$H_0: \mu_1 = \mu_2 = \mu_3$$

Let us take a problem. So this is the problem. So how to solve this? The score of a sample of 20 student pilots on their Federal Aviation Agency written examination are arranged according to the method used in their training. There are 3 methods used; video cassette methods, audio cassette methods, or classroom training methods. So these are 3 training

methods which were used on some students and then their scores were checked. So the Federal Aviation Agency is interested in evaluating the effectiveness of these training methods.

So which training method is the best, is it the first one, is it the second one, is it video, is it audio or the class room training method. It wants to test at the 10% level of significance. The hypothesis that the mean written exam scores are equal right, obviously so if I ask you to write the null hypothesis, what it would be? The null hypothesis would be H_0 is equal to there is no difference in the scores of the students who use the video method, so $\mu_{\text{video}} = \mu_{\text{audio}} = \mu_{\text{classroom}}$ okay.

Because we have more than 2 population involved, the Kruskal-Wallis test is appropriate method. Let us see how to do this. So the first thing is the values in the following table is ranked from lower to higher, in the Mann-Whitney also you have done that, you are doing some ranking.

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Written examination scores for 20 student pilots trained by three different methods

Video cassette	74	88	82	93	55	70	.	.	.
Audio cassette	78	80	65	57	89
Classroom	68	83	50	91	84	77	94	81	92

Okay let us see this. Video cassette; 74, 88, 82, 93, 55, 70, some values are given, the scores. People who follow the audio cassette method 78, 80, 65, 57, 89, so these values are not known to us. Who follow the classroom method, so these are the values given to us. Now the question is, is there is a difference in the score received by the students who followed these 3 different methods. So null hypothesis is saying at this moment no, right, so let us check it how do we do that?

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Written examination scores ranked from lowest to highest

Rank	Score	Training method	Rank	Score	Training method
1	50	C	11	81	C
2	55	VC	12	82	VC
3	57	AC	13	83	C
4	65	AC	14	84	C
5	68	C	15	88	VC
6	70	VC	16	89	AC
7	74	VC	17	91	C
8	77	C	18	92	C
9	78	AC	19	93	VC
10	80	AC	20	94	C

So first we do a ranking. So 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 up to 20, so 20 students are there. So the lowest value, we start in the lowest value 50, and which training method did they follow in the classroom. Second 55, what was the training method, now video cassette. Third 57, audio cassette. So we have done a ranking of these people. So all the students have been ranked, so out of which we can see there are may be C, classroom is 1, 2, 3, 4, 5, 6, 7, 8, 9; 9 people were in classroom method. So once we have done it, the ranks are given to you.

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Data and rank arranged by training method

Video cassette	Rank	Audio cassette	Rank	Classroom	Rank
74	7	78	9	68	5
88	15	80	10	83	13
82	12	65	4	50	1
93	19	57	3	91	17
55	2	89	16	84	14
70	6	-	-	77	8
-	-	-	-	94	20
-	-	-	-	81	11
-	-	-	-	92	18
Total	61	Total	42	Total	107

Then we arrange it. So video cassette, what was the score of the student, now let us say 74, what is his present rank 7 right. Somebody who scored 88, what is his present rank 15, so we have done it as per the method video cassette, audio cassette, and classroom. One who scored under video cassette method 82, his rank is 12; 93, 19; 55, 2 because he was the second

person; 70, 6. Audio cassette similarly, we have done it. So 78 was who scored got the ninth rank, 80 who scored got the tenth rank, similarly 89 who got was sixteenth rank.

Classroom also you can see. Now what we have done is we will take a total of these ranks. So 61, this total is 15+7+12+19+2+6 is 61. Audio cassette, the total is 42, ninth rank tenth rank 19, 23, 26, 32, 42. Similarly people who use the classroom method, the score for them is 107. It does not matter because automatically although these scores may be looking different to you, but the number if you see the n, it is also quite separate in all the 3 cases.

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Symbols used in Kruskal-Wallis test:

n_j = number of items in sample j ✓

R_j = sum of the ranks of all items in sample j ✓

k = number of samples

$n = n_1 + n_2 + \dots + n_k$, the total number of observations in all samples.

K- statistics ✓

$$K = \frac{12}{n(n+1)} \sum \frac{R_j^2}{n_j} - 3(n+1)$$

So what are the symbols used? n_j is the number of items in sample j, r_j is the sum of the ranks of all items in sample j, k is the number of sample. In ANOVA also we are using this number of groups, number of columns, number of items within the group, how many members were total there, so within the group how much and between group how many groups were there, so these things we are doing. Similarly total $n = n_1+n_2$ up to n_k .

So the case statistics which is to be measured like you measured in the Mann-Whitney U statistics, here we are measuring the K-statistics. How does it measure? $12/n \times n+1$ into summation of the rank square divided by n_j , the jth variable, sample j, $-3 \times n$, this is the formula.

