Marketing Research and Analysis-II
(Application Oriented)
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Lecture - 24
Hypothesis Testing – II
(Problem solving)

Welcome everyone to the lecture series of our course marketing research and analysis. So, we have a continuing with hypothesis testing in different cases.

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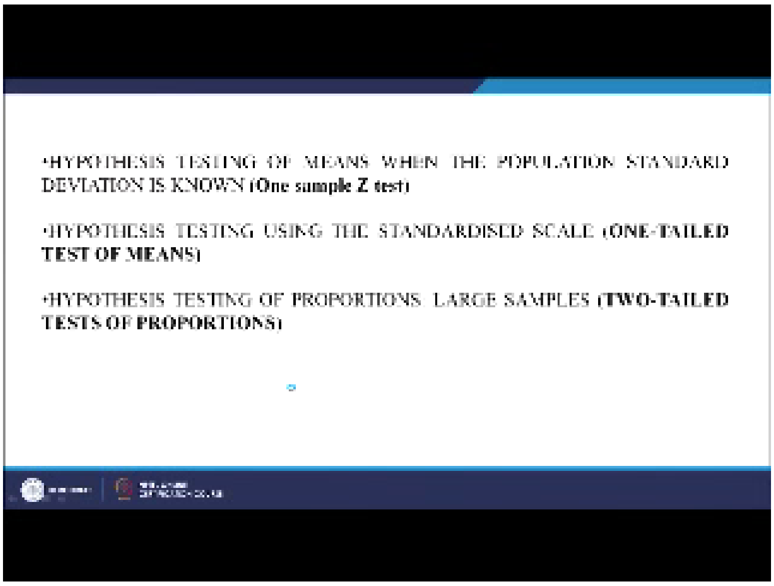

So, for example be in the last class recovered hypothesis testing of means when population standard deviation is known right. So, but as I said it is a very rare case you are not aware of the populations standard deviation most of the time. So, that is a very generally not applicable. Hypothesis testing using the standardised scale right second one tailed test of means and hypothesis processing of proportions. So, we dealt with these cases in the last lecture.

Today we are going to move ahead with different other cases and how does a researcher handle them right. So, is a very important I have been repeating all the time kindly have patients whenever you feel like you are not understanding, take a pause, stop it and try to go back and see whether you understand it or not right. Once you are clear then only you should move ahead ok.

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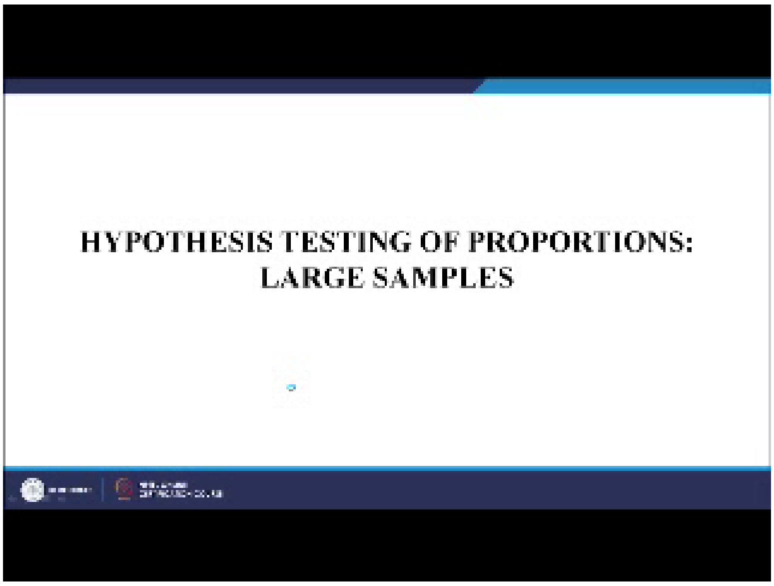

So, then the next case we are going to deal with it today is the hypothesis testing of proportions in case of large samples ok. So, we are talking this time about one tailed test of the proportions right.

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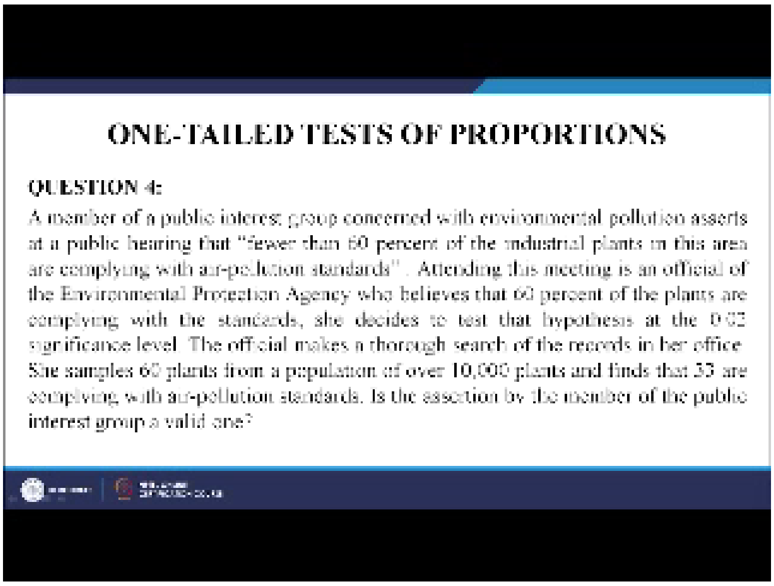

So, let us start the question a member of a public interest group concerned with environmental pollution asserts you know today nowadays people are talking about environmental problem and how it affects us and all. So, one of the public interest group that assert that fewer than 60% of the industrial plants less than 60% industrial plants in a particular area are actually complying with the air pollution standards so there worried that people are not complain to the air pollution standards less than 60% are actually following it.

Attending this meeting is an official of the environmental protection agency from the government maybe who believes that 60% of plants are complying with the standards it is not 60% less than 60% but this person from the Environment protection agency they believe that it is 60%. Despite to test the hypothesis at 2% level of significance how much 2% level of significance. The official makes the thorough search of the records in her office.

She samples 60 plants how many 60 plants from a population of over 10000 plants and find that 33 are only complying with air pollution standard this 33 is out of 60 ok. Is this assertion by the member of the public interest group who said it is less than 60% is it a valid one. So, now the researcher is facing a problem where he has to check whether the statement of the public interest group person whatever he had said that is less than 30% is it true or not true.\

So, fast kindly write the frame the null and alternative hypothesis in this case. So, what is of interest to you and what is the null hypothesis. So, kindly write it on your notebook or something if you are following the class looking listening to me.

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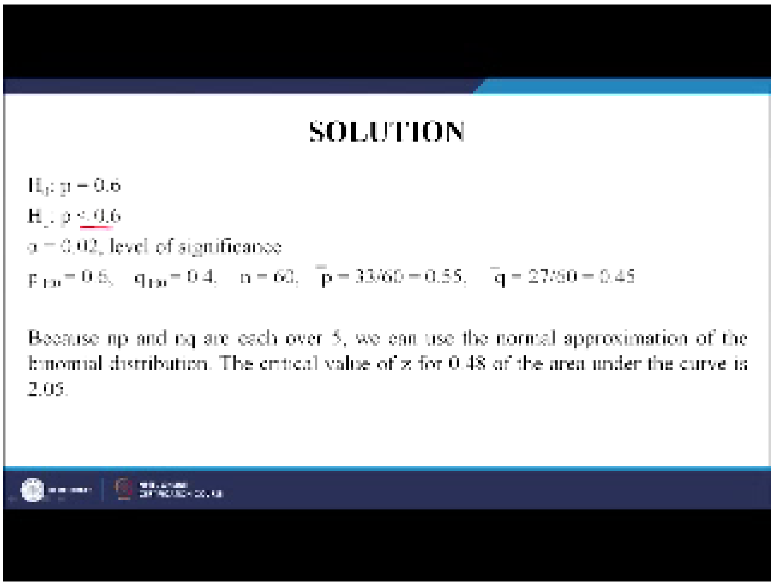

So, what is it saying? p < 0.6 the environmental agency is interested to seek is it really less than 60% ok so that is if they are trying to check. So, what is the null? Null = .6 fine α is 2% level of significance so the hypothesised proportion is .6. So q rate of failure you can say is 0.4 n is 60 so now the sample that they took was 33 in which you know 60 out of which 30 were found to be actually having a problem 33 are complying with the air pollution standard that means they were good.

So, what is the good one .55, 33 by 60 so what is the ratio of the proportion of the group which are not following the standard that q which is equal to .49 p and q are over 5 we can use the normal approximation of the binomial distribution. In this case again you see we have taken .48 of the area under the curve why the reason is again let us go back and think we are talking about only one tail.

So, the less than we are interested in the less than case right so in this side that means we are not interested in this side right this is of no interest to us we are only interested in this side. So, 2% is a level of significance so the 2% is the rejection part. So, out of two when you will -50 that mean the area is .48 for that the critical value for Z .48 of the area under the curve is how much 2.05 ok.

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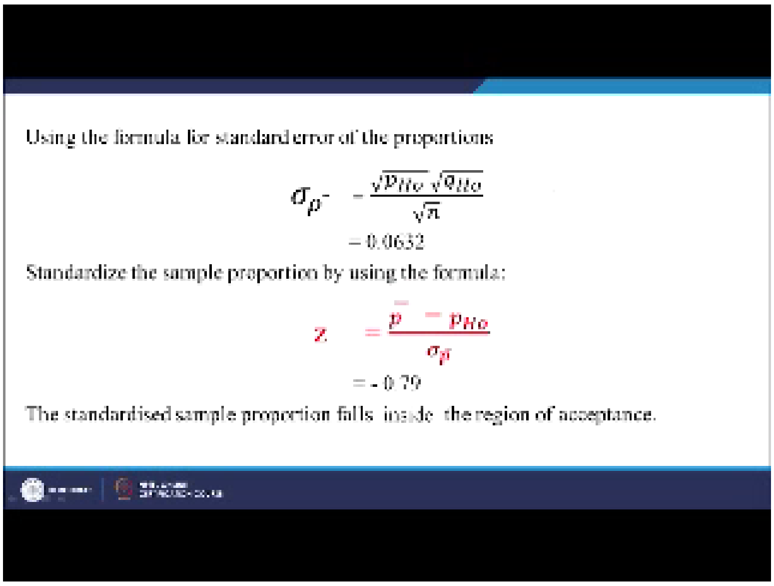

Now first you will calculate the as you remember standard error of proportion standard and proportion is root over of pq by n so by using this formula we have calculated that in this case 0.0632 so using this standard of proportion let us calculate the Z value. Z value is proportion of the sample minus hypothesised proportion upon the standard error of the proportion which is giving us the value of - 0.79 ok.

So, -0.79 the standardised sample proportion that means falls inside the reason of acceptance correct you see minus acceptable limit was -2.05 let us say and our case is -0.79 so that is not match with in the acceptance region.

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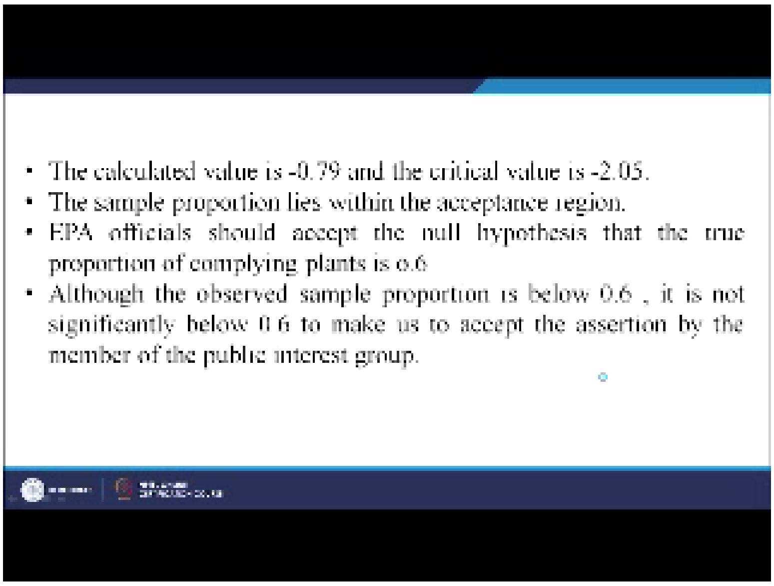

So, the calculated value is -0.7 the critical values - 2.05 so the sample proportion lies within the acceptance region right. EPA official this part is very important from researcher point of view this is what you need to write. So, EPA official should accept the null hypothesis to the truth true proportion of complying plants is 0.6 or 60%. Although the observed sample proportion is below 0.6 it is not significantly below 0.6 to make us to make us to accept assertion by the member of the public interest group.

That means what it says you are accepting the null hypothesis so what are the null hypothesis that the proportion of the plants complying with the standard is not less than 60% but it is 60% right.

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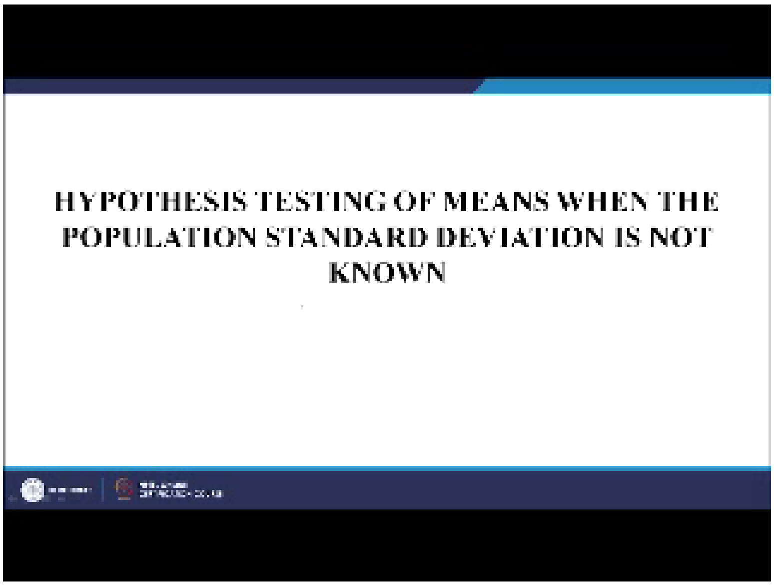

Now coming to the next case the next case is hypothesis testing when the population standard deviation is not known to you. This is of means now this case is again of means earlier we were doing with means and proportion again coming to means. So, this is the two tailed test of mean in which we are using let us see what is the case first.

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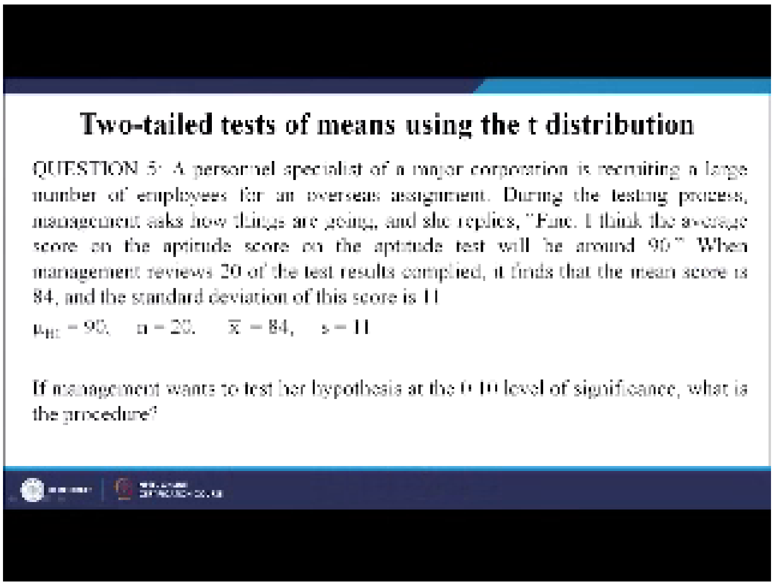

A personal specialist officer of a major Corporation a personal human resource officer of the corporation is equating large number of employees for an overseas assignment during the testing process management ask how things are going and she replies fine I think the average score of the attitude aptitude will be around 90 the average score of the sample right when management reviews are not sample the hypothesised means right so when the management reviews 20 of the test results this is invisible to 28 find that the mean score is actually 84 so this is the sample mean right.

As the standard deviation of the score is 11 so your hypothesised mean is 90 n = 20 sample mean is 84 and sample standard deviation is 11 ok. So, this is a case where the population standard deviation not known and you see the sample size is less than 30 so this follows the t-distribution right. If the management wants to test the hypothesis at 10% level of significance how should they go ahead? Again I would say take a break and write the frame the null and alternative first.

So, what is the researcher interested here they are interested to see whether the main score is 84 and whether the hypothesis is correct or not the word hypothesis that is was that average score will be around 90.