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$$\begin{aligned}
K &= 12 / n(n+1) \sum R_j^2 / n_j - 3(n+1) \\
&= 12 / 20(20+1) [\{61^2 / 6\} + \{42^2 / 5\} + \{107^2 / 9\} - 3(20+1)] \\
&= (0.02857) * (620.2 + 352.8 + 1272.1) - 63 \\
&= \underline{1.143}
\end{aligned}$$

So using this formula, let us see what is the K-statistics coming. So K is 12 divided by n is 20, 20 students are there, x 20+1, this much. Summation of the ranks, now much, 61 was the first, so 61 square/6, sample size was 6 for the first, this was I think the video cassette group. Then audio cassette group was 42, so 42 square/5+, classroom the total rank was 107, 107 square/9. Some of they have managed by themselves because it has been divided by the sample size right -3 x n+1. Now doing this, we get a score of 1.143.

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- The sampling distribution of the K statistics can be approximated by a chi-square distribution when all the sample size are at least 5.
- Because our problem meets this condition, we can use the chi square distribution for this test.
- In a Kruskal-Wallis test, the appropriate number of degree of freedom is k-1, which in this problem is 3-1=2 because we are dealing with three samples.
- The hypotheses can be stated as follows:

$$H_0 : \mu_1 = \mu_2 = \mu_3 \quad \checkmark$$

$$H_1 : \mu_1, \mu_2, \text{ and } \mu_3 \text{ are not equal}$$

$$\alpha = 0.10 \quad \checkmark$$

So the sampling distribution of the K statistics can be approximated by a chi-square distribution when all the sample sizes are at least 5, so this distribution K statistics is approximated by a chi-square distribution whenever the sample size is more than 5. Because our problem meets this condition, you can use the chi-square distribution for a Kruskal-Wallis

test. So the approximate number of degrees of freedom is $K-1$, so which in this case is $3-1 = 2$, so there are 3 samples, so in ANOVA also you are doing the same.

There are 3 suppose samples you are subtracting one from it. So the hypothesis is $\mu_1 = \mu_2 = \mu_3$ correct or alternate hypothesis is $\mu_1, \mu_2, \text{ and } \mu_3$ are not equal, at least one of them is not equal. What is my significance level, 10%.

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- The acceptance region for the null hypothesis (that there are no differences among the three populations) extends fro zero to a chi square value of 4.605.
- Obviously, the sample K value of 1.143 is within this acceptance region.
- **Therefore**, the FAA should accept the null hypothesis and conclude that there are *no differences in the result obtained by using the three training methods.*



The acceptance region for the null hypothesis that there are no differences among the 3 populations extends from 0 to a chi-square value of 4.605. Obviously, the sample K value is 1.143. Now if you look at for a 10% level of significance, it says the chi-square value would have been maximum up to 4.605 and here our value is only 1.143, which is well within the acceptance region. So the null hypothesis cannot be rejected.

So therefore the Federal Agency should accept the null hypothesis and conclude that there are no differences in the result obtained by using the 3 training methods. So this is one important element because it helps you to compare between groups, 3 separate groups and test how it is being done and what is the difference between the sample. You can also do it in a different way as I used to explain, I do not remember whether I have done it or not. While doing an ANOVA, you can distribute into paired, for example let us say sample 1, sample 2, sample 3 are in this case let us say.

So you can do a Mann-Whitney test between S_1 and S_2 , you can do a Mann-Whitney test between S_2 and S_3 , you can do a Mann-Whitney test between S_1 and S_3 and also see which

is the most effective method out of these 3, you can do that right. So when you are doing this, but one problem will occur. If you do multiple tests, as you do multiple suppose t-test also in case of a parametric test, what happens is the alpha gets inflated. So the alpha gets inflated and that creates an error.

So in such conditions, what about is advised is that you use a Bonferroni correction test. What is this Bonferroni correction test, that means suppose you take alpha of 5% and you have suppose 3 tests now, S1-S2, S2-S3, S1-S3 then this 5% should be divided among the 3 tests, this is called the Bonferroni adjustment test. So uh when you do this K value test and then you understand well it is more like the ANOVA only and whether there is difference between the groups exists or not.

Let me see if I can show you if time permits through the SPSS (**Video Starts: 19:27**) now again go to nonparametric. Now you see, here you can see this K independent samples. So K independent samples will be, why k independent samples because this is more than 2, so independent sample was for 2, but when it is more than 2, it is K independent sample. Now let us take any one, again the same thing the salesforce image for example and the grouping variable now I am taking is let us say the type of industry, I think x14 is the type of industry.

So what I am defining is, the minimum is 1 and the maximum is you can see here 3, continue, okay, do you want anything from here, I think we should follow the normal process. So if you want to use the descriptive, you can use. Now one interesting thing is you can also check for the median because why you see because we have said that the parametric test follow median, median is the most strongest parameter of measure of center tendency in case of a nonparametric test (**Video Ends: 20:37**).

But I hope you have understood that Kruskal-Wallis test is nothing but a very similar test like the ANOVA, only condition is this is a nonparametric test instead of a parametric one. I will wind up the lecture here. Thank you so much.