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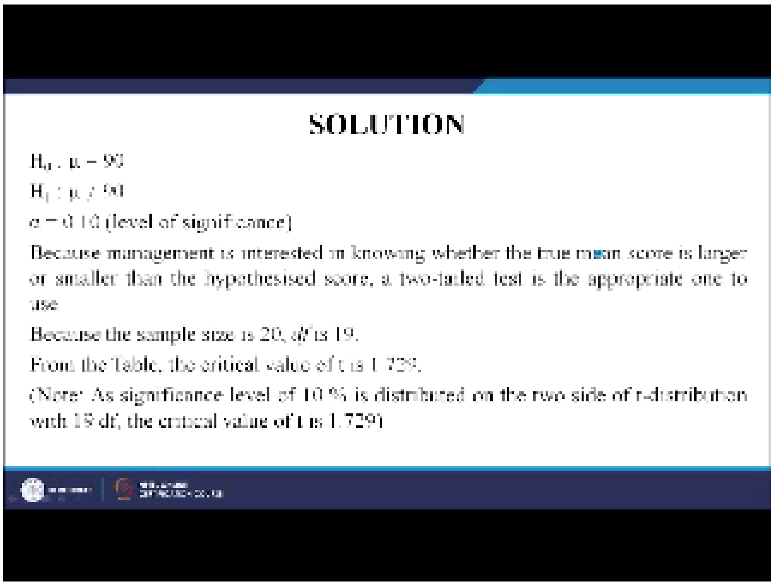

μ = 90 but what is the researcher interest no μ is not equal to 90 so this is what is of interest and automatically μ become is equal to 90 and these are null hypothesis. Because management is interested in knowing whether the true mean score is larger or smaller than the hypothesised score obviously a two-tailed test is appropriate. So, sample size is 90 now this is important I think I have explained already what is the degree of freedom right.

This degree of freedom is of critical importance especially in the case of t-distribution because when you look at the t-distribution the value the area under the curve when you come to calculate t check it this t is degree of freedom is way of vital importance to you. So, this degree of freedom is nothing but all the time the n -1 right. So, that means since the sample size is 20 and maximum it could go up to 30. The degree of freedom is this case is 19, 20 – 1.

From table the critical value of p for 19 that means for degrees of freedom right at 10% level of significance is equal to 1.729.

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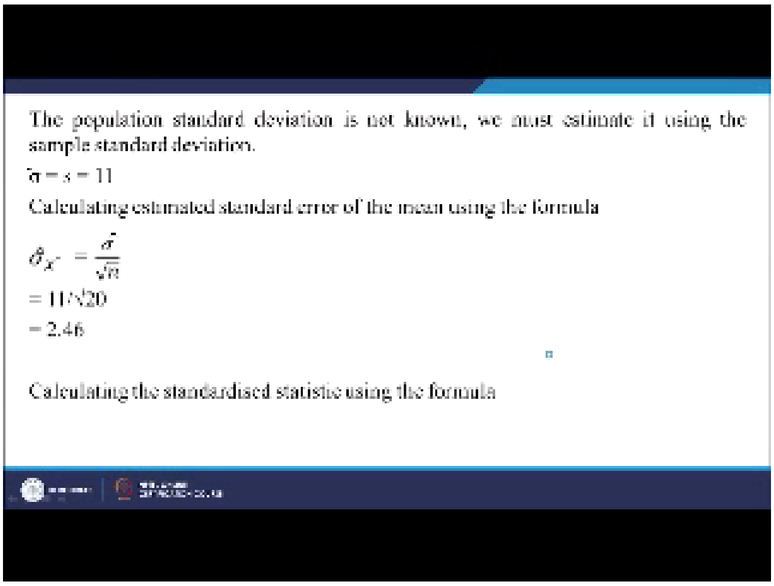

Now let us calculate the standard deviation, the standard error we know that the standard deviation in this case we have to use it for the sample because the population standard deviation is not known to us. So, let us use the sample standard deviation which is given to us is 11. So, from here estimated standard error this sign is estimated is equal to estimated standard deviation which is the sample standard deviation which is root over of n so that means this is equal to 2.46. Now using this we will use to calculate the t-value p or the Z value this is the same formula right

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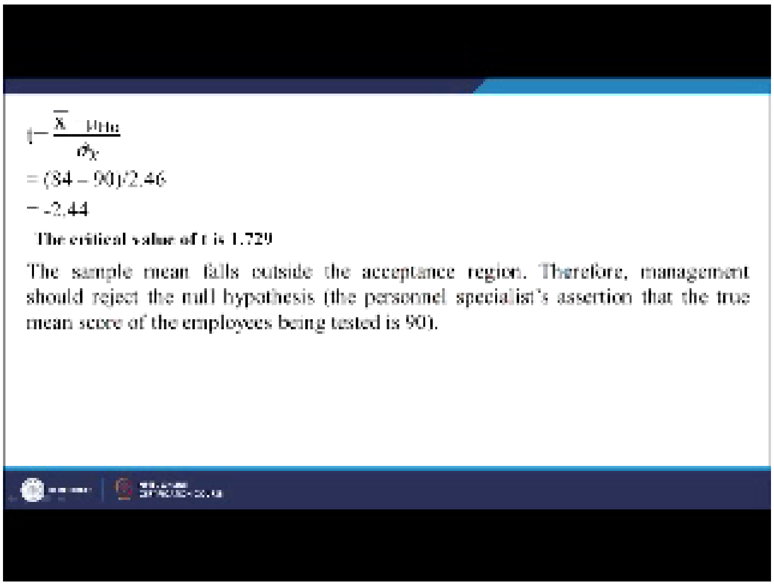

Z or t = X - µ upon standard error so this is coming -2.44 the critical value of p if we have seen how much now 1.729 t value from the table. So, now please interpret how does it look it is something like you are acceptable limit is minus sorry this ok - or + whatever - and + 1.729 what is your calculated value -2.44. So, -2.44, is beyond this limit right. The null sample mean first outside the acceptance region. So, this is acceptance region, therefore the management should reject the null hypothesis personal special is assertion whether true mean score of the employees is being tested is 90 is therefore rejected. I think this is clear ok.

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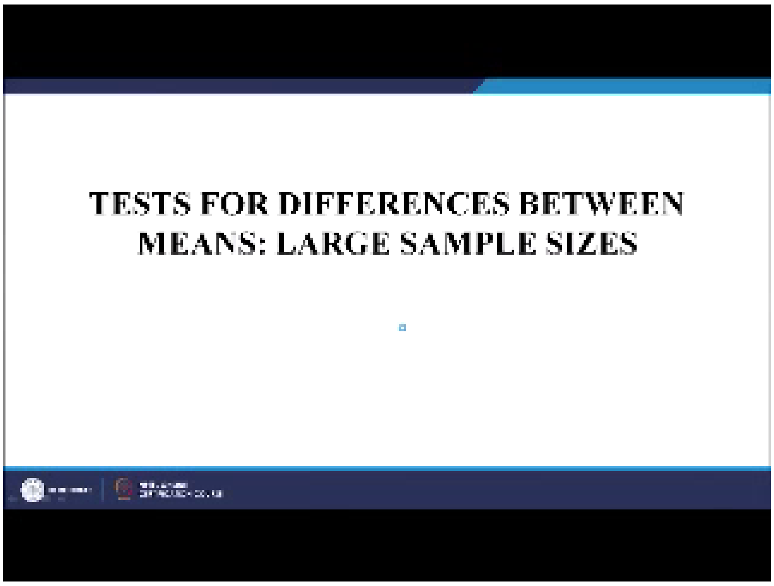

Let us go to the next case right we will test for differences between means of large sample sizes again.

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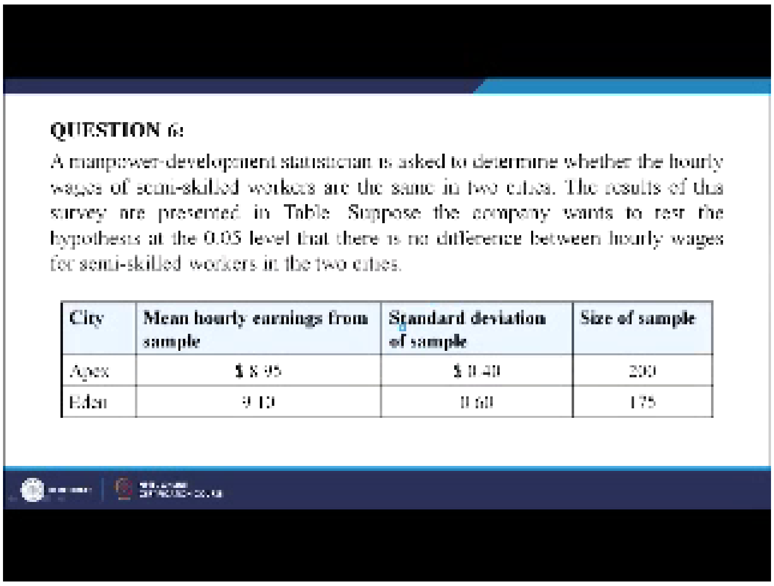

This is for large sample sizes again similar case but there are two samples here right. This is the case where a manpower development earlier you had to deal with one sample only is asked to determine whether the hourly wages of workers whether it is the same in two cities or not. See this is many a times you also of face in life that I want to know whether the HRA given in two cities that may help you in deciding whether you should change your location or not because the companies giving same HRA or different HRA and how much is that is it significant in different or not such things help you to decide right.

The result is given to you so that two cities are Apex and Eden right and the mean early earnings from the sample are 8.95 dollars, 9.1 dollar right standard deviation is also given to the size of the sample is given to you. Now earlier you are dealing with only one sample right now how would you proceed here in this case what should you do? Just think for a while would you approach this problem. There are two samples and you want to this is a case of a in simple terms is a independent sample test right.

So, first you will again right the null and alternative hypothesis. So, what is the null and alternative? Alternative is there is a difference obviously. Null there is no difference.

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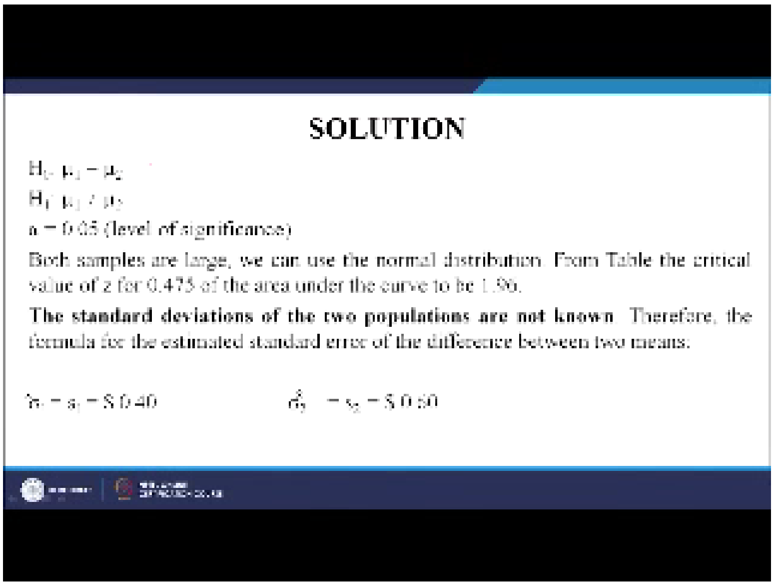

Let us see $\mu_1 = \mu_2$ there is no difference is my null hypothesis μ_1 is not equal to μ_2 is my alternate hypothesis right but since I am not telling who is which one is bigger so I do not know exactly which is the direction of the test right. So, therefore it is a two-tailed test. But had I said the mean of the; let us say Apex is more than the Eden but how much it is not sure then that case maybe I would have gone for a one tailed test ok.

Both samples are large why the sample size how much let us see 200 and 175 so more than 30 so we use a normal distribution so, the area under the curve is how much 95% /2, 95.95 / 2 = 0.4753 this value is 1.96 time and again we have been doing this the standard deviation of the two populations are not known to you. Therefore the formula for the estimated standard error of the different 2 means; we will assume will take these two values right the sample standard deviations when did the population standard deviation is not known we know that will taken sample standard deviation and then calculate the standard error.

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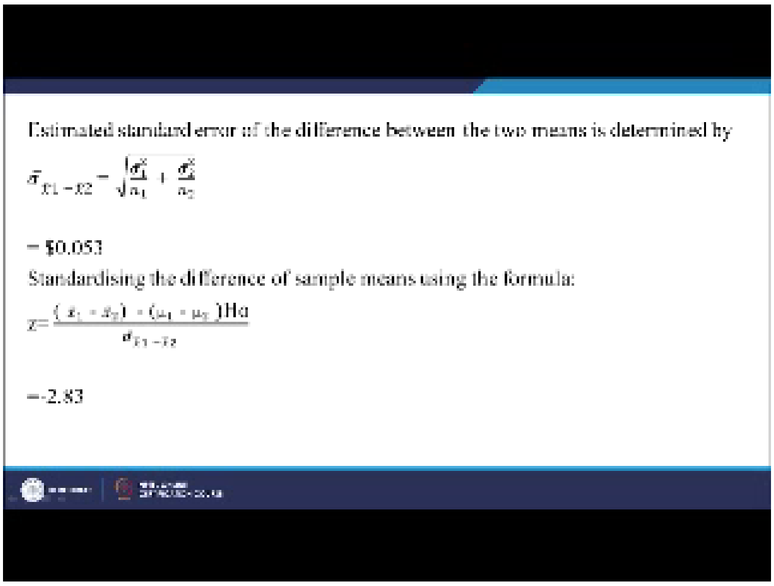

The standard error of the difference between two means now is something like this earlier what you are doing $\sigma_x = \sigma_x$/ root over of n but here what we have done if we have just taken combined the value. So, now this case σ_{x1-x2} right of the two samples is equal to root over of σ_1 square by n1 + σ_2 square by n2 so that is giving us a value of. 053 from after this the next step we all know that we need to calculate the Z.

So, how will you calculate Z earlier the Z was x- μ upon standard error and now we are just replacing like this X / X1 – X2 correct – again it is the same $\mu_1 - \mu_2$ correct same nothing has changed divided by instead of σ_x we are having σ_{x1x2}. So, now if you see this is interesting what is your null hypothesis? In this case $\mu_1 = \mu_2$, If $\mu_1 = \mu_2$ that means $\mu_1 - \mu_2$ is nothing but zero ok. So, what remains is Z = $\overline{x}_1 - \overline{x}_2$ bar upon the σ_{x1x2}. So, this is equal to -2.83.

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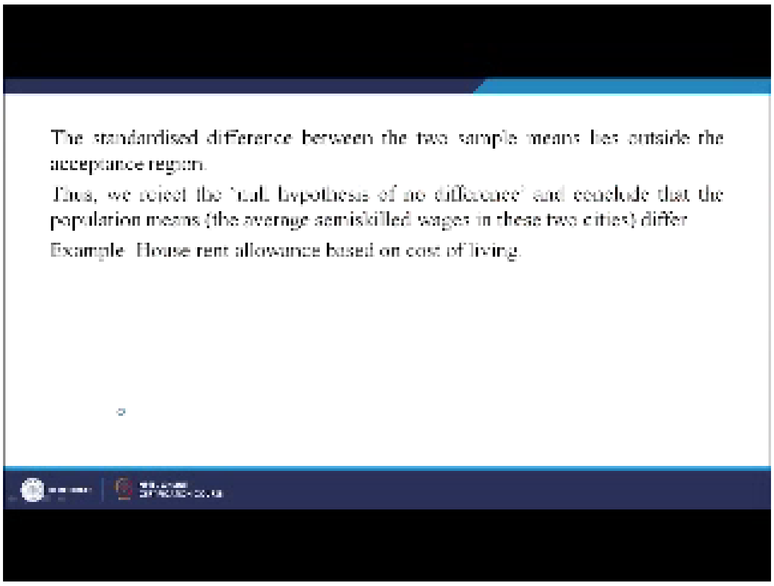

So, what you think the standard difference between the two sample means lies outside the acceptance region is it true let us see - 2.83 is much further away from the acceptable value of 1.96 ok, 1.96 we had yes therefore the null hypothesis is rejected. We reject the null hypothesis of no difference and conclude that the population means differ in the two cities as I said example you can think of a house rent allowances or the salaries between men and women right gender wise several studies where we are trying to seek you whether the two samples are they wearing significantly or not.

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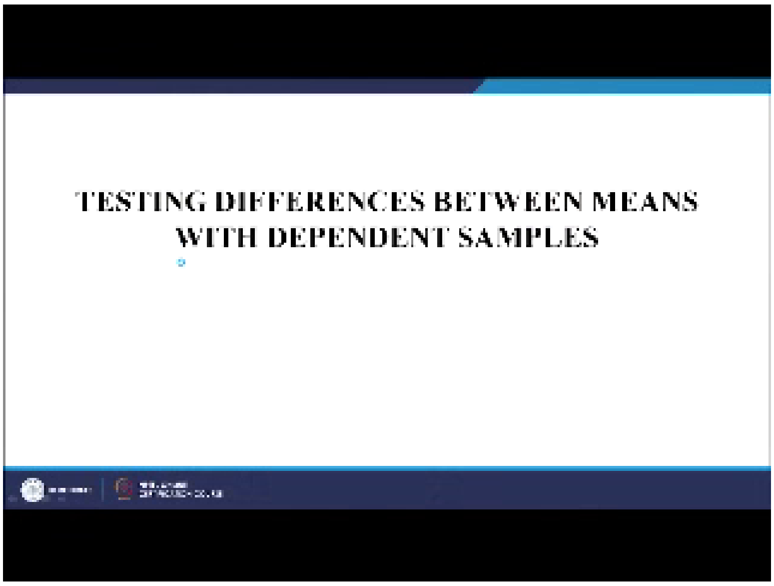

Now we will go the next case this is a testing differences between means with dependent samples that means what is a dependent sample? 1 sample which has been tested once again the same sample is tested again so the sample actually has not changed the sample is same but it behaves like 2 different samples but actually the sample is same right. As frequently we say in case of medicine or something you check the Drug on the particular sample give the medicine treatment right then check the efficiency of the drug ok.

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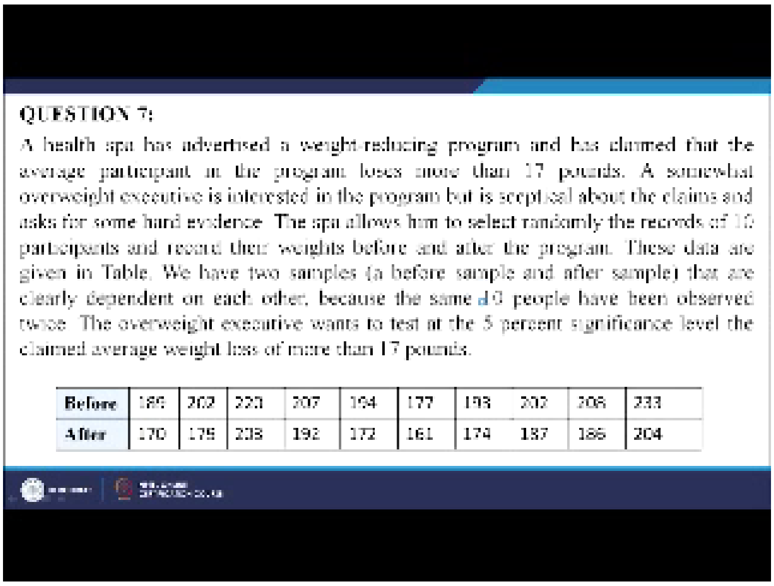

So, health spa as advertised weight reducing program and has claimed that the average participant in the program uses more than 17 Pounds what is reclaiming his claiming that the average participant in his program loses more than 17 pounds. Somewhat overweight executive is interested but he doubts the claims and ask for hard evidence. The spa allows him to take a record of 10 participants n = 10 so if n = 10 that means we understand this is small sample the t-distribution.

And measures there weight before and after the program so 10 people 10 participants before then they participate in the program exercise and then after that they again measure weights this data is given. So, before what was the weight so this is a weight before 189 202 go on till 233 after the going through the program the weight this is the weight. Now we have two samples that are clearly dependent on each other because same 10 people have been observed.

So, the over weight executive wants to test at 5% significance level whether the claimed average weight loss of more than 17 pounds is true or not right. Take a pause again frame the null and alternative ok let us see.

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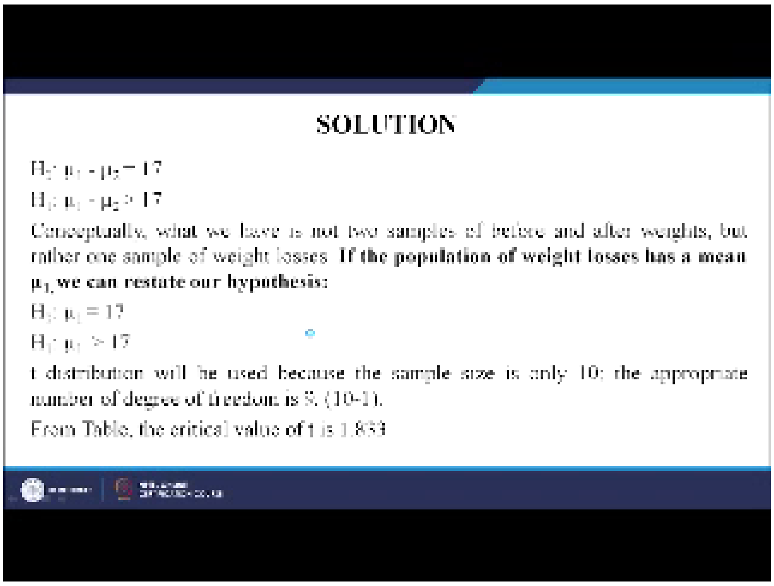

So what he is interested in the difference between the two is more than 17 Pounds that is what was a claim he wants to check whether this is true or not. Suppose it is not greater than 17 then the claim is false but what is the other side what is the null hypothesis μ_1 - μ_2= 17 he is not interested you know to look at the whether it is less or not he is saying it should be greater than the claim was there is greater than right.

Conceptually we have not two samples of before and after weight but rather 1 sample of weight loss in the population of weight loss has a mean of μ_1 we can restate hypothesis this much like μ_1 = 17 and what do you mean by this saying this that means what it means is the sample 1 whatever value let us say X1 X2 X3 sample to them is next time what the weight was X4 X5 X6 so this X1 - X 4 is equal to whatever let us say X11 let us put it this way X 21 X31 suppose I am saying.

This difference is it like it is behaving like one sample so this difference is what is of concerned us, so he said this difference μ_1 = 17 in the null hypothesis μ_1 is greater than 17 is the alternative hypothesis. The t-distribution is used because of sample size only 10 and the number of degrees of freedom is 10 - 1 so 9. From table the critical value of p is 1.833 so you need to check the t table.

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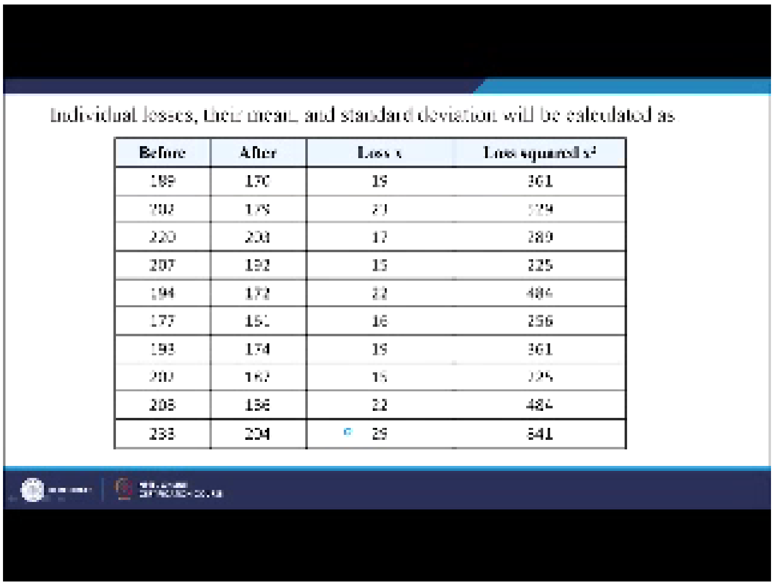

Before	After	Loss x	Loss squared x²
189	170	19	361
202	179	23	529
220	203	17	289
207	192	15	225
194	172	22	484
177	161	16	256
193	174	19	361
200	[illegible]	15	225
208	186	22	484
233	204	29	841

So, it is 1.83 so how do you calculate this individual losses that mean and standard deviation will be calculated after before after this is last this is what I was just talking about in the previous slide. Weight so you can that two ways you can do this so this is one you calculate this loss squared and use this formula.

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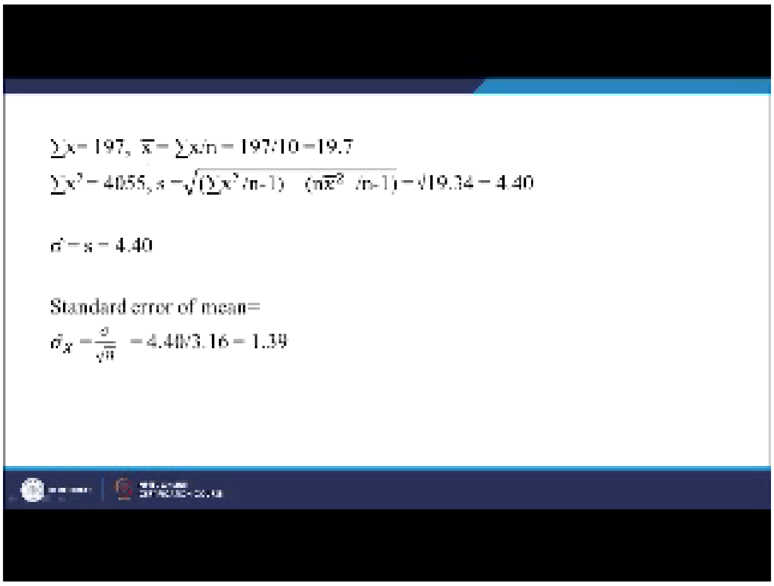

So, what is the formula so what is the ratio calculated so σ_x^2 /n -1 – n * $\overline{x}^2$ / n - 1 using this formula you can calculate the sample standard deviation which is coming as 4.4 you can do it in other way also very simple way is S= x – $\overline{x}$ squared / n -1 root over. So, if you use the same formula what is this X in this case and what is this in X bar. So, the X bar is nothing but the mean of this ok so 19 + 23 + 17 + 15 + 22 + 16 + 19 up to 29 so the mean divided by 10 right that is the X bar.

So this is the $\bar{X}$ you have calculated so X - $\bar{X}$ all the time 19 – $\bar{X}$, 23 – $\bar{X}$ so you find out this and square correct. So, using this you find the same value almost the same value right. So this is the summation you have taken just after doing, this is the summation you are taken and you calculated. The standard error of mean now has been calculated to be 4.4 which you calculate through this way σ_X =197, $\bar{X}$ is 19.7 σ_X^2 this value that is 4055 taking this we calculate the standard deviation which is equal to formula is root over of summation of X^2 / n - 1 - 10 $\bar{X}$ square / n – 1.

All you can simply what you can do is I would just you know you can use this formula I had already used it. So, this formula is nothing but summation of X - $\bar{X}$ square / n – 1. So, this root over when you take and calculate the sample standard deviation is the same value will get ok. So, X is given to you $\bar{X}$ is just you have to take the summation of this and the mean so I think we have calculated here $\bar{X}$.

So, now the question is virtual calculated then you find out the standard error of the mean so which is coming to be 4.4 / 3.16, 3.16 because n = 10 so 1.39.

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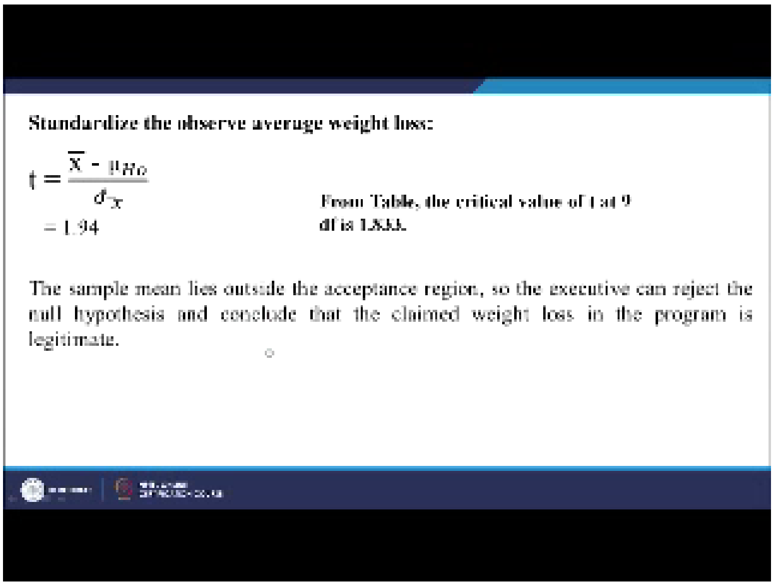

Now if it is 1.39 let us calculate the t value that the t values says the $\bar{X}$ which is $\bar{X}$- μ upon standard X. Now standard $\bar{X}$ was how much X bar was 19.7 – μ, μ was 17 divided by standard error so that is equal to 1.39. So, when you divide the value is 1.394 but your critical value is 1.833. If it is 1.833 and this is 1.94 that means the value is actually going beyond the critical value right it is lying somewhere outside the critical value.

So since it is lying outside the critical value the executive can reject the null hypothesis and conclude the claimed weight loss in the program is right is true that it means is actually a greater than 17 pounds ok.

So this is the case of dependent sample therefore almost all medical cases are all training type of things where a person is to be checked what happened after something has been done something training has been given or some medicine has been given and some surgery has been done in such cases we use dependent sample T test.

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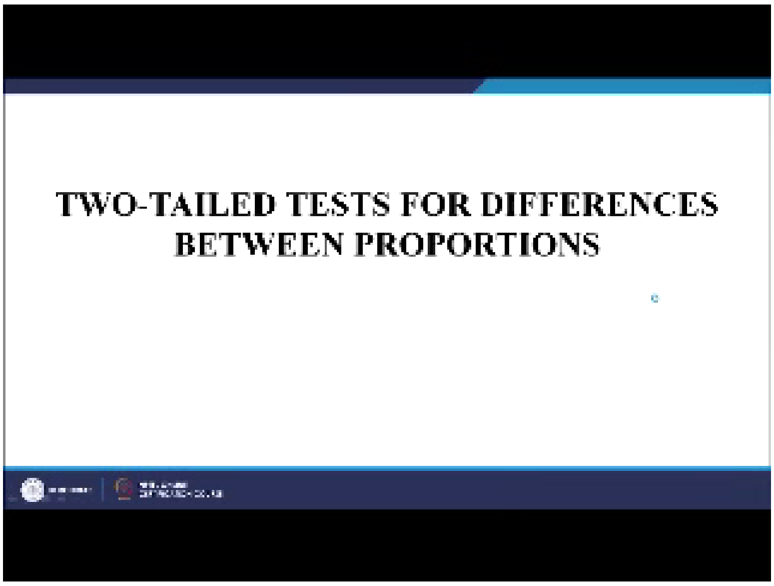

So, let us take two tail differences between proportions now. So you have talked about means again coming back to proportions. I hope second last problem will be doing.

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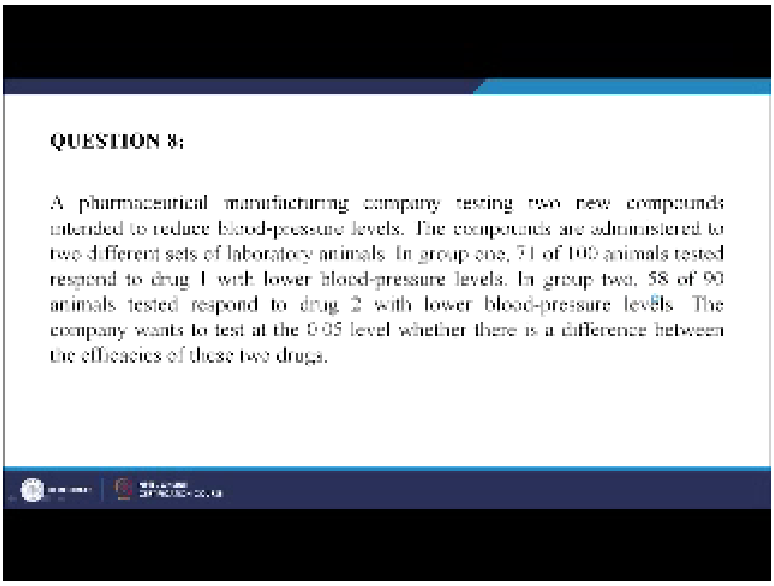

A pharmaceutical manufacturing companies testing new compounds to reduce blood pressure. The compounds are administered two different sets of animals in group 1, 71 of 100 animals tested responded to the first test. So, that means p1 = 0.71, in group 2, 58 of the 90 so that mean p2 = 58/ 90 ok the company wants to test at 5% level of significance whether there is a difference between the efficacy of the two drugs so kindly again the start writing your null and the alternate .

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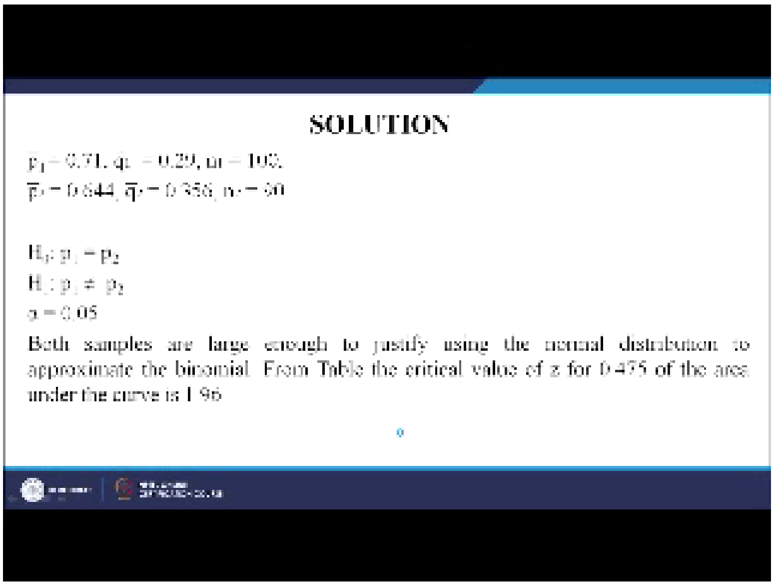

So p1 = .71 q = 1 – p1 that is equal to .29 and n1 is 100 in this case p2 = .644 that is that means what 58 by 90 ok is equal to 6.2 q2 = .356 and n2 is 90 our null hypothesis that there is no difference because the alternative which is of our interest is that there is a difference between the drugs but we do not know which drug is more efficient right at the moment. Alpha is 5% level of significance.

Both samples are having a large sample size to justify the normal distribution from the table the critical value of Z for .475 of the area under the curve is .96 ok that we have seen regularly from other problems.

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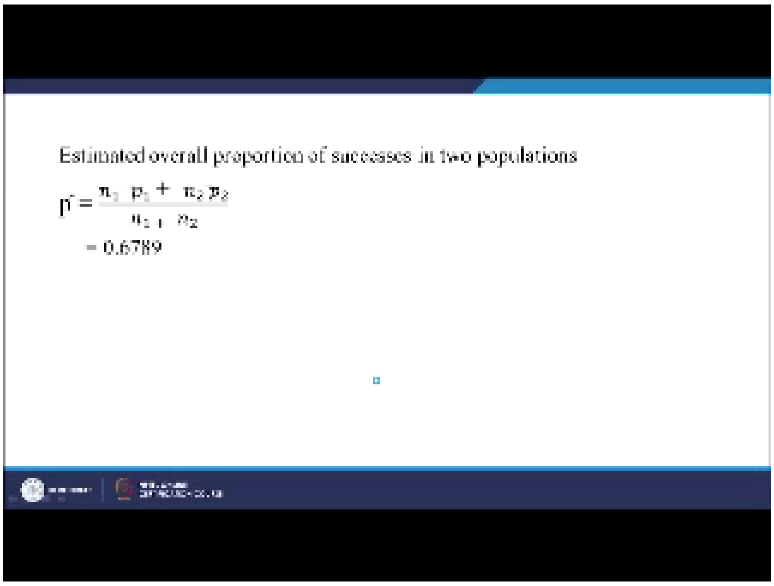

How do you deal with this condition so the estimated over proportion of success because there are two populations here, two population correct. so, one responding to drug 1 the other one is responding to drug 2 here you have to take a combined effect show the how do you calculate the compound effect the estimated proportions success in the two populations is n1 p1 + n2p2 / n1 + n2 when you do this division a combined it generally normalises effect ok. So, the combined proportion success is finally 0.6789.

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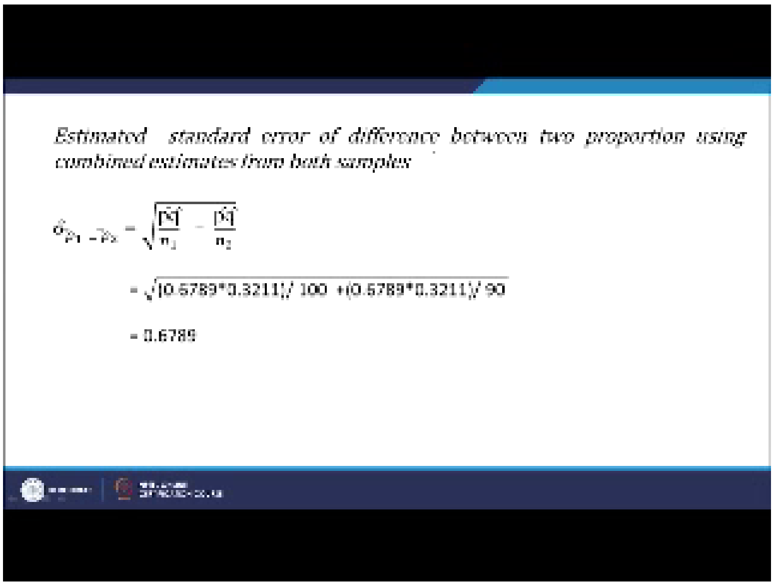

Similarly the estimated standard error between the two proportions is how much the σ_{p1p2}= pq / n which was the formula we had plus pq / n2. So, we had p1 q1/n1 + p2 q2/ n2 so this is what is of interest. So, you can do this and you calculate so again it is coming 0.6789 although the value is the same as the proportion of success. It is because we will be using this two values now this value plus this value also to finally calculate our Z value.

So, that is how much. Σ_{p1p2} = pq/n which was the formula we had +pq/n2. So, we had p1q1/ n1+ p2q2/n2 so this is what is of interests. So, you can do this and calculate so again it is coming .06789 although the value are same as the proportion success because we using these two values this value plus this value to finally calculate our Z value?

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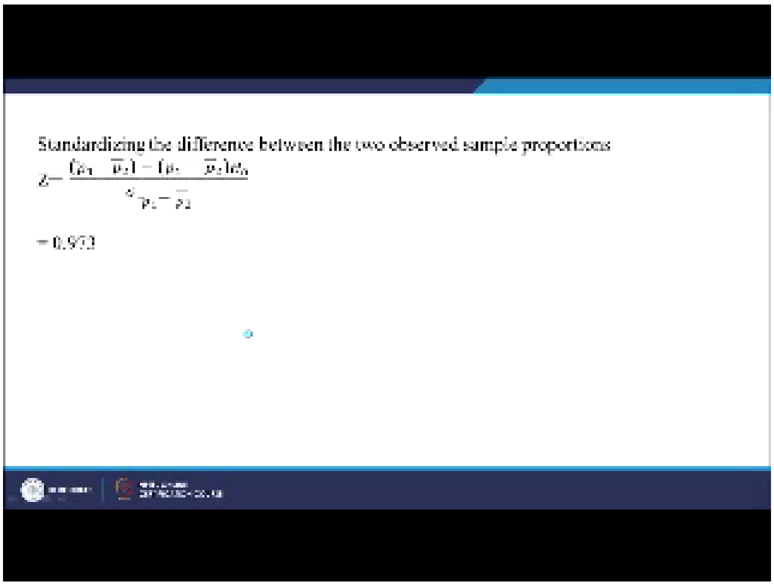

So, Z is how much p1 – p2 – p1 – p2 of the hypothesised proportion right this is the sample and this is the hypothesised. So, these two obviously you said p1- p2 is same correct. Actually you should; this is the hypothesised proportion so you should be taking the hypothesised proportion. So, we said that these two are same so when these two are same automatically the formula says p1 - p2 bar this is please do not take it bar this is just p1 – p2/ Sigma p1 p2 ok. So this part is automatically 0 ok they are equal that is why the Z values 0.973.

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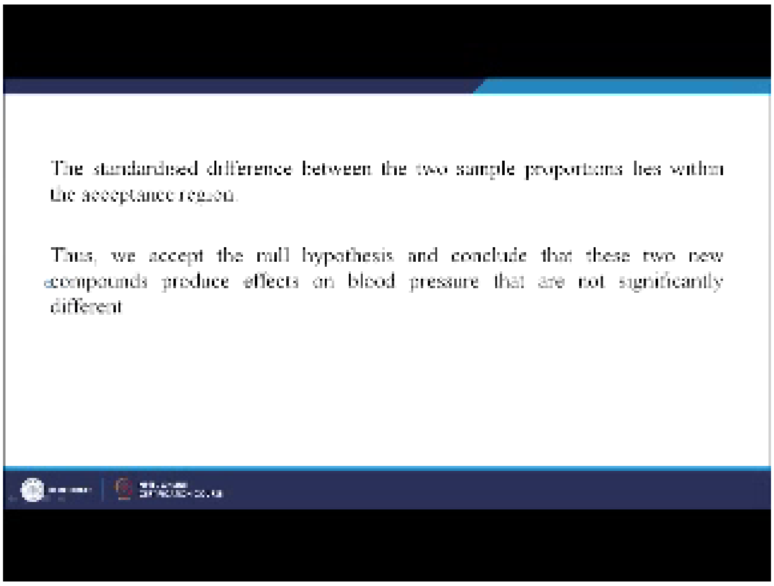

The standardized difference between the two sample proportions lies within the acceptance region is it true yes .96 lies within 1.96 so we are accepting the; not rejecting the null hypothesis. We accept the null hypotheses and conclude that these two new compounds produce effects on blood pressure that are not significantly different they are almost same ok. So, I hope you understood till now will come to the last case now.

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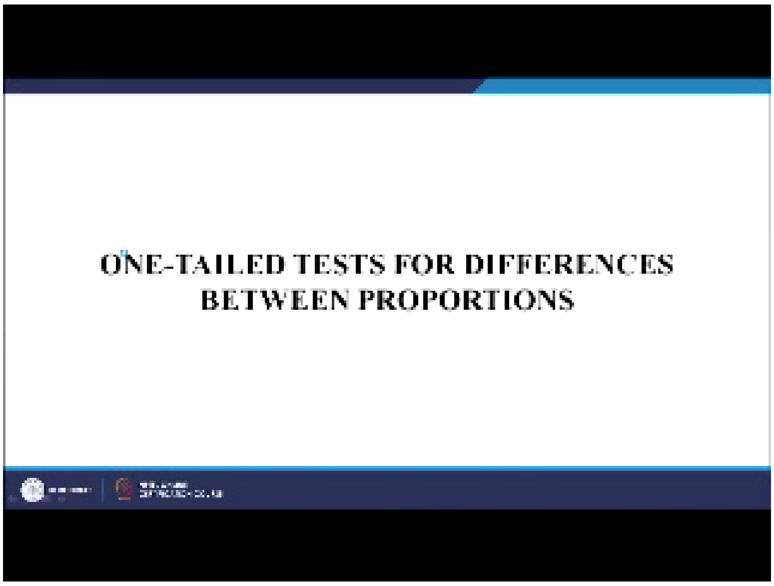

So, one tailed test for differences between proportions is the last case for the day.

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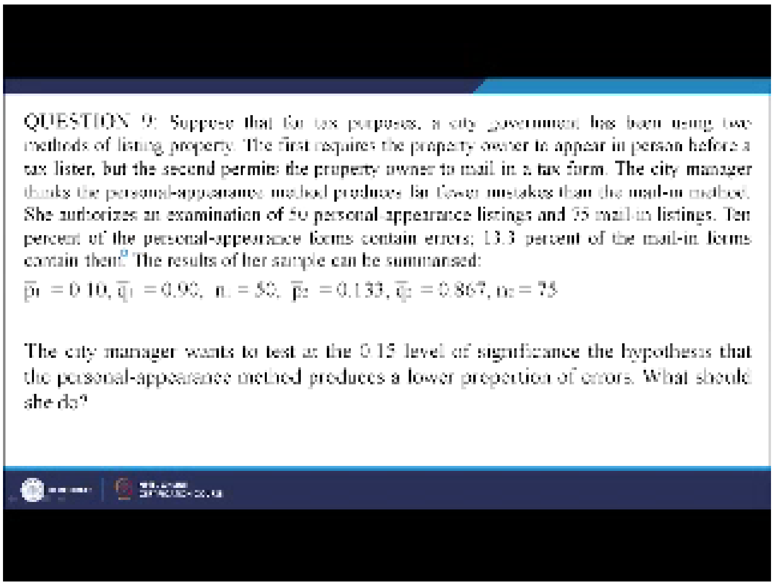

Let us take this case now here there is slightly different because this is a place where for tax purpose city government has been using two methods of listing property. The first requires the property owner to appear in person before a tax lister you have to come in person, you are to visit the office. Second permit the property owner to mail in a Tax Form you can email that the city manager things the personal appearance method produces less mistakes than the mail because obviously nobody is there to guide you so that makes more mistakes the mail method.

She authorises 50 personal appearance listings and 75 mail listings so personal is 50 and mail is 75 ok. 10% of the personal appearance form contains errors how much 10% and 13.3% of the mail contains error ok. The result of the sample can summerised as p1= 10% q1 is 90%. She is interested to check the mistake so 90 and their success is so actually the mistake this time q1 is 90, n1 is 50.

Similarly p2 is 13.33 q2 is no mistakes is .867 and n2 is 75 second sample the manager wants to test at 15% level of significance whether the hypothesis that personal appearance method produces lower proportion of errors is it true or not how should the project. So, again please pause take a pause and right the frame the null and alternate.

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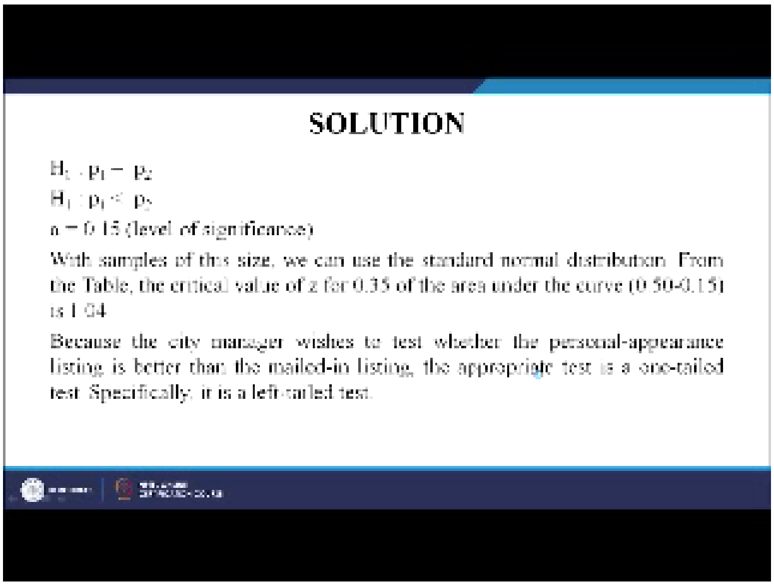

So, in this case whatever she is trying to see p1 which is p1 was your personal appearance had lower mistakes then the mail that is what they are interested. So, first right this portion this is one tailed test if this is not true then where is the null hypothesis is not there is no difference so p1 = p2 right sample of size, the critical value of Z for .35 of the area 15% that to one side is .5 the other side the left side let say it is .35 ok because .5 - .015 because is a one tailed test ok is 1.04. The city manager want to test which one is good right which there is a difference or not.

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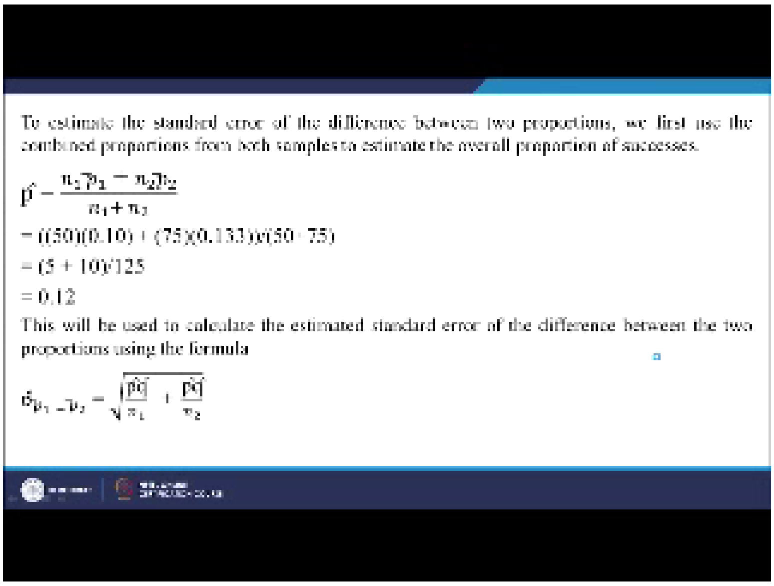

So, again let us calculate the proportion of successes and it is equal to n1 p1 + n2 p2/ n1 +n2 calculating this we get 0.12 over proportion of success using this you calculate the standard error of proportion so again this is p1q1 p2q2 / n2 right so please and this is slight technological error you can check this.

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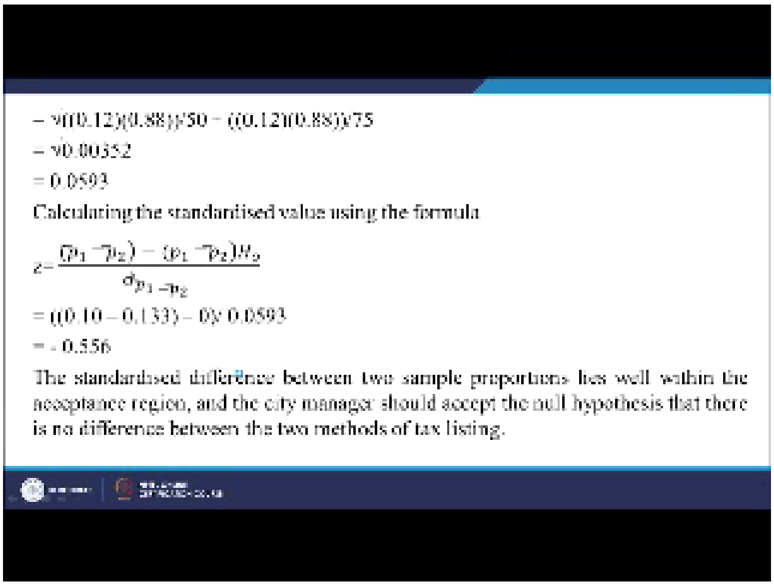

And then you calculate which comes to 0.0593 similarly you know with this one is again finally 0, so this is 0, so coming to this we calculate the final value is -0.056 the standardised different two sample proportion is lies well within the acceptance region which is how much what was the critical value? The critical value was 1.04, so therefore the city managers should accept the null hypothesis and that there is no difference between the two methods of tax listing.

At this moment you cannot say that there is a tax the differences in the claim that whether they appear personally are they appear as they send it through the mail. There is no difference at the moment we cannot make that claim right. So, these are the different ways the format of testing hypothesis when you have one sample or let us say 2 sample all the same sample getting repeated in a case of dependent samples test.

So, moreover this is what we are testing and I think you must be very clear the moment and some typological error which was there kindly correct them, you may correct it because you understood it and I hope it is clear to you the objective and the things are clear to you and wish you all the luck thank you so much.